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NAVIGATION AND ASTRONOMY FOR STUDENTS

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BY

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WITH MANY MAPS AND DIAGRAMS

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PREFACE

This book is designed for those students who desire an understanding of the principles of Navigation and their application to travel by sea or by air. It may be specially useful for students and pupils in post-primary schools, such as Technical Colleges, Nautical Schools, or the new Technical Secondary Schools, where some branches of the work help to prepare young students for a nautical or aeronautical career.

Many books on elementary Navigation have appeared during the last few years, but, so far as the writer is aware, none has treated the subject so comprehensively in one volume. An attempt has been made here to explain the relationship between all the components of Terrestrial Navigation, Astronomical Navigation, and Radio Navigation, while at the same time drawing contrasts as they arise between practice at sea and in the air. Instead of producing a mere statement of facts, this book endeavours to show Navigation as a vital science whose roots, reaching down to antiquity, cherish fruit of great value now ripe for the picking.

Wherever possible an appeal has been made to the student's resourcefulness in drawing and constructing, as a practical approach will do far more to impress the points than volumes of theoretical discourse. A general knowledge of Mathematics has been assumed, but the processes involved in the solution of plane and spherical triangles are given in detail. Graphical methods of solution are given where practicable as an alternative to or check on calculation, but spherical triangles cannot be drawn accurately to scale.

The haversine formula has been introduced in preference to others as the most useful general-purpose formula for satisfying all likely requirements in the solution of the spherical triangle used in Navigation. Haversine tables and tables of logarithms and logarithm ratios needed in these calculations will be found in any volume of nautical tables, such as Norie's, Inman's, or Burton's.

Chapters IV, IX, and XVI may be omitted at the first reading. Nautical students should omit Chapter II and also the sections on Wind Finding (Chapter VI) and the Air Plot (Chapter VII). Aeronautical students may omit Chapter VII except for the section on the Air Plot.

I am greatly indebted to Miss Dorothy Pinchbeck, formerly of the School of Art, Grimsby, for the redrawing of all the figures and drawings, and am grateful to those of my students who have assisted in checking the answers to exercises, and also to my wife for reading the proofs. I have been much encouraged by the kindly interest and understanding of Dr R. E. Richardson, M.Sc., Director of Education, Grimsby, and I wish to thank the publishers for the valuable advice given me during the preparation of this work and for their unfailing courtesy at all times.

Permission has been obtained from the Controller of H.M. Stationery Office for the reproduction of the included extracts from the *Nautical Almanac*, the *Air Almanac*, and Part I of the Admiralty Tide Tables. Permission to include tidal data for Liverpool was granted by the Liverpool Observatory and Tidal Institute. Figures 38 and 39, illustrating the values of Magnetic Dip and Magnetic Variation respectively, are reproduced from Admiralty Charts Nos. 2598 and 3598. The Directorate of Military Survey has permitted the reproduction of the section of Plotting Sheet "Scotland West." Information concerning the Distant-reading Gyro-magnetic Compass was kindly supplied by the makers, the Automatic Telephone and Electric Company, Ltd, and material on the Hyperbolic Method of obtaining Position, together with Fig. 162 and the Decca Lattice Chart contained in the endpaper envelope, was kindly provided by the Decca Navigator Company, Ltd.

I. E. A.

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ABBREVIATIONS

A/C: Alter course
A.P.: Air position
Alt.: Altitude
Az.: Azimuth
B.B.: Back bearing
C.: Compass
C.A.: Conversion angle
Ch. lat.: Change of latitude
Ch. long.: Change of longitude
Co.: Course
Co-lat.: 90°-latitude
C.W.: Continuous wave
Dec.: Declination
Dep.: Departure
Dev.: Deviation
D. lat.: Difference of latitude
D. long.: Difference of longitude
D.M.P.: Difference of meridional parts
D.R.: Dead reckoning
E.M.F.: Electro-motive force
Eq.T.: Equation of time
E.T.A.: Estimated time of arrival
E.T.I.: Estimated time of interception
G.D.: Greenwich date
G.H.A.: Greenwich hour angle
G.M.T.: Greenwich Mean Time
G.S.: Ground speed
H.A.: Hour angle
H.A.T.S.: Hour angle of the true sun
H.E.: Horizontal equivalent
H.W.: High water
H.W.F. & C.: High-water full and change
Km.: Kilometres
Lat.: Latitude
L.A.T.: Local apparent time
L.H.A.: Local hour angle
L.M.T.: Local mean time
Long.: Longitude
L.S.T.: Local sidereal time
L.W.: Low water
M.: Magnetic
M.P.H.: Miles (statute) per hour
M.pts.: Meridional parts
M.S.: Mean sun
M.S.L.: Mean sea level
M.T.L.: Mean tide level
n.m.: Nautical miles
P.: Port
P.L.: Position line
P.P.I.: Plan position indicator
Q.E.: Quadrantal error
R.A.: Right ascension
R.A.M.: Right ascension of the observer's meridian
R.A.M.S.: Right ascension of the mean sun
R.A.T.S.: Right ascension of the true sun
R.F.: Representative fraction
R. of A.: Radius of action
S.: Starboard
S/C: Set course
S.D.: Semi-diameter
S.H.A.: Sidereal hour angle
S.T.: Sidereal time
T.: True
T.A.S.: True air speed
T.E.: Track error
T.M.G.: Track made good
Tr.: Track
T.S.: True sun
T.T.T.: Time to turn
Var.: Variation
V.I.: Vertical interval
W/V: Wind velocity
Z.D.: Zenith distance
Z.T.: Zone time
ϕ In transit or in line with
∼ Difference
≠ Is not equal to

PART I

TERRESTRIAL NAVIGATION

CHAPTER I

NAVIGATIONAL GEOGRAPHY

The earth's shape—Geographical poles—Circles on the earth—Great circles—Meridians—Antimeridians—Equator—Small circles—Parallels of latitude—Position on the earth—Latitude—Longitude—Difference, or change, of latitude—Difference, or change, of longitude—International Date Line—Longitude and time—Time zones—Direction on the earth—Rhumb lines—Distance on the Earth.

The Earth's Shape. Most people nowadays accept as true the statement that the earth is round, although there was a time when this assertion was received with considerable doubt if not actual ridicule. The usual proof offered in support of the statement is the fact that when a ship is viewed approaching from a distance across open sea the upper parts, masts and funnel, are the first to become visible, and only slowly do the bridge and deck accommodation and finally the hull appear. If the earth were flat the hull, being the largest part, would appear first. A curved portion of the earth must, therefore, intervene to shut out from view the part of the ship normally most obvious. This observation multiplied many times at places far remote from each other leads to the conclusion that the earth's surface is continuously curved in all directions and the earth, being solid, must be approximately round. The sun and moon always appear to us to be round, and it is natural to assume that the earth is no exception, especially as the shadow of the earth when cast by the sun on to the moon on the occasion of a lunar eclipse shows the earth to have circular shape no matter how often the observation is made.

The Geographical Poles. The earth, however, is not quite a perfect sphere. It has been ascertained by experiment that a given linear measurement on the earth's surface does not always subtend the same angle at the earth's centre—*i.e.*, if in Fig. 1. $AB = HD = KF$, then $ACB \neq HCD \neq KCF$. There are two places at which the subtended angle is greatest, and these two points N and S (Fig. 1) must be nearer than any other point to the centre of the earth. They are in fact **the poles, the ends of the axis about which the earth rotates**—the rapid axial rotation being regarded as the cause of such flattening. To distinguish one from the other they are named North Pole and South Pole. If you stand

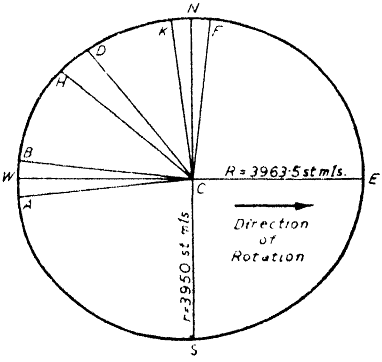

Fig. 1. THE EARTH'S SHAPE

facing the rising sun or stars you will face the direction of the earth's rotation, the pole to your left is then the North Pole and that to your right the South Pole. The direction towards which the earth rotates is named east, while that from which it turns is named west. Fig. 1 shows a cross-section of the earth through the poles, NCS being the axis of rotation. The ellipticity is exaggerated for emphasis. It will be noticed that there is a difference of only 13·5 statute miles between the maximum and minimum radii. The earth is thus almost a sphere, but is commonly known as an ellipsoid or oblate spheroid.

Circles on the Earth. The determination of two fixed points, the poles, on the earth's surface makes possible the construction of a network of circles, a graticule to which the position of any point may be referred. **Circles drawn on a sphere so that their planes pass through the centre of the sphere are known as great circles.** They divide the sphere into two equal parts and are the largest circles it is possible to draw on it. Each centre must coincide with that of the sphere. Arcs which link the diametrically opposite poles are semi great circles called **meridians,** so named because they divide the day into two equal parts, the sun being over the meridian of a place at midday (Latin, *medi diem*). In Fig. 2 NKS and NLS represent two meridians. Their counterparts NTS and NRS respectively completing the great circles on the reverse side of the earth are known as anti-meridians. A series of circles is thus built up dividing the earth's surface into regular intervals east and west. A second series of circles each with its plane at right angles to the earth's axis and therefore parallel to each other completes the graticule. These circles are known as parallels of latitude, and their planes cut the earth, as it were, into slices. Only one can be a great circle—*i.e.*, that which passes midway between both poles. This is the **equator.** The remainder are all small circles—those whose planes do not pass through the centre of the sphere. In Fig. 2 WKLETR represents the equator, while ABOD, FGHJ, and MPVX are small circles of latitude parallel to it. A circle, or parallel of latitude, may be traced on the surface of the earth by a revolution of the sphere's radius about the polar axis, provided the angle between the radius and the plane of the equator remain constant. In Fig. 2 the parallel FGHJ is traced out by the revolution of radius

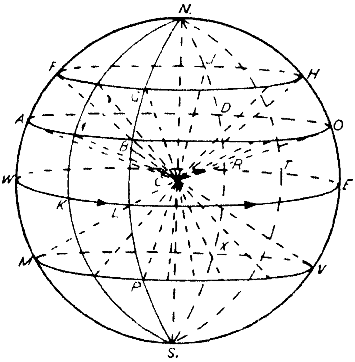

Fig. 2. Meridians and Parallels

CF, the parallel ABOD by the revolution of radius CA, and the parallel MPVX by the revolution of radius CM about the axis NCS.

Position on the Earth. The position of a point on the earth's surface is determined by the particular meridian and parallel of latitude passing through the point. Parallels are distinguished by their constant angular distance from the equator, the value of this angle being a measure of the latitude of any place on the parallel. **Latitude is,** therefore, best defined as **angular distance measured north and south from the equator,** from which it increases to a maximum of 90° at either pole. It is often given as a four-figure reference—*e.g.*, 00° 06′ N. Fig. 3 shows latitude measured as arc of a meridian from the equator QQ, or as angle at the centre of the earth subtended by this arc. The latitude of points V, K, and I is arc DK or angle DCK north, and the latitude of points F, M, and N is arc EF or angle ECF south.

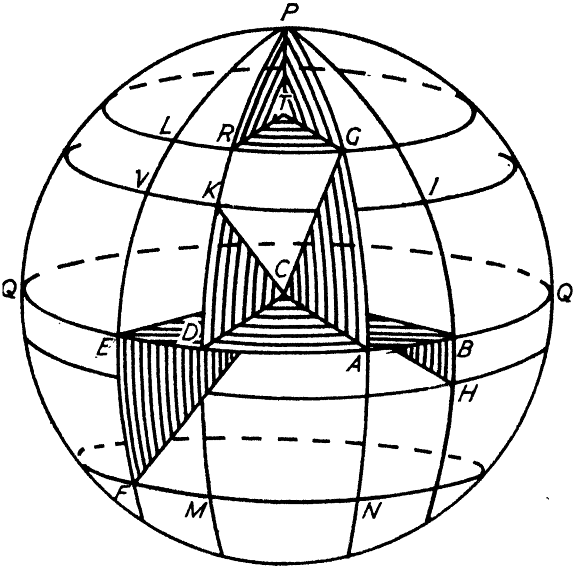

Fig. 3. Latitude and Longitude

The datum meridian is that which passes through Greenwich, and is commonly called the **prime meridian.** Other meridians are related to it by angular measurement made eastward and westward from it. This is known as **longitude: arc of the equator measured from the prime meridian east or west to a maximum of 180°.** In Fig. 3, G represents the position of Greenwich and PGAN part of the prime meridian. The longitude of points I and H is, therefore, arc AB or angle ACB east, the longitude of points R, K, and M is arc AD or angle ACD west, and the longitude of points L, V, and F is arc ADE or angle ACE west.

In order to compare the difference in position between two places or calculate their distance apart, we require to know the difference of latitude (d. lat.) and difference of longitude (d. long.)—referred to in air navigation as change of latitude (ch. lat.) and change of longitude (ch. long.)—between the two places. **Difference, or change, of latitude between two places is the arc of any meridian intercepted by the parallels of latitude passing through the places.** The difference of latitude from G to F is, therefore, LF or GN (south), and from H to K is HI (north). Since all parallels of latitude are circumscribed about the same axis, any two meridians will intercept equal arcs on them—*i.e.*, GR = AD because angle ACD = angle GTR. Similarly DE = RL = MF = KV. The angle measured eastward or westward between K and F is therefore equivalent to RL, DE, or MF. Because, however, the centre of each parallel of latitude is at a different point on the earth's axis it is convenient to refer to such angular distance as arc of the equator intercepted by the two meridians. **The difference, or change, of longitude between two places is the lesser arc of the equator intercepted by the two meridians passing through the places.** The change of longitude from F to H is, therefore, EB (east), and the change of longitude from G to K is AD (west). Numerically the difference of latitude or difference of longitude between two places may be found simply by taking the difference between their values when they have the same name—*e.g.*, both north or both east—and by finding the sum of their values when they are on opposite sides of the equator or prime meridian and have different names—*e.g.*, one north and one south or one east and one west—provided that if the sum be over 180° (possible only in the cases of difference of longitude) it is subtracted from 360° to find the lesser arc—*i.e.*, the shorter way round across the 180th meridian.

Same Names—Subtract
Opposite Names—Add

To ascertain in which direction the change is being made it is only necessary to note which value is increasing and give it that name—*e.g.*, a change of longitude from 5° E. to 10° E. is obviously taking place eastward, but a change from 5° E. to 5° W. takes place westward since it is the westerly longitude that increases. If there is only a decrease in value the change must be named opposite to that of the latitudes or longitudes concerned—*e.g.*, from 5° E. to 2° E. is a change westward, or from 174° W. to 169° W. is a change eastward.

Example. What is the difference of latitude and difference of longitude from Lisbon (38° 42′ N. 9° 08′ W.) to St Miguel, Azores (37° 36′ N. 25° 30′ W.)?

Lisbon	38° 42′ N.		9° 08′ W.
St Miguel	37° 36′ N.		25° 30′ W.
d. lat.	1° 06′ S.	d. long.	16° 22′ W.

Example. Give the change of latitude and change of longitude from Sydney (33° 52′ S. 151° 11′ E.) to Honolulu (21° 18′ N. 157° 51′ W.)

Sydney	33° 52′ S.		151° 11′ E.
Honolulu	21° 18′ N.		157° 51′ W.
ch. lat.	55° 10′ N.		309° 02′
			360° 00′
		ch. long.	50° 58′ E.

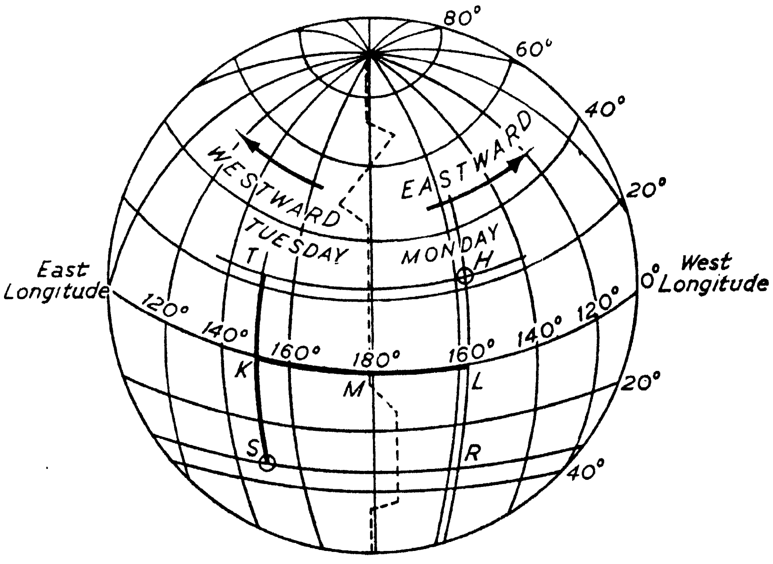

Fig. 4. The International Date Line

Example. The position of Brest is 48° 23′ N. 4° 27′ W. What is the position of Copenhagen if its d. lat. (ch. lat.) from Brest is given as 7° 17′ N. and its d. long. (ch. long.) 17° 02′ E?

Brest lat.	48° 23′ N.	long.	4° 27′ W.
d. lat.	7° 17′ N.	d. long.	17° 02′ E.
Copenhagen lat.	55° 40′ N.	long.	12° 35′ E.

Fig. 4 illustrates the second of the above examples in which the change of longitude takes place across the 180th meridian. S shows the position of Sydney and H that of Honolulu. The ch. lat. may be represented by ST, RH, or the arc of any other meridian intercepted by the circles of latitude passing through S and H. The ch. long is best represented by KL, the lesser arc of the equator intercepted by the meridians passing through S and H. KL may be regarded as being in two parts, K to M on the 180th meridian and M to L.

KM = 180° − 151° 11′ E. = 28° 49′ E. and
ML = 180° − 157° 51′ W. = 22° 09′ E. whence
KL = 28° 49′ + 22° 09′ = 50° 58′ E., as already found.

Exercise 1

Find the d. lat. (ch. lat.) and d. long. (ch. long.) from:

1. Oporto	41° 7′ N. 8° 33′ W.	to Odessa	46° 29′ N. 30° 46′ E.
2. Khartoum	15° 36′ N. 32° 48′ E.	to Kharkov	49° 59′ N. 36° 15′ E.
3. Bombay	18° 54′ N. 72° 49′ E.	to Mombasa	4° 03′ S. 39° 39′ E.
4. Bulawayo	20° 02′ S. 28° 58′ E.	to Benghazi	32° 06′ N. 20° 20′ E.
5. Freetown	8° 29′ N. 13° 14′ W	to Cape Town	33° 56′ S. 18° 29′ E.
6. Auckland	36° 50′ S. 174° 51′ E.	to Victoria B.C.	49° 30′ N. 123° 00′ W.
7. Valparaiso	33° 06′ S. 71° 40′ W.	to Manila	14° 35′ N. 120° 57′ E.
8. Palembang	3° 00′ S. 104° 39′ E.	to Durban	29° 40′ S. 30° 40′ E.
9. Reykjavik	64° 08′ N. 21° 59′ W.	to Liverpool	53° 24′ N. 3° 04′ W.

Exercise 2

Find the position of:

1. Swansea given d. lat. 0° 59′ S. d. long. 5° 40′ W. from Yarmouth
52° 36′ N.
1° 43′ E.
2. Athens given d. lat. 1° 51′ N. d. long. 29° 04′ E. from Gibraltar
36° 07′ N.
5° 21′ W.
3. Adelaide given d. lat. 7° 29′ S. d. long. 14° 24′ W. from Brisbane
27° 28′ S.
153° 02′ E.
4. St Louis given d. lat. 8° 36′ N. d. long. 00° 18′ W. from New Orleans
30° 00′ N.
90° 00′ W.

Identify the following places by reference to an atlas:

5. given d. lat. 25° 32′ N. d. long. 50° 34′ W. from Darwin
12° 28′ S.
130° 51′ E.

6. given d. lat. 0° 26′ S. d. long. 32° 02′ W. from Tahiti
17° 39′ S.
149° 30′ W.

7. given d. lat. 51° 52′ N. d. long. 133° 07′ E. from Hobart
42° 53′ S.
147° 21′ E.

8. given d. lat. 10° 53′ S. d. long. 128° 27′ W. from Los Angeles
34° 05′ N.
118° 18′ W.

9. given d. lat. 22° 34′ S. d. long. 30° 11′ E. from Hull
53° 45′ N.
0° 20′ W.

Date Line. The International Date Line is the 180th meridian modified to pass between Asia and America and bring islands belonging to the same group into the same time zone. The Date Line is made necessary because one rotation of the earth produces the time interval known to us as a day. Each day begins *first* at the Date Line when the sun is over the prime meridian and the time is twelve hours at Greenwich on the preceding day. As the day, say Tuesday, advances the midnight meridian moves from the Date Line westward across New Zealand, Australia, India, the continent of Europe and so to Greenwich, where Tuesday then begins while it is noon on the same day at the Date Line. Continuing its journey westward the midnight meridian brings Tuesday to America and the Eastern Pacific and finally

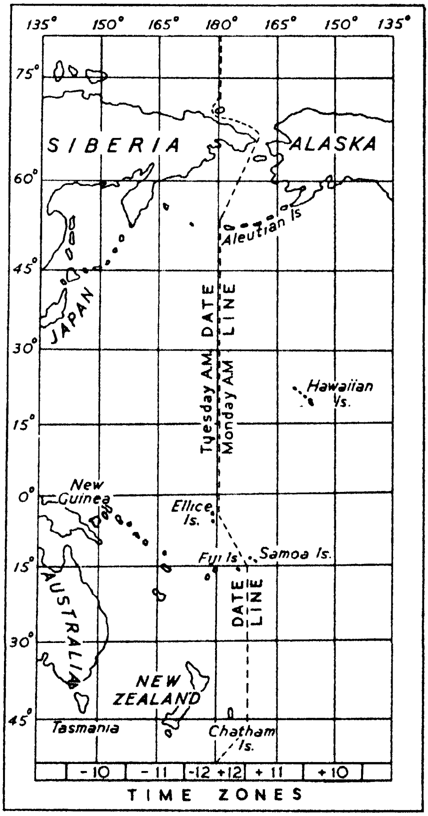

Fig. 5

reaches the Date Line again when for an instant it is Tuesday all over the world before the new day commences as before.

If we move from Greenwich along the earth's surface eastward against the movement of rotation, we go to meet the midnight meridian on its apparent rounds and hence shorten its journey to us by a certain difference of longitude. If this is done repeatedly until we have accumulated 180 degrees of longitude we shall have gained half a rotation or half a day—*i.e.*, it will be midday with us when it is midnight at Greenwich. If this process were continued we should gain a further half day in our journey eastward back to Greenwich and so arrive with a day in hand making that morning's papers appear a day old. To avoid such confusion the date is put one day back on crossing the Date Line eastward and one day forward on crossing the Date Line westward. There might seem to be some advantage or disadvantage in this, and such would probably prove to be the case if you were to have two birthdays on successive days when eastbound or no birthday at all when westbound. The dates, however, are levelled out before reaching home. The modified 180th meridian fills the role of demarcation line admirably, in that it need cross no land masses and can be arranged to by-pass island groups easily. The importance of Greenwich in fixing the prime meridian is, therefore, obvious, as the Greenwich meridian and this convenient Date Line are respectively each other's antimeridian.

Difference of Longitude and Time. Because one rotation of the earth produces the time interval known to us as a day, a difference of longitude of 360° must correspond to one day which we divide into 24 equal parts, or hours. One hour, therefore, must be equivalent to a difference of longitude of 15°. By multiplying the hours and fractions of an hour by 15 it is possible to convert the difference in time between two places into difference of longitude. Conversely, by dividing the d. long. in degrees and fractions of a degree by 15, we may obtain the corresponding difference in time. We may find it convenient to divide by 4 and multiply by 60 instead of multiplying by 15, and to multiply by 4 and divide by 60 instead of dividing by 15. The reason for this should be apparent from a consideration of the following examples.

Example. The local time at Vancouver is 10 hrs. 48 mins. when the local time at Quebec is 14 hrs. 15 mins. 8 secs. on the same day. What is the d. long. between the two places?

	hrs.	*mins.*	*secs.*
Local time at Quebec	14	15	08
Local time at Vancouver	10	48	00
Difference of time	3	27	08
Divided by 4		51	47
Multiplied by 60	51°	47′	00″
I.e., d. long.	51°	47′	00″

Example. What is the local time at Madrid (3° 41′ W.) when the local time at Rome (12° 27′ E.) is 7 hrs. 15 mins?

Longitude of Rome	12°	27′ E.	
Longitude of Madrid	3°	41′ W.	
Difference of longitude	16°	08′ W.	
		4	
	60)64°	32′	
Difference of time	1 hr.	4 mins.	32 secs.
Local time at Rome	7 hrs.	15 mins.	00 secs.
Local time at Madrid	6 hrs.	10 mins.	28 secs.

Because Rome is east of Madrid, the sun will reach Rome's meridian first, so that local time at Rome is always ahead of local time at Madrid and we must, therefore, subtract the difference of time in this case.

Example. When the local time at Brisbane (153° 02′ E.) is 15 hrs. 24 mins. what is the local time at Seattle (122° 00′ W.)?

Longitude of Brisbane	153°	02′ E.	
Longitude of Seattle	122°	00′ W.	
	275°	02′	
	360°	00′	
Difference of longitude	84°	58′ E.	
		4	
	60)339°	52′	
Difference of time	5 hrs.	39 mins.	52 secs.
Local time at Brisbane	15 hrs.	24 mins.	00 secs.
Local time at Seattle	21 hrs.	03 mins.	52 secs.

Because, however, the date line has been crossed eastward, the date at Seattle must be one day behind that at Brisbane.

Exercise 3

(*a*) Convert the following d. long. into difference of time:

1. 36° 21′; 2. 144° 17′; 3. 85° 42′; 4. 5° 36′; 5. 177° 02′; 6. 14° 55′; 7. 28° 19′; 8. 64° 46′.

(*b*) Convert the following difference of time into d. long.

	hrs.	*mins.*	*secs.*
1.	4	17	50
2.	11	29	16
3.	00	52	24
4.	6	20	43
5.	3	45	38
6.	9	06	58
7.	1	00	32
8.	5	48	12

Exercise 4

From the following details find the local time in the longitudes of B.

	A		A				B	
	Longitude			*Local time*			*Longitude*	
			days	*hrs.*	*mins.*	*secs.*		
1.	00°	00′	6	12	00	00	35°	16′ E.
2.	00°	00′	25	17	36	09	114°	27′ W.
3.	35°	15′ W.	18	5	42	06	77°	29′ W.
4.	48°	12′ E.	11	22	14	50	105°	42′ E.
5.	17°	04′ W.	30	8	20	34	10°	26′ E.
6.	114°	27′ E.	2	1	05	30	66°	32′ E.
7.	174°	52′ E.	12	14	53	12	152°	20′ W.
8.	150°	44′ W.	27	6	11	42	168°	06′ E.
9.	5°	16′ E.	4	1	08	40	13°	22′ W.

Time Zones. For the sake of convenience longitude is often divided into hourly intervals, or time zones, so that places within certain limits of longitude may keep the same time. This prevents the confusion which would arise, especially in connexion with time-tables, if every one kept the time appropriate to his own longitude. The time zones are numbered from 0 to 12 prefixed by + or —. Zone 0 extends from longitude 7½° E. to longitude 7½° W., within which limits Greenwich time is kept. Zone + 1 extends from 7½° W. to 22½° W., within which limits the time is one hour behind Greenwich time and one hour must be added to find it. Zone — 1 extends from 7½° E. to 22½° E., and here one hour must be subtracted to obtain Greenwich time. Zone 12 extending from 172½° E. to 172½° W. is named — for 172½° E. to the Date Line and + for 172½° W. to the Date Line. The time is the same over the whole zone, but the date is different on opposite sides of the Date Line. There are local modifications, however.

Direction on the Earth. A meridian can be drawn through any point on the earth's surface, and because it joins both poles it must indicate both north and south. North is generally regarded as the principal datum direction and every other direction is related to it in terms of angle measured clockwise from 000° to 360°. A line crossing the meridian at right angles will indicate east in the direction 090° and west in the direction 270°. **Lines drawn on the earth's surface to cross successive meridians at the same angle are called rhumb lines** (Greek, *rhumbos*, a whirl). When produced indefinitely they form a spiral round the earth's axis. This may be seen from Fig. 6, where YC is a rhumb line along which the angles PYC, PAC, PDC, PHC, etc., are equal. Because the meridians converge towards the pole the angle at which a line intersects them can only be kept constant by a gradual approach to the pole. After passing to the reverse side of the earth at C (Fig. 6) the rhumb line is seen to reappear at C_1. Parallels of latitude are rhumb lines, but because they cross each meridian at an angle of 90° they approach neither pole but complete a circle about the earth's axis. The advantage of rhumb lines to navigation is that they provide routes along which direction relative to true north is constant. Its disadvantage is that it does not provide the shortest route. The most direct route is provided by the arc of a great circle. In Fig. 6, YVF represents the arc of a great circle passing through Yokohama and San Francisco. It will be seen that the angle PYV is less than the angle PVF which is less than the angle PFB. The direction relative to true north thus changes when travelling along a great-circle route. Proceeding from Y to F (Fig. 6) the initial direction PYF of 054° increases until it becomes 090° at V, the vertex, the nearest point to the pole, and increases further, until, when F is reached, the final direction is 123°. Were we to follow the great-circle route still farther towards B, we should find that the direction would

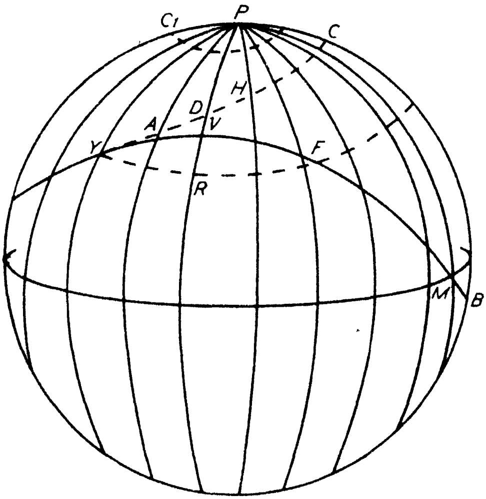

Fig. 6. GREAT CIRCLE AND RHUMB LINE

continue to increase until the equator was reached at M. In the southern hemisphere it would decrease. Yokohama and San Francisco are approximately in the same latitude so that the rhumb line joining them will almost coincide with the parallel of latitude. This is represented in Fig. 6 by the arc YRF which intersects each meridian in a direction of approximately 090°. The divergence from the great circle is seen to be considerable. The nearer the connected places are to the pole, the greater will become the divergence between rhumb line and great circle—other things being equal. Nearer the equator, however, the divergence is less until on the equator itself there is no divergence, as the equator is both great circle and rhumb line. The advantage of the great circle to navigation is that it provides the shortest route between two places. A piece of string drawn taut on a globe will demonstrate this. Its disadvantage is that direction along it relative to a meridian is not constant. Study the Yokohama to San Francisco great-circle route on a globe. Look along YVF and you will see that it is the shortest distance and the arc of a circle bisecting the globe.

Distance on the Earth. The unit of distance measurement in navigation is the nautical mile, although in aircraft the statute mile may often be used when flying over land and using a topographical map or if the air-speed indicator is calibrated in miles per hour only. In continental aircraft the kilometre is generally the unit used. A nautical mile is the length of a minute of latitude and is reckoned for convenience as 6080 feet. It contains 10 cables. Owing to the shape of the earth a minute of arc on a meridian measures less at the equator than it does at the poles. This can be seen from Fig. 7 where C is the

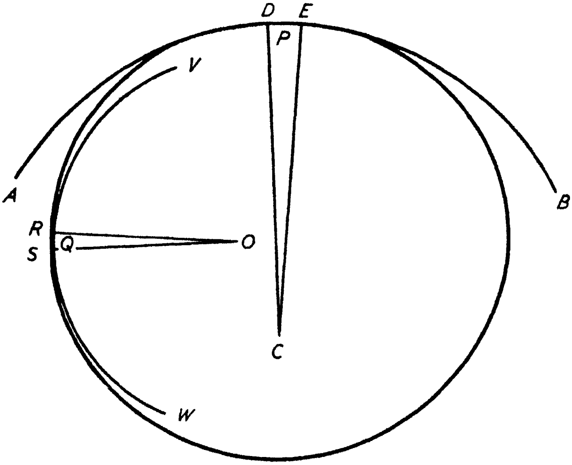

Fig. 7. The Nautical Mile

centre of curvature of the polar arc AB, and O the centre of curvature of the equatorial arc VW. The radius of AB is greater than the radius of VW, so that a given angle at C will subtend a greater distance on AB than the same angle will subtend on VW when measured at O. A degree of arc on a meridian at the pole is therefore greater in linear measure than a degree of arc on a meridian at the equator. The distance of a minute of arc at the poles is given as 6108 feet while a minute of arc at the equator is given as 6046 feet. Hence 6080 feet is a convenient average and is about the value for the English Channel. Because a meridian is for practical purposes a semi great circle, distance in nautical miles along any great circle may be measured in minutes of arc. A minute of arc of the equator is constant and referred to as a geographical mile whose length is 6087·2 feet. The statute mile introduced during the reign of Queen Elizabeth has 5280 feet.

The kilometre, a thousand metres, is almost the ten thousandth part of the length of a meridian measured from the equator to the pole. It corresponds to ·62137 of a mile, or 3280·8 feet. The nautical mile, statute mile, and kilometre are proportional to 76, 66, and 41.

Example. Convert 120 statute miles to nautical miles and kilometres.

$$120 \text{ st. m.} = \frac{120 \times 66}{76} \text{ n.m.} = \frac{1980}{19} = \mathit{104{\cdot}21} \text{ n.m.}$$

$$120 \text{ st. m.} = \frac{120 \times 66}{41} \text{ km.} = \frac{7920}{41} = \mathit{193{\cdot}17} \text{ km.}$$

General Exercises

1. Describe a circle to represent the earth, and on it mark the poles and draw meridians at intervals of 30°. Add the equator and parallels of latitude at intervals of 20° from it. Mark the following positions:

	Latitude	*Longitude*
(*a*)	40° N.	60° W.
(*b*)	60° N.	40° E.
(*c*)	20° S.	30° W.
(*d*)	50° S.	20° E.

Give the difference, or change, of latitude and longitude:

	from	*to*
(i)	(*a*)	(*b*)
(ii)	(*b*)	(*d*)
(iii)	(*c*)	(*a*)
(iv)	(*c*)	(*d*)

2. Define: Geographical poles; meridian; equator; parallel of latitude; latitude; longitude; difference, or change, of latitude; difference, or change, of longitude; prime meridian; antimeridian; Date Line.

3. Find the change of latitude and change of longitude from:

(*a*) Mildenhall (52° 21′ N. 00° 30′ E.) to Melbourne (37° 50′ S. 144° 58′ E.)
(*b*) Vancouver (49° 30′ N. 126° 00′ W.) to Vladivostock (43° 10′ N. 132° 00′ E.)
(*c*) Antofagasta (23° 30′ S. 70° 25′ W.) to Shanghai (31° 25′ N. 121° 30′ E.)
(*d*) Aden (12° 58′ N. 45° 01′ E.) to Fremantle (32° 01′ S. 115° 47′ E.)
(*e*) Cairo (30° 01′ N. 31° 13′ E.) to Cape Town (33° 58′ S. 18° 26′ E.)
(*f*) Bergen (60° 25′ N. 5° 20′ E.) to Baltimore (39° 23′ N. 76° 35′ W.)
(*g*) Sydney (33° 55′ S. 151° 12′ E.) to Seattle (47° 31′ N. 122° 15′ W.)
(*h*) Munich (48° 10′ N. 11° 36′ E.) to Moscow (55° 45′ N. 37° 36′ E.)
(*i*) Wellington (41° 15′ S. 174° 46′ E.) to Honolulu (21° 25′ N. 157° 55′ W.)

4. How many degrees of longitude correspond to one hour? Change the following measurements in arc into measurements in time:

(*a*) 17° 50′; (*b*) 29° 32′; (*c*) 42° 10′ 15″; (*d*) 79° 18′; (*e*) 4° 26′; (*f*) 153° 09′ 30″; (*g*) 211° 46′; (*h*) 285° 55′; (*i*) 336° 21′ 45″.

5. When it is 03 hours 44 minutes 30 seconds on May 6 at Greenwich, what will be the local time at:

		Longitude
(*a*)	Marseilles:	5° 25′ E.
(*b*)	Delhi:	77° 20′ E.
(*c*)	Moscow:	37° 36′ E.
(*d*)	New York	74° 00′ W.
(*e*)	San Francisco:	122° 30′ W.
(*f*)	Cape Town	18° 26′ E.
(*g*)	Honolulu	157° 55′ W.
(*h*)	Shanghai:	121° 30′ E.

6. When it is 05 hours 17 minutes on June 19 at Auckland, N.Z. longitude 174° 46′ E., what will be the local time at:

		Longitude
(*a*)	Perth:	115° 50′ E.
(*b*)	Tahiti:	149° 30′ W.
(*c*)	Colombo:	79° 58′ E.
(*d*)	Callao	77° 10′ W.

7. Contrast great circle and rhumb line.

8. What is the difference between a nautical mile and a geographical mile?

9. When does a rhumb line coincide with a small circle?

10. Convert to the nearest unit:

(*a*) 175 nautical miles into statute miles and kilometres.
(*b*) 534 statute miles into nautical miles and kilometres.
(*c*) 829 kilometres into nautical miles and statute miles.

CHAPTER II

MAPS

Topographical maps—Cassini's Rectangular Co-ordinate system—Bonne's projection—International Modified Polyconic projection—Direction on a map—Measurement of distance—Scale—Representative fraction—Time-speed scales—Position by (i) Graticule reference, (ii) Direction and distance, (iii) Grid reference—Grid deviation—Grid variation—Relief—Vertical intervals—Horizontal equivalent—Map reading—Circle of uncertainty—Conventional signs.

Topographical Maps. When you wish to go on a cycling or motoring tour you consult a map in order to plan your route unless you know the countryside very well. From time to time after you have set out you refer to the map for the purpose of identifying objects en route and so establish your position before proceeding farther. You then measure the distance to be covered before reaching the next landmark, and so estimate the time at which you will be there. The value you attach to the map will depend upon the accuracy with which it represents the features a road user requires. Your first consideration will probably be concerned with the way in which the different types of roads are distinguished, so that you may select the one best suited to your requirements. You would expect places to be clearly indicated by name, and you would look for details such as convenient resting places and warnings of road dangers. One of the most desirable features, however, is often taken for granted—*i.e.*, that the scale for distance is constant all over the map. To make this possible, while at the same time maintaining approximately correct direction between places, the framework of meridians and parallels on which the map has to be built up must be developed according to a particular scheme.

There is more than one way of achieving this balance, and a brief description of the methods adopted in constructing the framework, or graticule, for the Ordnance Survey map of (*a*) England and Wales and (*b*) Scotland and Ireland is given here.

For (*a*), a line representing the central meridian of the area to be mapped is drawn on paper and a reference point marked on it. The positions of regularly spaced graticule reference points—*i.e.*, the intersection points of parallels with meridians—are then put in on either side of the central meridian by measuring off two calculated distances for each point required. These are measured first from the original reference point along the central meridian and then at right angles to it. Suppose, for example, that the point of origin were at the intersection of 52° N. with 2° W. and you wanted to mark the position given by 53° N. and 1° W., your first measurement would be from the point of origin northward along the meridian to the point at which the great circle passing through 53° N. and 1° W. would intersect the central meridian at right angles. This would not be at the intersection of 53° N. with 2° W. as the parallel of 53° N. is a small circle and a rhumb line, not a great circle. The point would be in a slightly higher latitude than 53° N. The second measurement would be that along the great circle from its intersection with 2° W. to 53° N. and 1° W. The actual distances are calculated by spherical trigonometry and the representative distances on the map measured to scale to correspond.

When sufficient reference points have been marked their appropriate meridians and parallels are drawn through them and the graticule is ready for the insertion of ground features such as coastline, rivers, towns, railways, roads, and many other details.

This graticule does not allow shapes to be represented correctly nor may areas on it be compared, but it has almost a constant scale when covering small countries and a straight line on it represents approximately the arc of a great circle. This type of projection is known as Cassini's Rectangular Co-ordinate system.

For (*b*), maps of Scotland and Ireland, a central meridian is drawn straight and there is a standard parallel of latitude intersecting it represented correctly to scale by a curve concave to the pole. The other parallels are concentric with the standard parallel and spaced correctly to scale along the central meridian. The other meridians are drawn through points correctly spaced along the parallels and when complete are found to be curved concave to the central meridian.

Maps drawn on this projection represent areas correctly, and shapes may be regarded as correct if the areas concerned are not more than a few degrees of longitude from the central meridian. Scale is correct along the central meridian and all parallels, and may be regarded as approximately correct everywhere for small countries such as Scotland but not for continents. This is Bonne's projection.

Ordnance Survey maps are widely used for military purposes, and are drawn to a scale that will suit the particular requirements of the section concerned. For instance, infantry units prefer a scale of 1 inch to the mile, whereas for larger bodies, such as divisions, a smaller scale of 4 miles to the inch would probably be favoured. Artillery units require a larger scale than either of the others.

International Modified Polyconic Projection. Because of the correct directional and constant scale values of Ordnance Survey maps the 1-inch and ¼-inch series were adopted by the R.A.F. in the early days of flying. In later years, however, they have been supplemented by maps constructed to scales[1] of one in a quarter-million, one in a half-million, and one in a million, especially for use over Europe. This last scale has been selected by the International Commission for Air Navigation (I.C.A.N.) for local maps of general use based on the polyconic projection, the principle of which is illustrated by Fig. 8.

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[1] See Measurement of Distance, p. 310.

A single cone sitting on a globe would touch it along a small circle. If the axis of the cone were in line with the globe's axis the circle of contact would be a parallel of latitude, which when traced on to the cone would be accurately represented. It is desirable to have as much of the globe as possible represented accurately on a map, and the more parallels we can bring into contact with a cone the more accurately will the representation be achieved, provided that scale along the meridian is not thereby unduly sacrificed. The polyconic projection is the development of a series of such cones, each of which is a different shape, having a different angle at the vertex, and touches the globe along a different parallel of latitude. Fig. 8 shows that half the vertical angle has the same value as the latitude along whose parallel the base of the cone rests. V_1A being a tangent to the circle representing a globe meets it at an angle of 90°. Angle $V_1OA = 90° - \text{lat. } A = 90° - \text{angle } OV_1A$, so that angle $OV_1A = \text{lat. } A$. A similar relationship may be proved in the case of the other cones. For the sake of clarity only three cones are shown in the figure. Their bases are seen to touch the globe along the parallels of 28°, 40°, and 52° respectively. The graticule when developed will appear as shown in Fig. 9. As the parallels are accurately represented, scale along them will be correct, but only the central meridian is drawn accurately to scale, and distortion increases outward from it. It is most suitable, therefore, for large countries which are situated outside the tropics and have a comparatively small change of longitude—*e.g.*, U.S.A.

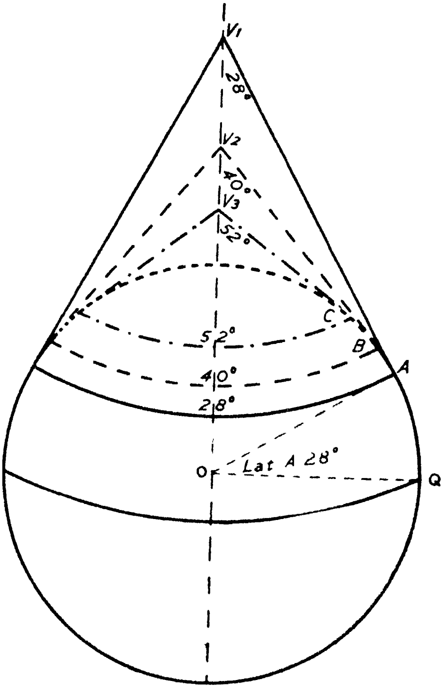


Fig. 8. Principle of the Polyconical Projection

Although admirable for map making from some points of view the polyconic projection could not be used widely for navigational purposes without modifying its scale along the meridians. The adjustment called for is achieved in the International Modified Polyconic projection, which consists of sheets covering four degrees of latitude and six degrees of longitude up to latitude 60°. Fig. 10 represents one of these sheets. The top and bottom parallels of latitude are still correctly represented to scale but not the central meridian. Instead the scale is correct along the two meridians two degrees removed from the central meridian. It will be noted that all the meridians are straight lines converging towards the pole. The edges of adjacent sheets fit both at top and bottom and at the sides. This enables strips to be built up on which the track of an aircraft may be marked by a continuous line. On

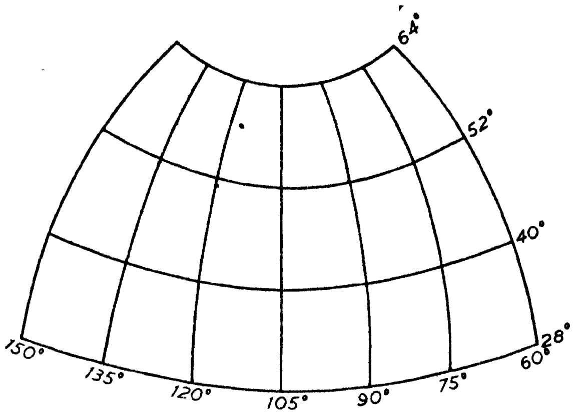


Fig. 9. Polyconical Projection

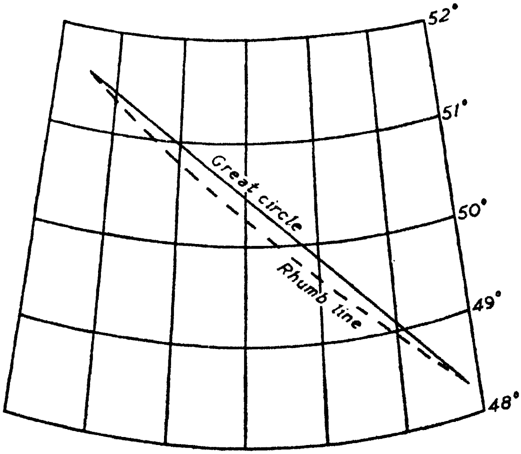


Fig. 10. International Modified Polyconic Projection

the polar side of latitude 60° each sheet covers four degrees of latitude and twelve degrees of longitude.

Direction on a Map. Maps are required for navigation by visual means. The direction of one position from another on the map must therefore correspond to that followed by the eye. Nature endows most of us with straight vision, and our line of sight is the line of least distance—*i.e.*, it follows a great circle on the earth. On topographical maps used in navigation, therefore, a straight line represents a great-circle arc or very nearly so. The amount of divergence of a straight line from a great-circle route is unlikely to exceed $\frac{1}{2}$°. Fig. 10 shows how a great-circle arc and a rhumb line between two points would appear on the International Modified Polyconic projection. It is obvious that because the meridians converge towards the pole a rhumb line to cross each meridian at the same angle must be a curve concave to the pole.

Measurement of Distance. Topographical maps are designed to show surface features, and the more detail that is required the larger must be the representation as a whole. This factor determines the proportion which dimensions on the map must bear to dimensions on the ground.

A given length of line on the map is made to correspond to a fixed distance on the ground. This definite relationship constitutes the **scale** to which a map is constructed. It may be that one inch on the map represents one mile on the ground or one inch on the map represents four miles on the ground. Whatever the relationship is it applies to any part of the map, and we know that if we find the distance between two points on the map to be 6 inches the distance between their corresponding points on the ground will be 6 miles on the 1-inch map and 24 miles on the $\frac{1}{4}$-inch map. It is thus a simple matter to convert scale distances to actual distances and vice versa. Distances on the ground, however, are not always measured in statute miles, and the use of inches is limited to a few countries. The relationship can be stated much more satisfactorily for all units of length by giving it in the form of a ratio—*i.e.*, a given distance on the map represents a certain fraction of the corresponding distance on the ground. This is known as a **representative fraction** (R.F.) when the numerator is one—*e.g.*, $\frac{1}{500,000}$ meaning that one unit of any sort on the map corresponds to 500,000 of the same units on the ground.

The distance on the ground corresponding to a given length of line on the map may then be found by simple proportion as shown by the following worked example.

Example. Find the distance on the ground corresponding to a distance of 7 inches on a map whose R.F. is $\frac{1}{250,000}$.

1 inch on the map corresponds to 250,000 inches on the ground.

7 inches on the map corresponds to 7 × 250,000 inches on the ground.

I.e., 7 inches on the map corresponds to $\frac{7 \times 250{,}000}{12 \times 5280}$ st. m. on the ground.

I.e., 7 inches on the map corresponds to 27·6 st. m. on the ground.

To find the length of line on a map corresponding to a given distance on the ground the process of calculation is reversed.

Example. What length of line on a map, R.F. $\frac{1}{500{,}000}$, corresponds to a distance on the ground of 40 kilometres?

500,000 cm. on the ground correspond to 1 cm. on the map.
40 km. = 4,000,000 cm.

$$\therefore 40 \text{ km. on the ground} = \frac{4{,}000{,}000}{500{,}000} = 8 \text{ cm. on the map.}$$

Exercise 1

Supply the missing values in the following table:

	Representative fraction	*Distance on map*	*Distance on ground*
1.	$\frac{1}{500{,}000}$	4·0 inches	
2.	$\frac{1}{250{,}000}$	6·5 inches	
3.	$\frac{1}{1{,}000{,}000}$	7·5 cm.	
4.	$\frac{1}{250{,}000}$		35 km.
5.	$\frac{1}{250{,}000}$		26 st.m.
6.	$\frac{1}{1{,}000{,}000}$		48 n.m.
7.		5·5 inches	22 st.m.

Time-speed Scales. When an aircraft is flying over the ground at a constant speed in a fixed direction the distances covered on the map in equal intervals of time will be the same. The speed determines the distance covered over the ground in a given interval of time, and distance on the map corresponds to distance over the ground according to the scale of the map. The navigator of an aircraft in flight often needs to know how far he has proceeded on a map in a certain interval at a given speed. He could convert his speed per hour over the ground into miles covered in any particular interval, and this in turn could be converted into corresponding distance on the map as already explained. As the navigator has little time for calculation when in the air, he provides himself with a time-speed scale on which he can measure immediately the distance covered on his map according to his speed and the interval. Suppose the R.F. of his map to be $\frac{1}{250,000}$ what distance on the map would correspond to the actual distance covered over the ground in one minute at a speed of 120 M.P.H.?

In 1 minute the distance covered would be $\frac{120}{60}$ —*i.e.*, 2 miles. This would correspond on the map to $\frac{2 \times 12 \times 5280}{250,000}$ —*i.e.*, 0·50688 ins.

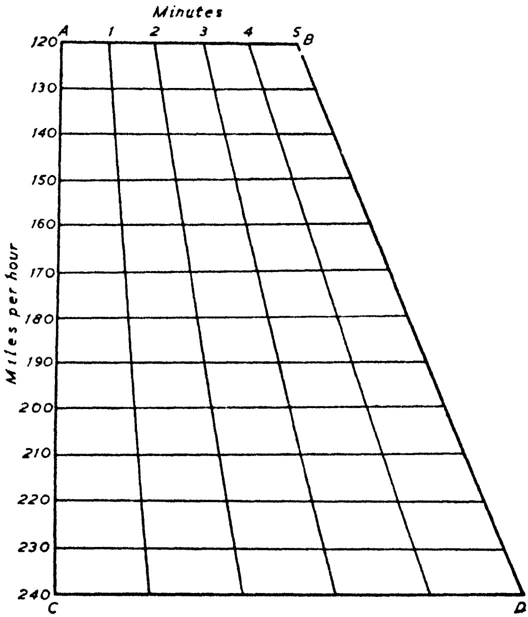


Fig. 11. TIME AND SPEED SCALE

Multiples of this distance could then be marked on a straight line and later taken off as the distance covered in any required interval of minutes.

The top line AB of Fig. 11 illustrating a time-speed scale shows how this has been done for multiples of minutes up to 5. Distances covered in longer intervals could be taken as multiples of that covered in 5 minutes or less. Distances covered at other speeds would, of course, not be the same in the same interval of time. Lines representing these may be drawn similarly however. At a speed of 240 M.P.H. the distance covered would be just twice that at 120 M.P.H. The bottom line CD of Fig. 11 has been drawn parallel to AB to represent the distance covered in 5 minutes at this speed. To complete the scale for speeds between 120 M.P.H. and 240 M.P.H. we need only divide the vertical line AC into convenient proportional parts corresponding to, say, intervals of 10 M.P.H. each and draw lines parallel to AB and CD to meet BD. Lines joining the minute-intervals marks on AB and CD will then cut off distances on the other lines representing those covered at their respective speeds. For example, the distance covered on this map at 160 M.P.H. in 3 minutes would be 2·03 ins. and that at 210 M.P.H. in 2 minutes would be 1·77 ins. These should be verified.

Exercise 2

1. Construct a time-speed scale for a bicycle between the speed limits of 7½ M.P.H. and 15 M.P.H. using a time unit of 2 minutes. Scale of map 1 inch to 1 mile or $\frac{1}{63,360}$.

Find the distance covered on the map in 20 minutes at 10 M.P.H.

2. Construct a time-speed scale for a car between the speed limits of 20 M.P.H. and 40 M.P.H. using a time unit of 2 minutes. Scale of map 4 miles to 1 inch. Find the distance covered on the map in 8 minutes at 35 M.P.H.

3. Construct a time-speed scale for an aircraft between the limits of 150 M.P.H. and 280 M.P.H. using a time unit of 1 minute. R.F. $\frac{1}{500,000}$. Find the distances covered on the map in 5 minutes at 200 M.P.H., 7 minutes at 230 M.P.H., and 3 minutes at 260 M.P.H.

Position. All maps are drawn on a framework, or graticule, consisting of meridians and parallels of latitude set at intervals of 1° on many topographical maps, although for rapidity of reference the R.A.F. often use a 10′ interval for both meridians and parallels. Position may be indicated by three distinct methods on a map—*i.e.*, (i) by graticule reference, (ii) by direction and distance, and (iii) by grid reference. Reference to the graticule is probably the most common method. It implies giving firstly the value of the latitude in degrees and minutes marked at the sides of the map, and secondly the value of the longitude marked at both top and bottom. In the northern hemisphere the parallel next below the place is noted, and then the difference in minutes between the parallel and the place is measured by dividers and added to the value of the parallel as shown at the side. This constitutes the latitude. In the southern hemisphere the latitude increases in value downward and the parallel next above the place would be noted. To this would be added the difference in minutes between the parallel and the place as measured by dividers.

Longitude is measured similarly but sideways. If the degrees and minutes increase in value to the right you are in east longitude, and the meridian next to the left of the place is noted before the change of longitude in minutes is added. If the degrees and minutes increase to the left you are in west longitude and must note the value of the meridian next to the right before adding the change of longitude in minutes. Because meridians converge, the mean of values read at top and bottom may sometimes have to be taken for places situated about midway between these edges.

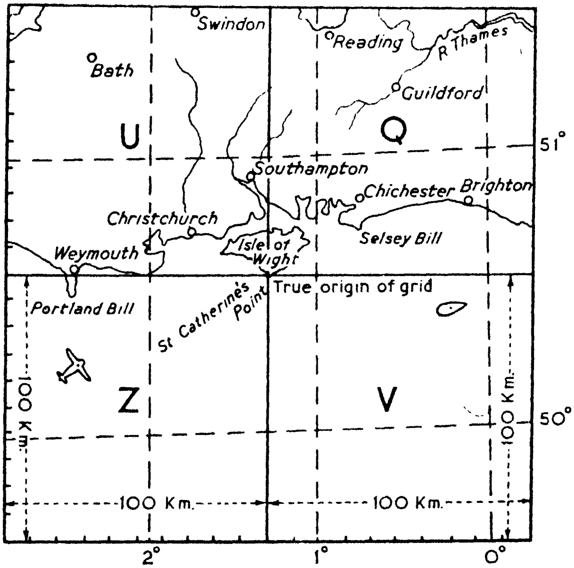


Fig. 12. Modified British Grid

Fig. 12 is a section of a map of southern England. To find the latitude and longitude of Southampton we note that the parallel below it is 50° N. The change of latitude measured by dividers between the 50th parallel and Southampton is 54′ N. The latitude of the port is, therefore, 50° 54′ N. The meridian to the right of Southampton is marked 1° and that to the left 2°, so that longitude increases to the left and is therefore westerly. The ch. long. between the meridian of 1° W. and Southampton found by transferring the distance to the bottom of the map is 23′, making the longitude 1° 23′ W. Sometimes in the R.A.F. for purposes of secrecy each integral degree of latitude and longitude is indicated by a pair of code letters. Then, whether in northern or southern hemispheres, the minutes of latitude are measured upward from the parallel next below a place, and the minutes of longitude are measured to the right from the meridian next to the left of it. Discrimination between hemispheres and between east and west longitude is thereby ignored. The minute measurements are taken from the graticule intersection to the south-west. If we refer to 50° N. as AB, 1° W. as KL, 2° W. as PQ, and 3° W. as XY, the position of the aircraft by the above method would be ABXY 1534. Both pairs of letters being given first, beginning with latitude and then the minutes in the same order.

The position of a craft may also be given in terms of direction and distance from some known point including, when necessary, a graticule intersection point. The position of the ship relative to St Catherine's Pt. would thus be given as 100° St Catherine's Pt. 41 n.m.; the direction from being given first, and measured in degrees clockwise from 000° at true north, and the distance given last in nautical miles. Using this technique in conjunction with the coded graticule we should give the position of the ship as a direction and distance from any near-by graticule-intersection point—*e.g.*, 047° ABKL 39 n.m. A military map has for the purpose of indicating position a system of squares superimposed on it and is thus said to be gridded. The largest square, a primary square, has a side of 500 km. and is identified by a letter of the alphabet. Each primary square is subdivided into twenty-five secondary squares of 100 km. side lettered from A to Z omitting I. Fig. 12 shows an area covered by four of these squares in the British Modified Grid system, squares U and Z belonging to the primary square V and squares Q and V to the primary square W. To give a position by grid reference to the nearest kilometre, the side of the secondary square containing the point in question is subdivided into 100 parts and measurements are taken from the south-west corner of the square, first to the right and then upward. The position of Southampton would be given by this method as U9335. When the identity of the secondary square is obvious, as on large scale maps, only the figures need be given, and if a greater degree of accuracy than to 1 km. is desired the squares may be further subdivided and the reference given in six or even eight figures, remembering always to give the measurement to the *right first*—not, as in the R.A.F. Lettered Co-ordinate system, upward first and then to the right.

It will be noticed that the grid lines diverge slightly from the meridians or parallels. The angle formed by the intersection of a north-south grid line with a meridian is known as **grid deviation,** while the angular difference between grid north and the direction of magnetic north is called **grid variation.**

Exercise 3

Using Fig. 12, complete the following table by describing the given positions in three other ways where possible, remembering that one degree of latitude = 60 nautical miles.

	Name of Place	*Graticule Reference*	*Direction and distance from St Catherine's Pt.*	*Grid Reference*
1.	Reading			
2.		51° 23′ N. 2° 21′ W.		
3.	——		200° – 30 n.m.	
4.				Q4870
5.	Weymouth			
6.		50° 44′ N. 1° 46′ W.		
7.	——		150° – 45 n.m.	
8.				U5599
9.	Brighton			
10.		50° 50′ N. 0° 45′ W.		
11.	——		110° – 60 n.m.	
12.				Z2591

Give the approximate scale of the map as:

(i) a representative fraction;
(ii) statute miles to the inch.

Relief. Horizontal measurements, though of extreme importance on a map, are not the only ones required; a knowledge of differences in height is also desirable. The airman in particular needs to know whether he is likely to meet high ground during the course of his flight and, if so, how high he must climb to pass safely over it. We are all more or less familiar with the atlas type of map on which different shades of colour indicate different ranges in height above mean sea-level. Brown is used for the higher ground, and the deeper the shade the higher the ground. On British air maps the colour used to indicate high ground is purple, the shade deepening with height. **Layer tinting,** as this is called, gives the general picture, but the gradations in shades required for more detailed information of height would be so slight as to be almost indistinguishable. Shades may vary sufficiently to indicate differences of 200 ft. or sometimes 100 metres. Where greater detail is desired lines joining points of equal altitude are drawn at **vertical intervals** (V.I.) of 100 ft. or 50 ft. These are known as **contour lines,** and their disposition gives as accurate an indication of slope as can be produced on a horizontal map. If you were able to remain stationary over an island in tidal waters, you would notice a difference in the shape of its outline as the tide rose and fell. If you drew the shape at low water and again after each successive rise of, say, five feet, you would have a series of shapes one inside the other gradually dwindling in size to a point which would be the peak. This is just the impression which contour lines are intended to give with a greater vertical interval. Where contour lines are far apart we know there is a large horizontal distance compared with the regular vertical interval. Before a true comparison may be made, however, we must convert the distance between contours into actual distance—*i.e.*, find the **horizontal equivalent** (H.E.). This, as explained earlier, depends upon the scale of the map. The ratio $\frac{\text{V.I.}}{\text{H.E.}}$ then gives the gradient. The highest point of a region is usually indicated by a spot beside which the height is given in either feet or metres. This is known as a **spot height.** A more indefinite method of indicating slope is that formed by a series of short lines in the direction of a fairly steep change in height. This is known as **hachuring,** and is found on the Mercator plotting sheets.

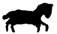
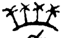
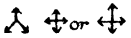
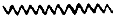
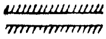
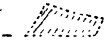
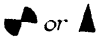
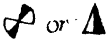
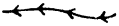
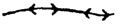
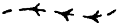
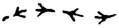
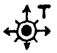
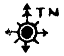
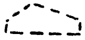
Exercise 4

Find the gradient from the following values:

	Scale of map	*V.I.*	*Distance between contours*
1.	1 inch = 1 mile	50 ft.	0·20 in.
2.	4 miles = 1 inch	100 ft.	0·35 in.
3.	$\frac{1}{500,000}$	200 ft.	0·25 in.
4.	$\frac{1}{250,000}$	100 ft.	0·42 in.

Map Reading. Proficiency in the use of an air map depends to a large extent upon the ability of the navigator to interpret the printed language of the map in terms of visible objects on the ground. Only those objects should be represented on the map which are large enough to be seen easily from the air in clear weather and distinctive enough to be readily recognizable. Some ground features due to their nature, shape, or colouring stand out as landmarks and these are represented on the map by tinting or symbols. Water features are tinted blue, wooded country green, towns and main roads brown, and railways black. National boundaries are in red, while the conventional signs listed on the opposite page may be printed in either red or black. From a moderate altitude the ground appears flat—it is seen in plan—so that ability to recognize horizontal shapes is of great importance. Water features stand out well, and the coast provides a line extremely useful in helping to locate the position of an aircraft when this is in doubt. The direction of the coastline may be compared with a similar direction of coast on the map and thus assist considerably in establishing the aircraft's whereabouts. Lakes are useful in helping to fix a

IMPORTANT CONVENTIONAL SIGNS GIVEN ON MAPS FOR USE IN THE AIR

Land Aerodrome (height above M.S.L. indicated in feet or metres)		◉
Landing Ground (height above M.S.L. indicated in feet or metres)		○
Airship Base (position indicated by spot)		
Water Aerodrome (position at centre of circle)		⚓
Seaplane Mooring Area or Anchorage (position at centre of stock)		
Hangar for Airships		
Mooring Mast for Airships		L
Conspicuous Object (showing as clearly as possible its appearance as seen from the air) e g ,	White horse	
	Clump of palms	
	General symbol	
Radio-electric Station		
Aerial Light		✹
Marine Light		★
Light-vessel (position at centre of star)		
Overhead Electric-power Cable		
Aerial Corridor		
Prohibited Area		
Unlighted Obstruction to Air Navigation (height above 200 feet)		
Lighted Obstruction to Air Navigation (height above 200 feet)		
Obligatory Air Route: One-way traffic		
Obligatory Air Route: Two-way traffic		
Recommended Air Route: One-way traffic		
Recommended Air Route: Two-way traffic		
Aerodrome Controlled Zone		⬡
Airport with Customs Facilities		NAME OF AERODROME
Land and Water Aerodrome		
Land Aerodrome with Aerial Light and Aeronautical Radio-electric Station		
Land Aerodrome with Aerial Light, Aeronautical Radio-electric Station, and Non-directional Radio-beacon		
Danger Area		
Explosives Area (white cross at centre of area)		✕
Balloon Obstruction Beacon		

position but they may change shape due to a raising or lowering of the water-level. Woods are not so reliable; their shape may be altered by the felling of trees, and if the wood is extensive its shape is scarcely recognizable at all. Rivers form very useful guides, and the point at which a railway bridge crosses it, for example, often gives the navigator a valuable clue to his position on the map. Such a crossing also provides him with a sort of sign-post, as from his map he is able to discover to which places the four directions lead.

Towns sometimes look very much alike, and unless there is some distinctive feature about them their identity may be mistaken. Details which help to distinguish one town from another are situation—whether on a coast, river, or canal—especially if there is a confluence, proximity to a lake; and an unusual number of railway lines forming a junction, as well as the trend of railway lines. Roads, especially when tarred, are of little use as guides; there is often a multiplicity of them, and they may be shaded by trees.

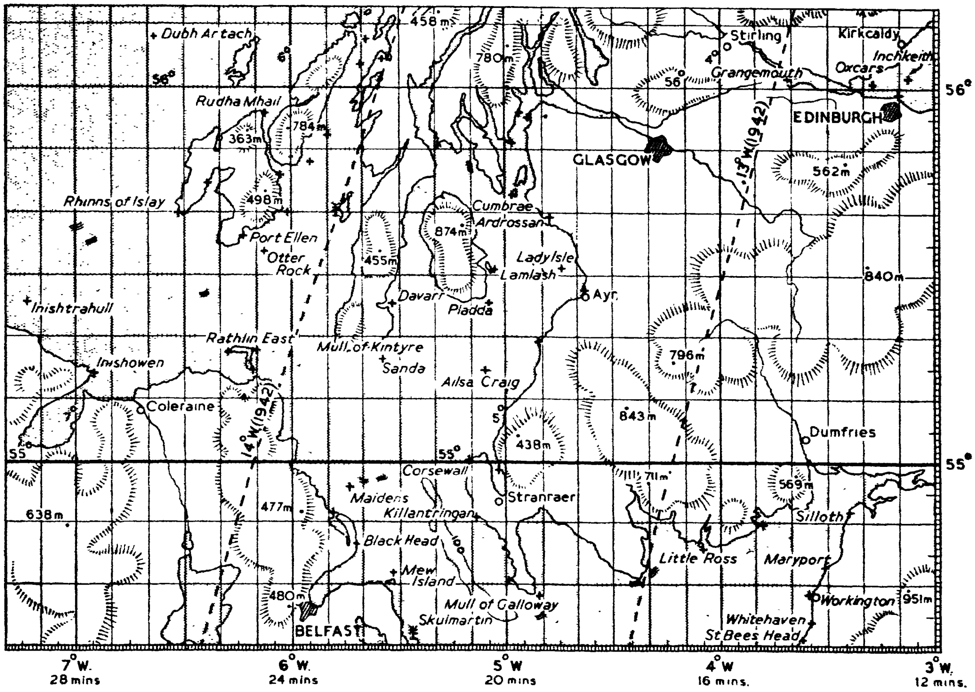
Horizontal objects are of little use at low altitudes; an aircraft is too close to them to appreciate their shape and loses sight of them much more quickly than when at higher altitudes. Hills, especially peaks, are then of greater importance for indicating position.

To obtain the best advantage from a map it should be set so that the navigator is looking along it in the same direction as that in which the aircraft is travelling. This is known as **orientating** the map—*i.e.*, placing it so that its directions are the same as those on the ground. The navigator should know beforehand by reference to the map just those places and prominent features over or near to which his aircraft is likely to pass during the course of its flight, allowing for a possible error in direction made good of about 5°. He will then know immediately whether the aircraft is following the required direction over the ground or not, and be able to rectify any discrepancy.

Similarly the time taken to pass from one object to another will provide him with a check on his speed over the ground. Should he fail to sight his destination he must estimate his error in miles, and with this as radius and his calculated position as a centre construct a circle within whose circumference he can then reasonably expect to be situated. This is his **circle of uncertainty,** and careful observation of the ground within its limits should bring into sight some object enclosed by it on the map.

Always remember that a navigator is never lost, but only more uncertain of his position.

SECTION OF PLOTTING SHEET "SCOTLAND WEST"



CHAPTER III

CHARTS

Historical background—Projection requirements—Gnomonic projection—Polar gnomonic chart—Equatorial gnomonic chart—Oblique gnomonic chart—Mercator chart—Latitude scale—Meridional parts—Construction using meridional parts—Graphical construction—Rhumb-line distance—Marine chart—Plotting sheet—Table of meridional parts—Characteristics of maps and charts.

Historical Background. Early representations of the earth's surface on a plane consisted mostly in delineating the boundaries between land and water and appear to us very crude in the light of modern knowledge. Not only was there the difficulty of representing a curved surface on a plane, but a regular system of establishing positions in terms of co-ordinates was slow in developing. Latitude was interpreted at first as angular measurement from the pole, due to early advances made in astronomy, by which it was known that the angular distance of any place from the Pole Star remained fairly constant. Considerations of longitude came later and were due probably to the observations of stars moving at a regular rate across the sky. Very approximate values of longitude were obtained by early Alexandrians who used a crude lunar method, but navigators had to be content either to sail coastwise or proceed to a certain latitude and then steer along the parallel to their destination. The star maps of Hipparchus (about 150 B.C.) were among the first to be provided with a network of arcs and lines for reference, and this, no doubt, provided geographers with their model. A certain Marinus of Tyre is regarded as being the first to use a graticule on which to construct a terrestrial map, and Ptolemy's map of the then known world (about A.D. 200) has the system quite well developed. As better methods of measurement developed and the earth became finally recognized as round, the framework, consisting of imaginary lines of latitude and longitude, was reproduced on a globe by Behaim in 1492. The Elizabethans used it on their voyages of exploration and discovery as it was convenient for great-circle sailing, which was then the practice over long distances. Such a small-scale replica was, however, by no means satisfactory for accurate navigational purposes, and various devices began to be introduced for the production of a large-scale representation of the earth's surface on a plane.

Projection Requirements. The faithful reproduction of a curved surface on a plane is unattainable. If you were to take a section of a rubber ball and attempt to flatten it out on a table you would not succeed without stretching it at the edges and compressing it at the centre. Distortion in some form of the surface features is unavoidable, and the problem facing a cartographer—as the map designer is called

—is how to avoid distortion of the particular characteristics he wishes to represent. In navigation the two most important lines required on a chart are the great circle and the rhumb line. The representation of either of these as a straight line is a distinct navigational advantage. Charts have, therefore, been designed to achieve these objects. A circle will appear as a straight line when looked at edgeways. When you are directly behind a rapidly moving hoop it appears straight to you, and if the sun, instead of you, were directly behind it, the hoop's shadow would appear as straight as that of a rod—*i.e.*, the hoops would be projected on to the ground as a straight line because the sun, the point of projection, would be in the same plane as the circle. To represent all circles as straight lines simultaneously the point of projection would have to be at the point of intersection of all their planes. This is made possible in the case of great circles on a sphere by using the sphere's centre as the point of projection. A rhumb line is a line that crosses all meridians at the same angle. If it were to be made straight it could cross all meridians at the same angle only if they were made straight too, and parallel. The case for the great circle is the simpler and will, therefore, be discussed first.

The Gnomonic Projection. If in a room you were to place a point of light at the centre of a transparent globe on whose surface meridians and parallels of latitude were already drawn, you would cause shadows of these lines to appear on the surrounding walls, ceiling, and floor. You would, in fact, be projecting the globe's graticule on to the flat surfaces of the room in a form known as the gnomonic projection (Greek, *gnomon* = style, or index, of a sundial), so named because when the earth's imaginary axis is treated as a style the projection acts as a dial, the shadow of the axis marking the hours. The arrangement of meridians and parallels on the ceiling would be different from that on each wall. On the ceiling and floor the meridians would be seen radiating from a central point in line with the globe's axis and representing the pole, while the parallels of latitude would appear as concentric circles circumscribed about this point. On each wall the meridians would be projected as straight parallel lines, increasing in distance from each other outward, while the parallels would appear as curves convex to the straight line representing the equator.

This is one way in which the cartographer reproduces the earth's graticule on a plane, but instead of the surface of projection, or primitive, as it is called, being maintained at a distance from the globe it is brought into contact with it. There is but one point of contact in the case of the gnomonic projection, and it may be anywhere on the globe. There are, however, three particular cases:

(i) polar, when the point of contact is at either pole;

(ii) equatorial, when the point of contact is along the equator;

(iii) oblique, when the point of contact is between either pole and the equator.

The Polar Gnomonic Chart. Fig. 13 illustrates the principle of construction. From this it can be seen how the meridians become straight lines radiating from the pole, at which the angles between them are correct, and how the parallels of latitude become concentric circles about the pole, the radii increasing progressively outward. Points L and R represent the projected positions of points M and K

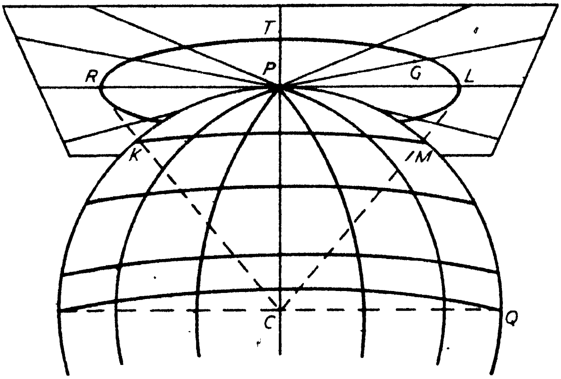


Fig. 13. Principle of the Polar Gnomonic Projection

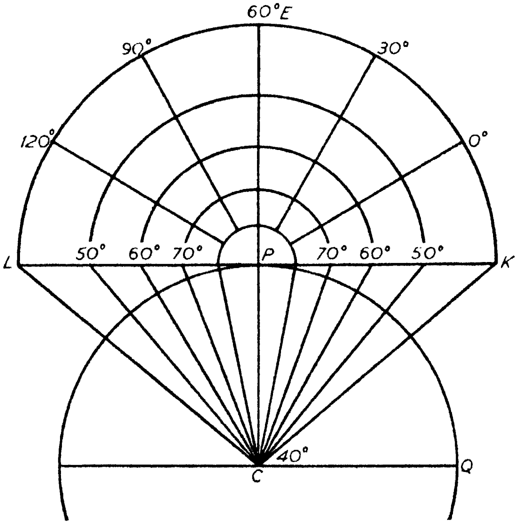


Fig. 14. Construction of the Polar Gnomonic Chart

respectively, the parallel of latitude which is seen to be represented on the tangent plane by the circle LTR.

The method of construction is simple, and is illustrated in Fig. 14. Draw a circle to represent the globe. Mark the position of the pole (P in diagram) and draw a tangent LPK. From C, the centre of the circle, draw radii at angular intervals from the pole corresponding to the parallels of latitude it is desired to represent and produce them to intersect the tangent to the pole. Then, with centre at the pole, construct the parallels of latitude for the chart by circumscribing a semicircle through the corresponding points of intersection. From P draw radii at suitable intervals to represent meridians. A graticule has then been completed covering 180° of longitude. The remaining 180° could be added by removing the constructional lines and producing the existing meridians across the pole to whatever parallel of latitude had been chosen as the limit. It will be noticed that the distance between successive parallels increases with distance from the pole so that beyond latitude 40°, the limit in Fig. 14, expansion becomes excessive. The equator can never be represented on a polar gnomonic chart because the equator and the tangent plane to the pole are parallel.

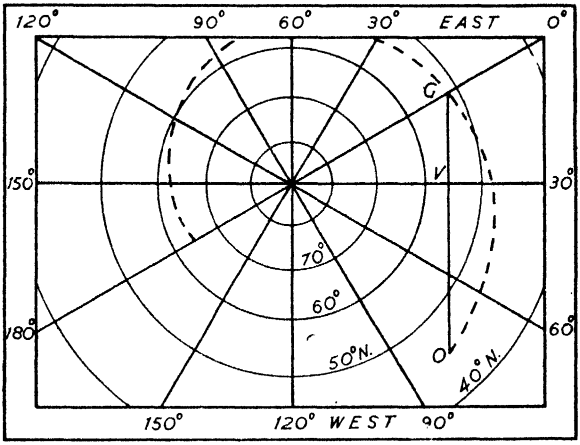


Fig. 15. Polar Gnomonic Chart

The radii of the parallels of latitude bear a definite relationship to the radius of the globe. In Fig. 14 the radius of parallel 40° is PK.

$$\text{Now } \frac{\text{PK}}{\text{CP}} = \text{tan. KCP}$$

$$= \text{tan. } (90° - \text{lat. of K})$$

$$= \text{tan. co-lat. of K, or cot. } 40°.$$

The radius to scale of each parallel is, therefore, radius of globe × cotangent of latitude. Because cot. 0° = infinity, the equator cannot be represented. The projection can never be used for a hemisphere.

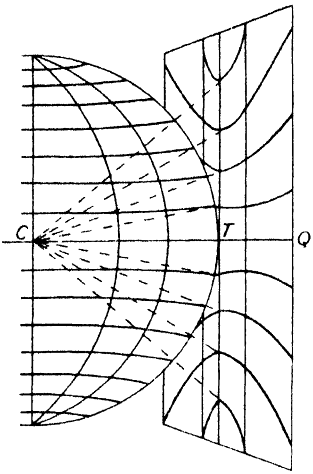


Fig. 16. Principle of the Equatorial Gnomonic Projection

The scale for distance is correct at the pole but increases outward, being constant only along a parallel of latitude. Fig. 15 shows the chart in a completed form. The positions of Ottawa and Greenwich are marked, and joined firstly by a straight line representing the arc of a great circle, and secondly by a rhumb line, which is seen to be a curve concave to the pole, and on being extended produces a spiral around the pole.

The Equatorial Gnomonic Chart. Fig. 16 shows how the equatorial gnomonic projection is produced. The point of tangency may be anywhere on the equator, but, as with the polar projection, the area projected must be always less than a hemisphere. In this case the poles cannot be shown. The meridians are, of course, straight, but their distance apart increases progressively to the right and to the left of the central meridian. The equator, being a great circle, is straight, but the parallels of latitude are curved, increasing their distance from both central meridian and equator.

Fig. 17 shows how this projection may be constructed quite easily. C represents the centre of the sphere and CD or CF its radius to any convenient scale. DTF represents the semi-equator with T as the point of tangency. Radials from C are drawn at an angular interval corresponding to the interval required between the meridians, in this case 10°. From their points of intersection with the tangent representing the equator perpendiculars are erected to represent the meridians —*e.g.*, 10°, 20° E. and W. The drawing of the latitude curves demands a little more patience. Constructional perpendiculars are drawn to each radial at its point of intersection with the tangent—*e.g.*, KL is drawn perpendicular to CK—and if the other radials be continued they will intersect KL, the perpendicular concerned with meridian 50° E., at points whose distances from K along the perpendicular to the tangent, say to M, give the scale for latitude along that meridian. Whether the division be for latitude 10° or more, depends upon which radial has made the intersection. In the case of point L it is the fifth radial from K, hence the division is for latitude 50°, and is transferred

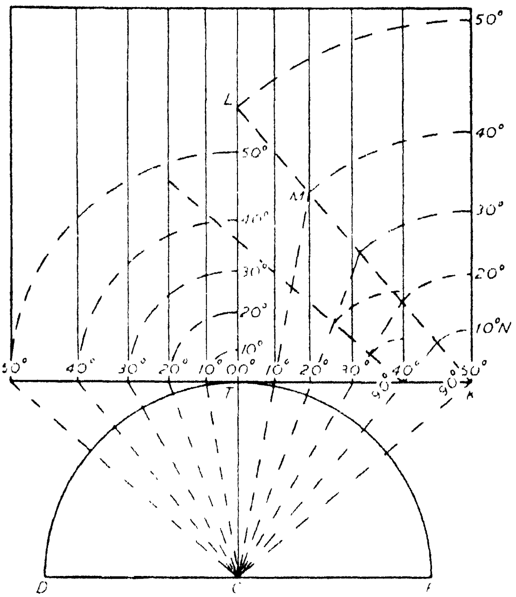


Fig. 17. Construction of the Equatorial Gnomonic Chart

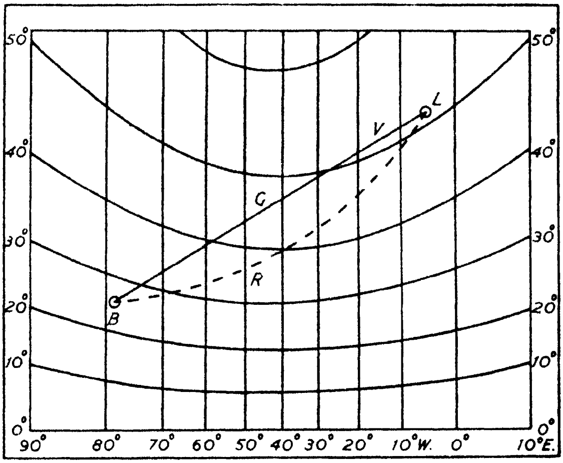


Fig. 18. Equatorial Gnomonic Chart

by radius as shown. M marks the point where the fourth radial from K intersects the perpendicular, and is, therefore, the measure of the latitude scale from K for latitude 40°. To avoid confusion due to multiplication of lines the early constructional lines may be erased as the work proceeds. Only one half need be done, as the meridians on the other side of the central meridian will have latitude divisions corresponding to those of their opposite meridian. A chart for the southern hemisphere is a repetition of that for the northern reversed. When the points of intersection of all lines of latitude with the meridians have been established the constructional lines should be removed and the points representing the same line of latitude joined by a smooth curve. The result will then be similar to that shown in Fig. 18 on which BGL has been added to represent the great circle, and BRL the rhumb line, linking the Bahama Islands with Land's End.

Scale on the equatorial gnomonic chart is nowhere regular, not even along a line of latitude, and differs at the same place in different directions.

The Oblique Gnomonic Chart. The oblique gnomonic chart has its point of tangency somewhere between the equator and the pole.

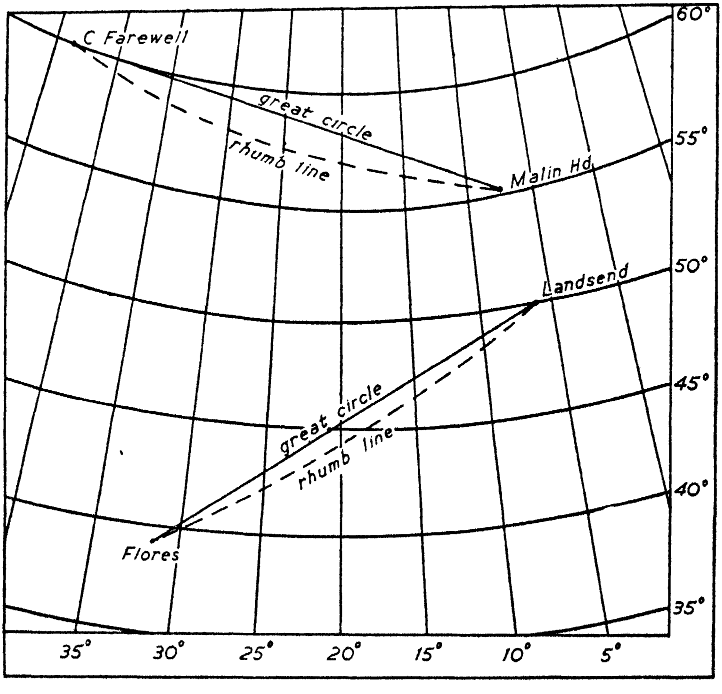


Fig. 19. Oblique Gnomonic Chart

This is the type in most common use but is much more difficult to construct than either of the other two projections. An explanation need not be entered upon here.

Fig. 19 illustrates its appearance when constructed for a point of tangency in latitude 40° N.

It will be noted that the meridians are straight and converge towards the pole and that the parallels of latitude are curves concave to the pole. Scale is nowhere constant.

The navigational value of charts constructed on the gnomonic principle lies in the fact that arcs of great circles can be represented as straight lines because the point of projection is the centre of the globe and the planes of all great circles pass through that centre. Gnomonic charts are of particular use in planning long voyages and flights, when the difference between great-circle and rhumb-line distance is often considerable.

The Mercator Chart. The growing need for simpler methods in navigation caused Gerhard Kremer (1512–94), a Belgian and cartographer to Emperor Charles V, to produce, in 1569, the now well-known Mercator projection. Mercator (merchant) is the Latin equivalent of Kremer and the name by which he was best known. The projection was designed to permit rhumb lines to be represented as straight and to do this the meridians had to be drawn straight, parallel, and equidistant. Without adjustment, however, this characteristic in itself would not have allowed direction on the earth to be represented correctly on the chart, and as this was equally desirable it could be obtained only by an expansion of distance between parallels of latitude corresponding to that already made between the meridians. Kremer, however, does not seem to have known the laws regulating the sizes of degrees of latitude and longitude, as above latitude 40° his graticule is inaccurate. It was left to Edward Wright, an Englishman, to develop the graticule on a mathematical basis. He discovered that the successive increases in the length of a meridian vary as the secant of the latitude.

The essential characteristics of a Mercator chart are:

(1) That rhumb lines on the earth—*i.e.*, lines which cross successive meridians at the same angle—appear as straight lines on the chart.

(2) That the angles between rhumb lines on the earth remain the same on the chart.

The chart is developed according to mathematical laws designed to give it these characteristics.

Latitude Scale. A comparison of Figs. 20 (*a*) and 20 (*b*) will show that the latitude scale on the Mercator chart must vary as the secant of the latitude when the longitude scale is constant. In Fig. 20 (*a*) AB represents an arc of the equator intercepted by the two meridians $PEAP_1$ and $PFBP_1$. EF represents the corresponding arc of a circle of latitude whose angular value is given by the angle BCF, which is equal to angle CFD. Now CB and CF, being radii of the same circle, are equal, so that in the triangle CDF, CF represents the radius of the equator and DF the radius of the given circle of latitude, while angle CFD represents the latitude and angle CDF = 90°.

$$\text{Then } \frac{\text{CF}}{\text{DF}} = \text{secant CFD}$$

$$\textit{I.e.,}\quad \frac{\text{radius of equator}}{\text{radius of circle of lat.}} = \text{secant of latitude}$$

$$\text{or,}\quad \text{radius of equator} = \text{radius of circle of lat.} \times \text{secant lat.}$$

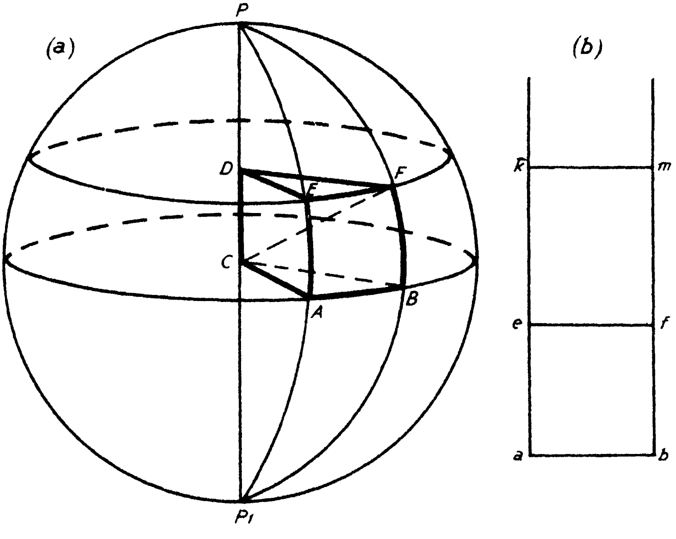


Fig. 20. The Latitude Scale

The radius of a circle is proportionate to the circumference and, therefore, proportionate to a part of the circumference. Because AB and EF are arcs intercepted by the same meridians, they correspond in angular measure—*i.e.*, angle ACB = angle EDF, and because they are proportionate to their radii, AB = EF × sec. lat.

In Fig. 20 (*b*) the meridians kea and mfb fulfil the first of the required conditions for the Mercator chart by being straight and parallel so that a rhumb line crossing them may be straight. This means that ef equals ab—*i.e.*, equals AB (Fig. 20 (*a*)). AB, however, equals EF × secant latitude, so that ef = EF × secant latitude. To fulfil the second condition of the Mercator chart, that angles between rhumb lines on the earth remain the same on the chart, correct shape must be maintained locally—the chart must be orthomorphic—and that is why the scale along a meridian—*i.e.*, the latitude scale—also must vary as the secant of the latitude. Therefore, because ef = EF × secant latitude—that is, the longitude scale has been multiplied by the secant of the latitude concerned—so must the latitude scale in the immediate vicinity of that particular latitude be multiplied by the secant of the latitude concerned. The cumulative effect of this is to stretch the distances between the equator and successive parallels by a progressively increasing amount.

Meridional Parts. Because the meridians on the Mercator chart are straight, parallel, and equidistant, the longitude scale is constant and may be used for measuring distances along the meridians. Minutes of longitude so used are termed meridional parts, and tables are available giving their value from the equator to any desired latitude. These tabulated values are deduced from the formula:

$$\text{meridional parts} = 7915{\cdot}7 \times \text{log. tan.} \left(\frac{\text{lat.}}{2} + 45^\circ\right).$$

This gives a summation of the secant series from the equator to the latitude concerned. The same value could be obtained by adding the secants of each minute of latitude from the equator to whatever latitude was chosen—*e.g.*, meridional parts for lat. L° = sec. $0^\circ\ 01'$ + sec. $0^\circ\ 02'$ + sec. $0^\circ\ 03'$ . . . . + sec. $(L^\circ - 01')$, but such a method would be extremely laborious.

Construction using Meridional Parts. In constructing the graticule of a Mercator chart the scale for longitude is chosen, and after a base-line has been drawn to cover the range of longitude,

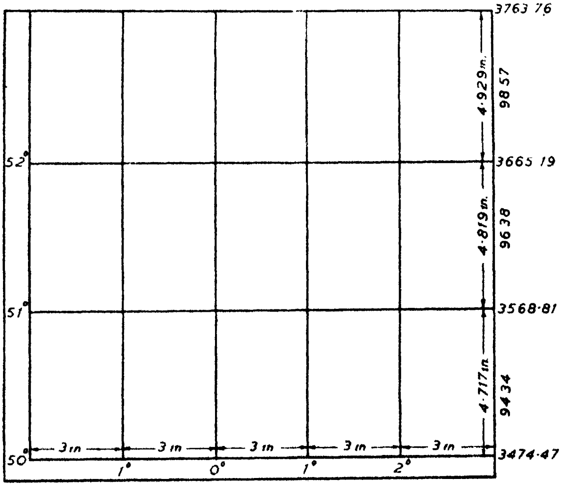


Fig. 21. THE CONSTRUCTION OF A MERCATOR CHART USING MERIDIONAL PARTS

perpendiculars are erected to represent meridians. The positions at which the lines of latitude cross the meridians, are found by measuring along the meridians from each line of latitude, beginning with the base-line, the difference of meridional parts (D.M.P.), between the two latitudes, generally one degree, the process being repeated until the upper limit is reached.

Example. Construct a Mercator chart from latitude 50° N. to latitude 53° N. and from longitude 2° W. to longitude 3° E., using a scale of 3 inches to 1 degree of longitude.

Latitude	*Meridional parts*	*D.M.P.*	*Latitude scale*
50°	3474·47		
		$94{\cdot}34 \times \frac{3}{60}$	= 4·717 inches
51°	3568·81		
		$96{\cdot}38 \times \frac{3}{60}$	= 4·819 inches
52°	3665·19		
		$98{\cdot}57 \times \frac{3}{60}$	= 4·929 inches
53°	3763·76		

A base-line representing lat. 50° N. is drawn 15 inches long (see Fig. 21), and at each extremity and every third inch a perpendicular is erected to represent a meridian. These are numbered 2° W., 1° W., 0°, 1° E., 2° E., and 3° E. from left to right. 4·72 inches above the base-line and parallel to it, a line is drawn to represent latitude 51° N.; 4·82 inches above this, a parallel line will represent latitude 52° N., and a parallel line 4·93 inches higher still will represent latitude 53° N. and the upper edge of the chart. The graticule is then complete and the degrees may be subdivided evenly at the edges into minutes.

If a table of meridional parts is not available the latitude scale may be obtained by multiplying the longitude scale by the secant of each mean latitude, *i.e.*,

$$3'' \times \text{secant } 50\tfrac{1}{2}^\circ = 3 \times 1{\cdot}5721 = 4{\cdot}716 \text{ inches}$$
$$3'' \times \text{secant } 51\tfrac{1}{2}^\circ = 3 \times 1{\cdot}6064 = 4{\cdot}819 \text{ inches}$$
$$3'' \times \text{secant } 52\tfrac{1}{2}^\circ = 3 \times 1{\cdot}6427 = 4{\cdot}928 \text{ inches}$$

giving the same results as the meridional-parts table to the nearest hundredth of an inch. If the chart were required for a part of the southern hemisphere, the measurements for latitude scale would have to be made downward. The longitude scale, being constant, may be used for any part of the world, care being taken, of course, to number the meridians correctly. In high latitudes the latitude scale increases rapidly and must, therefore, be calculated for shorter intervals than one degree.

Graphical Construction. The following graphical method of constructing a Mercator chart may often be found convenient.

The base-line of latitude and the meridians are drawn as described above, but instead of making latitude scale measurements by calculation a line is drawn from one of the intersection points—*e.g.*, 50° N. and 1° W.—making an angle with the base-line or any subsequent line of latitude equal to the mean latitude between that line and the next—*e.g.*, 50·5°. The length of this line intersected between adjacent meridians will be a measure of the latitude scale for one degree at that point and may be transferred as a radius to the meridians, so that the points reached may be joined by a horizontal line to represent the next line of latitude. Fig. 22 shows how this has been done to satisfy the above example.

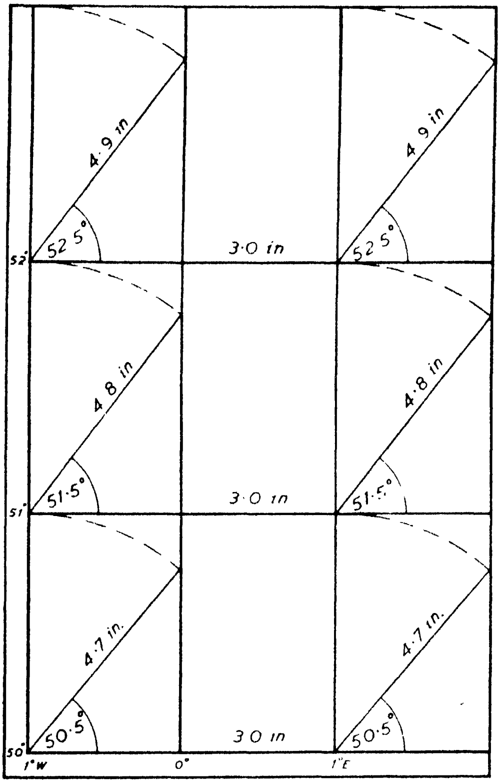


Fig. 22.

Rhumb-line Distance. Fig. 23 shows a complete graticule of a small-scale Mercator chart between latitude 0° and 60° N. and longitude 10° E. and 90° W. on which have been marked the positions of Land's End (L) and Great Bahama (B), joined by both a rhumb line

BRL and a great-circle arc BGL whose vertex is marked V. The main features of the chart are seen to be straight, parallel, and equidistant meridians and straight parallels of latitude, the distance between whose line increases progressively from the equator towards the poles. As a nautical mile has been defined as a minute of latitude, it will be obvious that there can be no constant scale of distance except along a parallel of latitude. This means that in measuring distance along a rhumb line the scale for latitude opposite the rhumb line's mid-point must be used. Compare the appearance of great circle and rhumb line in Figs. 18 and 23.

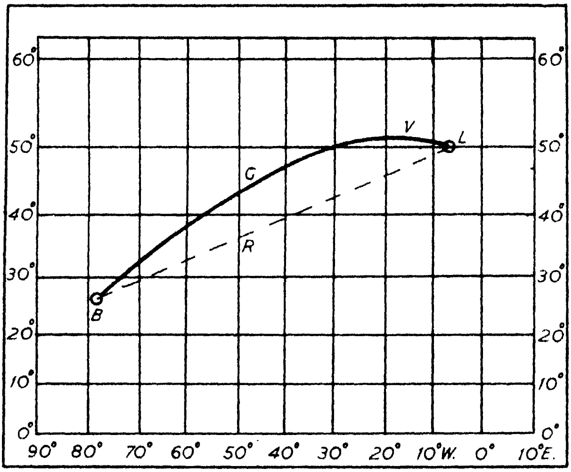


Fig. 23. The Mercator Chart

Marine Chart. The chart supplied for exercise with this volume is constructed on a larger scale and the minutes of latitude and longitude inserted, but, for the sake of convenience in construction, many features of the ordinary marine chart have had to be omitted. These include nature of coast, fathom lines, depth of water, nature of sea bottom, height of ground, characteristics of lights, state and direction of the tide, etc. A list of the most important chart abbreviations is given here for reference.

Plotting-sheet. The Mercator chart for use in the air is known as a plotting-sheet and has little detail. In addition to the graticule at 10′ intervals, the coastline, rivers, lights, some towns, and prominent heights are indicated. A compass rose, isogonal lines, and an abac are also found. A general idea of its appearance may be obtained from the reproduction (see p. 41). Much detail would obstruct plotting.

IMPORTANT ABBREVIATIONS USED ON MARINE CHARTS

DESCRIPTION OF LIGHTS

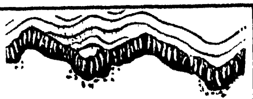
Alt. (Alternating)	Light changing colour
F (Fixed)	Steady and unbroken light
Fl. (Flashing)	Period of light less than period of darkness
Occ. (Occulting)	Period of light greater than or equal to period of darkness
F.Fl. (Fixed and Flashing)	Continuous steady light with regular short periods of greater intensity
Gp.Fl. (Group Flashing)	Groups of two or more flashes at regular intervals
F.Gp.Fl. (Fixed and Group Flashing)	Continuous steady light with regular groups of more intense light periods
Gp.Occ. (Group Occulting)	Groups of two or more breaks in a continuous steady light

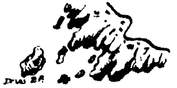
(U.)	Unwatched	(No keeper in attendance)
S.B.	Submarine Bell	(Sounded by wave action)
S.F.B.	Submarine Fog Bell	(Sounded regularly by electrical action)
L.B.S.	Lifeboat Station	
L.S.S.	Life-saving Station	
C.G.	Coast Guard	
P.D.	Position Doubtful	
E.D.	Existence Doubtful	

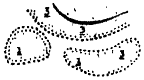
NATURE OF BOTTOM

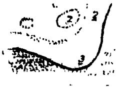
brk or bk	broken	peb or P	pebbles
chk or Ck	chalk	r or R	rock
g or G	gravel	s or S	sand
m or M	mud	sh or Sh	shells
oz or Oz	ooze	shin or Sn	shingle
wd or Wd	weed	st or St	stones

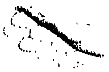
IMPORTANT CONVENTIONAL SIGNS USED ON MARINE CHARTS

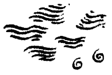
Cliffs 

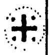
Rocky Coast 

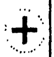
Sandy Beach 

Mud Banks 

Breakers 

Tide Rips 

Rock awash at Mean Low-water Springs or 

Rock with less than 6 feet of Water over it at Mean Low-water Springs + or 

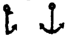
4-fathom Line ....

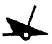
6-fathom Line --- --- --- ---

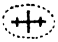
20-fathom Line -..-..-..-..-..-..-

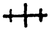
100-fathom Line ....

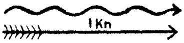
Anchorage for large vessels

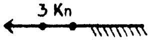
Anchorage for small vessels 

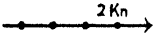
Wreck of which any part of the hull or superstructure is visible above Low Water 

Wreck over which there is a depth of 10 fathoms or less at Low Water 

Wreck with more than 10 fathoms over it at Low Water 

Current 

Flood Tide Stream (dots indicating hours after L.W.) 

Ebb Tide Stream (dots indicating hours after H.W.) 

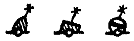
Depths marked in figures on the chart are given in feet or fathoms below Mean Low Water of Spring Tides, unless otherwise stated.

Underlined figures express heights in feet above Mean Low Water of Spring Tides.

No bottom found at depth expressed is indicated thus:

Position of Lights ★ ✱ •

Light-vessels (position at centre of base)

Light-buoys (position at centre of base) 

The height given against a light indicates the height in feet of its focal plane above Mean High Water of Spring Tides.

The use of tide-arrows was discontinued in 1933 in favour of tabulated values, but they are still found on many charts.

TABLE OF MERIDIONAL PARTS FOR EACH DEGREE OF LATITUDE FROM 40° TO 60°

Latitude	*Meridional parts*
40°	2622
41°	2702
42°	2782
43°	2863
44°	2946
45°	3030
46°	3116
47°	3203
48°	3292
49°	3382
50°	3474
51°	3569
52°	3665
53°	3764
54°	3865
55°	3968
56°	4074
57°	4183
58°	4294
59°	4409
60°	4527

For use in the construction of Mercator charts.

A COMPARISON OF CHARACTERISTICS OF MAPS AND CHARTS

	Topographical air map	Gnomonic projection	Mercator's projection
Meridians	Straight, converging towards the pole	Straight	Straight, parallel, and equidistant
Parallels of latitude	Concave to the pole	Concave to the pole	Straight and parallel
Scale	Constant for practical purposes	Not constant	Increases with latitude
Shape of land features	Correct for small areas	Not preserved	Correct locally
Area	Approximately correct	Incorrectly represented	Exaggerated as latitude increases
Great circle	Approximately straight	Straight	Convex to the pole
Rhumb line	Concave to the pole	Concave to the pole	Straight
Advantage	Visual relationships between ground objects preserved	Straight lines represent shortest distances	Straight lines represent constant directions relative to True North

General Exercises

1. Compare the principle and purpose of the Mercator and gnomonic charts.

2. State the advantages and disadvantages of the rhumb line and the great circle. When do a rhumb line and great circle coincide?

3. On the Figure constructed for Question 1, General Exercises, Chapter I, join positions (*a*) to (*b*) and (*b*) to (*d*) by great circle and rhumb line.

4. What are the two fundamental properties of a Mercator chart, and what are its essential characteristics?

5. For what regions is the Mercator chart unsuitable? Give reasons.

6. Construct by a graphical method a Mercator chart between the limits of latitude 50° S. to latitude 55° S. and longitude 64° W. to longitude 72° W. and check your measurements by the meridional-parts method.

Scale: 1 inch = 1° longitude.

On the chart place the following positions:

		Latitude	*Longitude*
(*a*)	Staten Is. lighthouse	54° 39′ S.	64° 07′ W.
(*b*)	Cape Virgins lighthouse	52° 50′ S.	68° 21′ W.
(*c*)	Dungeness lighthouse	52° 24′ S.	68° 26′ W.
(*d*)	Cape Possession lighthouse	52° 18′ S.	68° 57′ W.
(*e*)	Cape Gregory lighthouse	52° 39′ S.	70° 12′ W.
(*f*)	Sta Magdalena Is. lighthouse	52° 55′ S.	70° 33′ W.
(*g*)	Cape San Isidro lighthouse	53° 47′ S.	70° 58′ W.

Give the ch. lat. and ch. long. between Cape Virgins lighthouse and each of the other positions in turn.

7. Construct a Mercator chart between the parallels of 30° N. and 70° N. and between the meridians of 70° W. and 30° E.

Scale: 1 inch = 10° longitude.

What is the scale as a representative fraction in latitude 50° N?

Mark the following pairs of positions, join them by a rhumb line, and give the rhumb-line direction and distance:

(*a*) Hamilton (32° 15′ N. 64° 50′ W.) to Stockholm (59° 00′ N. 18° 00′ E.)
(*b*) Halifax, N.S. (44° 25′ N. 63° 55′ W.) to Lisbon (38° 42′ N. 9° 08′ W.)
(*c*) Land's End (50° 04′ N. 5° 42′ W.) to C. Race (46° 39′ N. 53° 04′ W.)
(*d*) Fastnet (51° 23′ N. 9° 36′ W.) to Belle Is. (51° 53′ N. 55° 22′ W.)
(*e*) St John's, N.F. (47° 34′ N. 52° 40′ W.) to Butt of Lewis (58° 31′ N. 6° 16′ W.)
(*f*) Paris (48° 50′ N. 2° 21′ E.) to Harbour Grace (47° 39′ N. 54° 10′ W.)
(*g*) C. Finisterre (42° 53′ N. 9° 15′ W.) to C. Farewell (59° 43′ N. 43° 53′ W.)

8. What are meridional parts, and why is their use necessary in the construction of a Mercator chart?

9. What advantages has a gnomonic chart, and what are its limitations?

10. Construct a polar gnomonic chart down to latitude 50° N. for a sphere of radius 4 inches. Put in the meridians at intervals of 30°. Mark the positions of London 51° 30′ N. 00° 00′, Churchill 58° 45′ N.

94° 00′ W., and Moscow 55° 45′ N. 37° 36′ E., and join them by lines representing great circles. Give the position of each vertex.

11. Construct an equatorial gnomonic chart extending up to latitude 60° N. between the meridians of 70° W. and 30° E. Plot the positions of the places given in Question 7 above, link each pair by a straight line representing a great circle, and, by noting the latitudes in which they cross the meridians, transfer each to the Mercator chart constructed in that question. Find the approximate initial and final directions of each great circle, and give the position of each vertex. Similarly transfer the rhumb-line tracks already found there to the gnomonic chart.

12. Using a tracing of the Mercator chart provided for exercise purposes, find the rhumb-line direction and the distance in nautical miles from A to B

	A		*B*	
(*a*)	54° 07′ N.	0° 05′ W.	52° 56′ N.	1° 20′ E.
(*b*)	52° 36′ N.	1° 43′ E.	52° 28′ N.	4° 35′ E.
(*c*)	51° 32′ N.	3° 26′ E.	52° 20′ N.	1° 42′ E.
(*d*)	52° 57′ N.	4° 44′ E.	53° 34′ N.	0° 05′ E.
(*e*)	52° 12′ N.	0° 09′ E.	53° 13′ N.	0° 33′ W.
(*f*)	53° 45′ N.	0° 21′ W.	52° 03′ N.	1° 09′ E.

13. Using a tracing of the Mercator chart provided, find the position of a craft after covering the following distances along the given rhumb line from A.

	A		*Rhumb-line direction*	*Distance* (*n.m.*)
(*a*)	52° 08′ N.	0° 20′ E.	045°	50
(*b*)	54° 29′ N.	0° 34′ W.	120°	105
(*c*)	51° 58′ N.	4° 06′ E.	310°	63
(*d*)	53° 20′ N.	4° 50′ E.	230°	141
(*e*)	51° 55′ N.	0° 10′ E.	350°	82
(*f*)	52° 45′ N.	0° 30′ W.	140°	78

CHAPTER IV

TIDES

Cause of tides—Tidal waves—Spring tides—Conjunction—Opposition—Age of the tide—Neap tides—Quadrature—Priming—Phase inequality—Phase inequality of times—Phase inequality of heights—Lagging—Effects of declination—Effects of distance—High-water lunitidal interval—High-water full and change—Rise of tide—Range—Height of tide—Stand of tide—Tide prediction—Harmonic method—Standard Ports—Admiralty method—Non-harmonic constants—How to read the tide tables—How to find the height of tide at any time—Reduction to soundings—How to find the time at which there will be a given depth of water—Tidal streams—Slack water—Flood and ebb—Eddies—Overfalls—Tide rips—Tide race—Tidal peculiarities—Bore—Equinoctial tides—Practical and commercial importance of tides.

THERE can be few people living in these islands now who have not at some time or other stayed sufficiently long near the coast to notice that the level of the sea, as compared with that of the land, is not always the same. Indeed, many will be well acquainted with the periodical rising and falling of the water and will know that the marks reached by the tops of successive rises are not at the same height, and, similarly, that the depths reached by subsequent declines in level are by no means always the same. The maxima and minima levels fluctuate because the forces producing this regular upheaval of water vary. As early as the fourth century B.C. Aristotle recorded the observation that "many ebbings and risings of the sea always come around with the moon." The Romans who navigated to Britain and beyond noted that variations in the time and the height of maximum water levels were connected with the moon's change in appearance —*i.e.*, with the phases of the moon. Patient observation over many years since then has established the fact that the moon plays the leading role in this water raising game and that the sun alternately supports and opposes her efforts. They each act separately, sometimes in harmony but more often at variance.

The Cause of Tides. The moon and the earth are kept in position relative to each other by the interaction of forces produced by themselves. Each body attracts the other in direct proportion to its mass and in inverse proportion to the square of the distance separating them, in accordance with laws of universal gravitation. Each body endeavours to leave the other by means of centrifugal force produced by its revolutionary movement about their mutual centre of gravity, the point through which their combined mass acts, G in Fig. 24. Both C, the centre of the earth, and M, the centre of the moon, revolve about G. The result is a compromise, the gravitational and centrifugal forces balancing each other, so that the bodies remain in equilibrium at approximately the same distance apart. Because the earth is very much larger than the moon their mutual centre of gravity lies below the earth's surface, and the moon thus revolves around a point within the earth. The surface of the earth, unlike that of the moon, is not solid, and, although the earth as a body resists any direct movement towards the moon, the water on that part of the earth facing the moon is overcome by the latter's attraction and is drawn up towards her in the form of a shallow heap.

On the opposite side of the earth, however, centrifugal force predominates and the water tends to move away from the moon, thus producing another heap on that part of the earth diametrically opposite to the moon. A heap is thus raised on the earth not only under the moon, as at A, Fig. 24, but also on the opposite side, at B —*i.e.*, two heaps are produced simultaneously by the influence of the moon. The one under the moon is known as the superior tide, while that on the opposite side of the earth is the inferior tide.

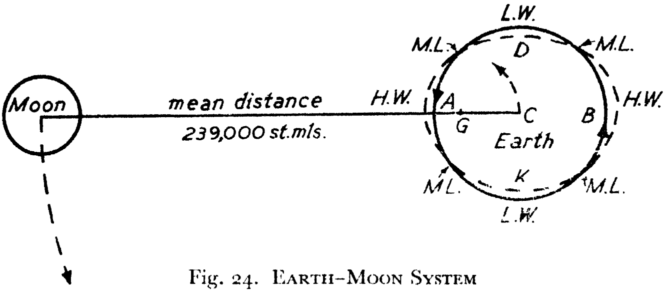


Fig. 24. EARTH–MOON SYSTEM

Just as the moon is linked by forces to the earth, so is the earth linked to the sun, which although much larger than the moon is much farther away and affects the water on the earth in a similar way but to a less degree than the moon. Two further heaps of water are, therefore, produced on the earth by the sun, but their height is only about three-sevenths that of those produced by the moon. Owing to the preponderance of the moon's influence, the solar heap becomes merged with the lunar heap and the sun's influence is consequently masked in the formation of two composite heaps.

Tidal Waves. If the earth were covered entirely by deep water these heaps would be domelike in shape, one vertex being at A (Fig. 24) and the other at B. The water at these two points would be at its greatest height above the mean level (M.L.), while along the great circle passing through D and K, the coincident bases of the two heaps, the water would be at its greatest depth below the mean level. We could then regard the earth as having the shape of a spheroid with its major axis in the direction of the moon. It would tend to conform to the shape of a rugby football rather than to that of an association football. As the earth rotates from west to east—the direction indicated by the arrows at A and B—the water levels cannot theoretically remain constant at any one place, except perhaps near the poles. As A moves eastward, its high-water level, like the crest of a wave, appears to move westward followed by a low-water level, or trough, which is itself followed by a further high-water level as A passes from K to B. This alternation is thus continuous, and means that all places, except those near the poles, should experience the passing of two high levels of water in the course of approximately one rotation, or day. The regularity of their appearance has led to such movements of water being known as tides (Anglo-Saxon, *tid*, time. *Cf.* Dutch, *tijd*; German, *Zeit*).

Owing to the existence of land masses on the earth the uniform shape of the heaps undergoes considerable modification. Only in the southern ocean has the tidal movement uninterrupted scope around the earth. There is reason to believe that this movement is at least to some extent transmitted to the other oceans, in which it gradually assumes a northerly rather than a westerly movement and develops into the form of a wave. There is little doubt, however, that the tide-generating forces of moon and sun also act directly on the waters of these oceans producing a wave which may be modified by the influence of the transmitted wave. The modern theory is to regard each ocean as a sort of dish in which the water is rocked to and fro in different directions by periodically repeated impulses of the tide-generating force, thus developing a rotatory movement of the wave about a nodal, or amphidromic, point situated near the centre. The level of water is regarded as being constant at the nodal point but varying progressively outward along the radii. An approximate mental picture of what the theory implies may be obtained by imagining a wheel with spokes at varying vertical angles rotating horizontally. The hub represents the nodal point and the spokes the progressive stages of wave development.

The difference between maximum and minimum levels at different places varies considerably. In mid-ocean it is small. At St Helena in the South Atlantic the difference amounts to only about three feet, at the Azores in the North Atlantic the greatest difference is about five feet, but closer to the land, and especially in shallow water where there is a narrowing channel, the maximum difference becomes very much greater. At London it is eighteen feet, in the Bristol Channel it may be as much as thirty-seven feet, while on the other side of the Atlantic, in the Bay of Fundy, the greatest difference so far recorded is found, *viz.*, fifty feet. In the land-locked and comparatively small area of the Mediterranean Sea the tide-generating forces nowhere succeed in raising the water level by more than a foot or two. Lakes show no tidal effect at all.

To afford a basis from which to pursue the study of tidal waves in more detail we have to consider the effects of the moon and sun separately and treat these bodies as if their movements were quite regular. We assume, therefore, that the moon and the sun move round the earth at a constant speed and at a constant distance away in the plane of the equator. The tidal effect of such bodies can be calculated, and the waves they produce are accurately represented by cosine curves with two maxima and two minima values per day. The solar day consists of 24 hours but the lunar day extends on the average to 24 hours 50 minutes. For this reason the two constituent waves cannot remain in step. The mean wave produced by the moon is referred to as M2 and the mean wave produced by the sun as S2, the suffix indicating the wave's twice daily, or semi-diurnal, appearance. Variations in the actual movements of moon and sun take place over regular periods, and their effects may be represented by other but much smaller constituent waves.

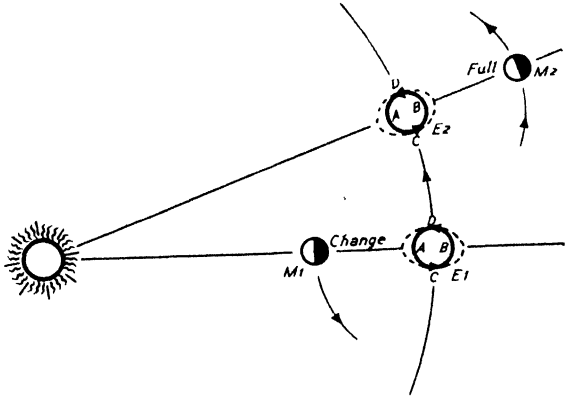


Fig. 25. Spring Tides (i)

Spring Tides. It will be readily understood that the composite wave formed by moon and sun together must be higher at certain times than the others. The moon and sun are not always in the same position relative to the earth. They may be on the same side of the earth—*i.e.*, have the same longitude, when they are said to be in **conjunction**—or they may be on opposite sides of the earth and differ respectively in longitude by 180°, when they are said to be in **opposition**, or they may be at any intermediate stage. When in conjunction, as seen in Fig. 25 (E_1), their waves are combined, and the maximum height of water will be experienced both at A, because the effect of their individual attractive forces are then added, and at B, because the effects of the centrifugal forces are then added. The moon is then said to be **new,** or at **change,** and is not normally visible to us, firstly because the side illuminated by the sun is necessarily turned away from us, and secondly because the sky around it is too bright. The moon is new at about noon of the places at which conjunction occurs. New moon, or change of moon, cannot occur near noon for all places. It would be near midnight at B for example, although the earth in rotating would bring B round to the moon before the latter had been able to move more than about 6° on her journey round the earth.

When the moon and sun are in opposition, as seen in Fig. 25 (E_2), the waves due to attraction are on opposite sides of the earth and so are the waves due to centrifugal force. This means that the wave due to the attractive effect of one is combined with the wave due to the effects of centrifugal force caused by the other. The moon then shows us a fully illuminated side and is said to be **full.** This occurs at about midnight of the places at which the moon is in opposition. On the other side of the earth the time will be about midday when the opposition takes place and the moon is actually full. Elsewhere the times differ.

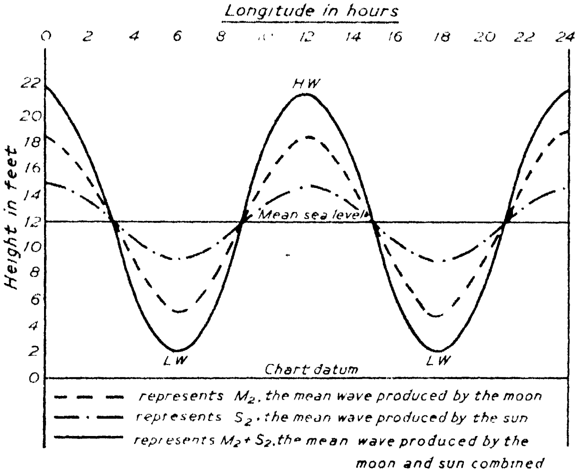


Fig. 26. Spring Tides (ii)

From what has been said up to now it is natural to assume that the highest tides occur when the moon is either new or full, as at any other times the moon and sun cannot pool all their tide-generating resources. Owing to fluid friction and the inertia of water, however, the wave lags behind the moon and the highest tides occur from approximately one to three days after new or full moon. These are known as spring tides, and the interval which elapses from new or full moon to the time of spring tides is known as the **age of the tide.** Waves oscillate about a mean level as shown in Fig. 26, so that an excessively high crest is followed by an excessively low trough or vice versa. Spring tides, therefore, indicate not only the highest but the lowest water levels for any particular place. The difference in height between these levels is known as **spring tide range.** From Fig. 25 it will be seen that if the water level is at its highest at A it is correspondingly low at C. As the earth rotates, A will move round to C and will then experience a low-water level as far below the mean level as the high water was above it. The low level at D would, of course, be equally low. Spring tides then are those which rise to the highest and fall to the lowest levels at about one to three days after new or full moon. Fig. 26 shows how the constituent waves M2 and S2 can combine to produce a spring tide.

Neap Tides. When the moon is new it is on the same side of the earth as the sun, and as the earth rotates they appear to revolve round it together. Because the moon does actually revolve round the earth in the same direction as the earth rotates, although at a much slower rate, the moon slowly parts company from the sun, moving to the eastward. When the earth has made one rotation, one day has elapsed and the sun is again on the meridian passing through A (Fig. 25 (E1)). The moon, however, has moved on, and A must continue round through an angle of about 12½°, corresponding to approximately fifty minutes of solar time, before the moon is overtaken and appears on A's meridian again. As the earth continues to rotate the moon moves still farther round until, after nearly 7½ days, the angle between sun and moon has increased to 90° and they are said to be in **quadrature.** The moon does not then come on to A's meridian until about six hours after noon, when the sun is in the western sky or may have set. We then see the moon as a half-illuminated disc called the **first quarter.**

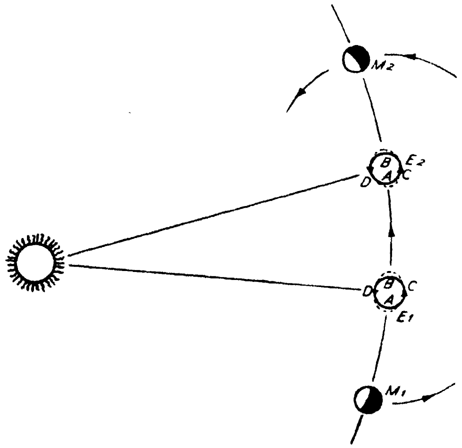


Fig. 27. Neap Tides (i)

On referring to Fig. 27 (E1) it will be seen that the tidal waves set up by the moon and the sun can no longer combine. On the contrary, the crests produced by the sun occur in the same positions as the troughs produced by the moon, and the crests produced by the moon occur with the troughs produced by the sun, but, as the moon's influence predominates, the result is that the sun's effect reduces that of the moon to a minimum. The maximum level of the tide, the high water, is then reduced to its lowest value, and the minimum level, the low water, is raised to its highest value. See Fig. 28, in which M2 and S2 are again combined. Tides produced under these conditions are called neap tides, and the difference in height between high- and low-water levels is the **neap-tide range.** Owing to fluid friction and inertia the wave lags behind the moon and neap tides do not occur exactly at quadrature but from approximately one to three days afterwards, as in the case of spring tides.

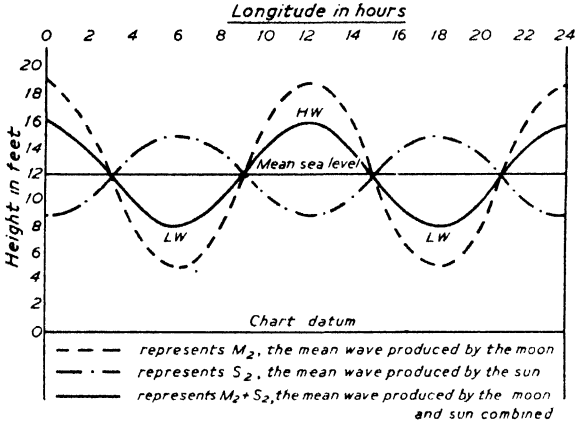


Fig. 28. NEAP TIDES (ii)

About 7½ days after full moon the sun and moon are again in quadrature. This is the **third quarter,** and neap tides will occur again as shown in Fig. 27 (E2), although, as already explained, not exactly on the date of quadrature.

Priming. From conjunction to first quarter the moon, in revolving round the earth, moves ahead of the sun and as it changes in phase, takes its tidal wave with it. The crest resulting from a combination of the lunar and solar waves must therefore be created between the individual waves. Owing to the moon's preponderating influence the crest will occur nearer to the moon's original wave than to the sun's. On referring to Fig. 29 it will be seen that at E1 this direction is indicated by a heavier line, high waters occurring at A and B, not under the moon as at conjunction, opposition, or quadrature. At conjunction (Fig. 25) high water occurs, at least theoretically, when the moon is on A's meridian. Now A (Fig. 29), in rotating with the earth, passes the position of high water before reaching the moon—*i.e.*, high water occurs before the moon's transit, a condition known as the **priming** of the tide. Both the interval between successive high waters, and the height of each high water as compared with that of spring tides, must naturally vary. Such differences are known as **phase inequalities.** Assuming the moon to revolve at uniform speed, the interval between successive high waters will be less than the average for the first three or four days after conjunction, but then greater.

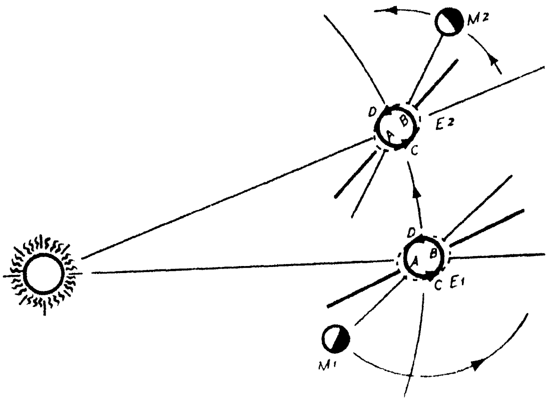


Fig. 29. Priming.

This difference due to the moon's phase is known as the **phase inequality of times.** The intervals may decrease to as little as thirty-eight minutes at springs, but be as much as sixty-six minutes at neaps. The range of the tides will decrease from springs to neaps. The difference in height between high water at springs and the high water of any other tide is known as **the phase inequality of heights.**

From opposition to third quarter similar conditions prevail, as can be seen on referring to E2 of Fig. 29. For whereas high water occurs under the moon at opposition (Fig. 25), the moon has moved on and the combination of lunar and solar waves creates a crest between the two, as at B, and, incidentally, at A. High water will again occur before the moon's transit, and the intervals between high waters will vary as they did when the moon passed from conjunction to first quarter. The ranges will decrease as before.

Lagging. Between first quarter and opposition, and between third quarter and conjunction, high water occurs after the moon's transit, and the tide is then said to lag. The interval between high waters will tend to be greater than the average for the first three or four days after quadrature, and then decrease to less than the average. Fig. 30 shows how at E_1 a wave opposite the sun combines with a wave under the moon to produce high water between them at A, and how similar waves opposite the moon and under the sun combine to produce a high water between them at B. As the earth rotates places such as A and B on being carried round will meet the moon before the wave —*i.e.*, the moon's transit will occur before high water. Fig. 30 illustrates similar conditions at E_2. This time the wave produced under the sun combining with the wave produced under the moon causes high water to appear between them at a point B. On the opposite side high water will occur at a point A between the inferior waves of the moon and the sun. As already explained for conditions during the second quarter, it will be seen that places such as A and B of E_2, in rotating with the earth, will meet the moon before they meet the wave produced principally by the moon but partially by the sun.

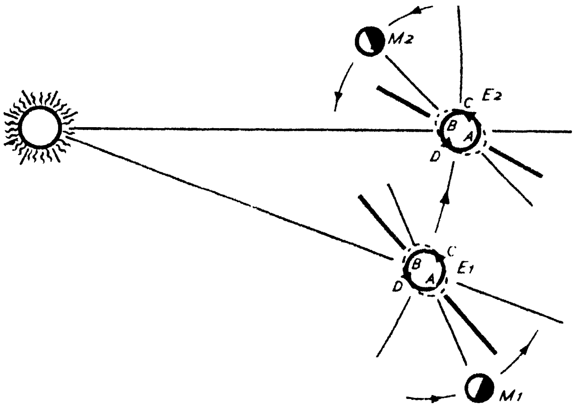


Fig. 30. Lagging

The Effects of Declination. In describing the foregoing figures it has been assumed for simplicity of explanation that both the moon and the sun have been approximately in the same plane as the earth and over the equator. It is well known that because we experience seasonal changes the sun must be sometimes north of the equator and sometimes south of it, crossing it on only two occasions during the year—*i.e.*, at equinoxes which occur in March and September. At all other times its direction makes an angle with the equator, reaching a maximum of 23½° north in June and 23½° south in December. An angle measured at the centre of the earth perpendicularly between the plane of the equator and the straight line joining the centre of the earth to the centre of a celestial body is called declination. It is as if the earth in Figs. 25 and 27 were to remain on the page, while the sun moved towards you to create say northerly declination or away from you to create southerly declination. No such noticeable feature as the seasons causes us to become aware of the moon's change of declination, unless we measure the angular distance of the moon above the horizon; but just as the sun crosses the equatorial plane so does the moon, not twice in a year, however, but twice in a lunar month, or lunation—the interval from one new moon to the next, about 29½ days. So the moon changes her declination much more rapidly than the sun, although her maximum is never more than 5° from that of the sun.

These movements upset the nicely balanced tidal waves we have discussed so far. They introduce irregularities into both the heights of maximum and minimum water levels and the times at which they would occur under uniform conditions. Because the moon has the preponderating influence over the tides, the effect of her change in declination is more prominent than that of the sun's. The effect of each body in this respect is really twofold. The effect on the mean tide of each body would be to increase the range when the body is near the equatorial plane but decrease it when near maximum declination. This produces a constituent wave known as K_2. But, on the other hand, the tendency of the body at maximum declination is to raise the water level both beneath it, A in Fig. 31, and at the diametrically opposite point B. This means that only one really high water can occur daily from this cause at a given place, one high level would occur in, say, latitude 20° N. and another in 20° S. producing only one high water in each hemisphere per rotation or day. Such tides are frequently referred to as **tropic tides,** or single day tides. These daily, or diurnal, tides caused by moon and sun are treated as constituent waves with a period of about one day. The constituent wave due to the combined declinational effect of moon and sun is known as K_1. Their individual declinational effects produce constituent waves known as O_1 for the moon and P_1 for the sun.

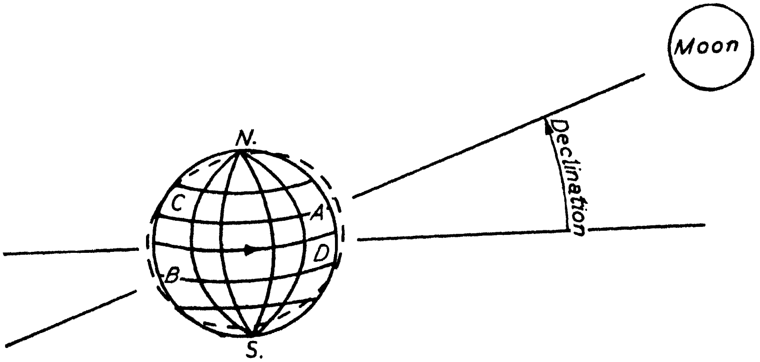


Fig. 31. The Effect of Declination

The moon's daily wave predominates over that of the sun, but is usually much smaller than the principal constituent M_2. When the waves are compounded, therefore, the daily constituent is not usually responsible for the whole tide, although at Victoria, British Columbia, the daily range may be more than nine feet. Usually the daily constituent modifies the principal wave to the extent of making one of the semi-diurnal high waters greater than the other. Fig. 31 shows that the greater high water at A can occur once only in that latitude during a complete rotation of the earth. A second high water occurs at C, but its height is less than that at A, so that in rotating with the earth A and C, or any other place along that parallel of latitude, will experience a higher and a lower high water alternately. This happens also at and near the parallel of B and D.

The Effect of Distance. The orbit of the moon around the earth is an ellipse not a circle, and the earth is situated at one of the foci. Once a lunar month the moon reaches its minimum distance from the earth, when it is said to be in **perigee,** and once a lunar month it recedes to its maximum distance from the earth, when it is said to be in **apogee.** The distance of the moon from the earth at perigee is 226,000 miles and at apogee 252,000 miles, giving a mean distance of 239,000 miles. The gravitational pull of a body varies inversely as the square of its distance away—*e.g.*, if the distance were halved the pull would be quadrupled, and if the distance were doubled the pull would be reduced to a quarter of what it is. When the moon's distance away is less than the mean distance her average tide-generating force will be increased, and when the moon's distance away is greater than the mean distance her average tide-generating force will be decreased accordingly. These variations are allowed for by the addition of a further constituent wave.

The earth's orbit around the sun is also an ellipse at one of whose foci the sun is situated. Once a year, in January, the earth reaches its minimum distance from the sun, when it is said to be in **perihelion,** and once a year, in July, it recedes to its maximum distance from the sun, when it is said to be in **aphelion.** When the earth's distance from the sun is less than the mean distance the sun's average tide-generating force will be increased, but when the distance is greater than the mean the average tide-generating force will be decreased. The earth's average distance from the sun, 93,000,000 miles, is so great, however, and the sun's tide-generating force so comparatively small in consequence, that any slight variation in distance from the average has little appreciable effect on the sun's gravitational pull as a whole.

Lunitidal Interval. High water at any place does not usually occur at the time of the moon's transit there for two reasons. Firstly, friction due to the configuration of the coast and the nature of the sea bottom, especially in shallow water, causes the crest of the wave to lag behind the moon. Secondly, priming tends to make high water occur before the moon's transit while lagging tends to make it occur after the moon's transit. The result is that at any particular phase of the moon the interval between the time of moon's transit and the time of the following high water is always the same at a given place. This interval is known as the **high-water lunitidal interval.** At springs and neaps there is no inequality due to phase of moon and the lunitidal interval is due to frictional delaying action alone. Because the tide primes and lags equally during a lunation, the interval at springs and neaps must be the average interval and is thus called the **mean high-water lunitidal interval.**

As the phase inequality is the same for both new moon (change) and full moon, the lunitidal interval on these occasions must be a constant for any given place. This interval is known as the **high-water full and change** (H.W.F. & C.), and because the transit of the moon when full occurs at or about midnight, and when new at or about noon, the H.W.F. & C. must also be the times of A.M. and P.M. tides respectively on those dates. When recording these times the hours, except 2 hours, are given in Roman numerals and the minutes in Arabic; *e.g.*, $\text{IV}^{\text{h}}\ 57^{\text{m}}$ is the H.W.F. & C. at Falmouth.

Rise and Range. Tides are oscillations about a mean level whose value is obtained by taking the average of all the high waters and all the low waters occurring at a particular place over a long period. It requires a large number of observations, but has already been calculated for many places and is given in the Admiralty Tide Tables under the heading of M_0. The value given for London Bridge is 11·0 feet, which means that the tide there oscillates about a level of 11·0 feet above chart datum, the lowest level to which it is anticipated that tides will normally fall. In the last chapter it was seen that depths marked on the chart are depths below chart datum, and now we see that tides are, so to speak, superimposed on chart datum, a combination of the two providing the actual depth of water according to the state of the tide. The height which any high water reaches above chart datum is known as the **rise** of that tide. Fig. 32 shows that by spring rise, neap rise, and mean rise is meant the heights of their respective high waters above chart datum. The heights of low waters above datum are also given for certain ports. Mean low-water springs —by which is meant the average low water of all spring tides—mean low-water neaps, and mean low water generally are illustrated in the diagram. Only very occasionally do low waters fall below chart datum and then attention is drawn to the fact by an asterisk placed in the tide tables alongside the value concerned. The difference in height between any high water and its preceding or succeeding low water is known as the **range** of that particular tide. Each tide will have its own range. In Fig. 32 the range for mean springs—*i.e.*, the average range for all spring tides—mean-neap range, and the mean range for all tides are illustrated. The difference in height between chart datum and the level of the water at any state of the tide is known as the **height of tide** for that time. The depth of water at any time is the charted depth plus the height of tide when above datum but minus the height of tide when below it.

At high water the tide does not cease rising abruptly but gradually.

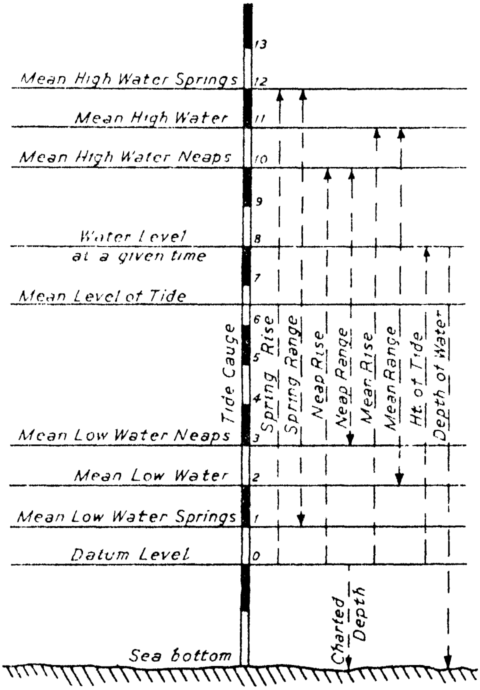


Fig. 32. The Tide Gauge

So gradual is the change in level at the 'top' of the tide that no appreciable change takes place, and the tide is said to **stand.** The time of high water is then taken to be mid-time between the instant at which the water ceases to rise and that at which it begins to fall. There is a similar stand at low water. The rate of change of depth is greatest at mean tide level.

Tide Prediction. The need for advance information on the times at which tidal waters will rise to their highest and fall to their lowest levels, together with the heights of those levels, is of the greatest importance to all those responsible for the navigation and manœuvring of surface craft. It has been shown that the moon and the sun are together responsible for the production of our tides, and it follows that a knowledge of their future positions relative to the earth is indispensable to the predicting of the tides they produce. Astronomers can forecast the positions of the moon and sun for years ahead, but, due to the varying relationships of these bodies with the earth, no simple method of predetermining their effect on it is available.

If we imagine the moon to move round the earth at its average rate in the plane of the equator and at its mean distance from the centre, we have a general basis from which to begin calculating the particular effects of the moon. This imaginary body moving thus round the earth in 24 hours 50 minutes is regarded as the principal tide-producing agent due to the moon, and because it causes two high waters to occur within that time the constituent wave, known as M_2, has a period of 12·42 hrs. Similarly, if we imagine the sun to move round the earth at its average rate in the plane of the equator and at its mean distance from the centre, we have a general basis from which to begin calculating the particular effects of the sun. The sun appears to move round the earth once in 24 hours, during which time it creates two high waters. The constituent wave of this principal solar agent, known as S_2, has a period of 12 hours.

Variations on these mean positions, due to change in declination, distance, etc., are numerous, but they have regular periods, and their effects may be represented by subsidiary bodies moving uniformly round the earth at speeds corresponding to those periods. We have, then, an earth surrounded by probably thirty imaginary satellites, all moving at different speeds round the earth in the plane of the equator and each developing its own harmonic wave. The combination, or synthesis, of all these waves produces the actual wave due to all predictable causes. This is the principle of the **harmonic method of tide prediction.** It would be extremely laborious to calculate the time of high water or height of tide from such a wealth of detail, but fortunately this is not necessary as, due to the ingenuity of the late Lord Kelvin, a machine exists which combines the curves and computes tidal data automatically.

Tides are predicted in this way for many places all over the world. The predicted times and heights of high and low water for 130 ports appear in the Admiralty Tide Tables, Part I, 45 European ports in Section A, and 85 non-European ports in Section B. Such ports are known as **Standard Ports.**

Data for predicting the times and heights of high and low water at ports other than standard ports are given in Part II of the Admiralty Tide Tables. The phase lag (g), or interval, between the theoretical and actual time of high water caused by a constituent wave and the amplitude (H), or half-range, of that wave are given for each of the four principal constituent waves—*viz.*, M_2, S_2, K_1, and O_1—together with the mean tide level (M_0) or mean sea level (Z_0). Constituents K_1, and O_1 are single day waves produced by the declinations of the moon and sun as explained under Effects of Declination (p. 69).

By means of these values, together with certain astronomical details, the time and height of high or low water at a given place may be found by the Admiralty method of tide prediction. It requires the use of a form specially drawn up for the purpose, and is fully explained and illustrated by a worked example in Part III of the Admiralty Tide Tables.

A quicker but less accurate method of predicting the times and heights of high or low water is that offered by the use of a time difference and a ratio of ranges on a standard port. Results obtained from it should be regarded as only approximate at places where the tide is largely semidiurnal, but at places where the tide is largely diurnal they are liable to considerable inaccuracy and should be avoided.

When tide tables are not available non-harmonic tidal constants found on charts may be used. They are: (i) Mean High-Water Lunitidal Interval; (ii) Mean Spring Rise; (iii) Mean Neap Rise; and (iv) Mean Tide Level. To find the time of high water the lunitidal interval should be added to the time of moon's transit at Greenwich given in the *Nautical Almanac*. Low water is then assumed to occur 6 hours 12 minutes after the time of high water. Spring tides must be assumed to occur 1 to 2 days after new or full moon, and neap tides 1 to 2 days after quadrature.

The *Atlas of Tides and Tidal Streams* contains charts showing co-tidal lines along which high water occurs at the same time, and co-range lines, along which the mean range of the tide is the same. A time difference on a standard port and a mean high-water interval are given against each co-tidal line, while against each co-range line is found a ratio of ranges on a standard port and the mean range of the tide. Such co-tidal charts should be regarded as giving only approximate values, and be used only for predicting tides at sea.

How to Read the Tide Tables. Part I of the Admiralty Tide Tables (A.T.T.), extracts from which will be found at the end of this book (p. 392), contains the predicted times and heights of high water and low water for every day of the year at standard ports. High-water values for the day appear first followed by those for low water. There are usually two high waters and two low waters a day, but as the interval between successive high waters and low waters generally exceeds twelve hours, there are days on which only one high water or only one low water occurs. A line is then drawn in the space where the other high water or low water would otherwise be given. Times are given in the 24-hour system based on the zone time kept at the place. Heights are given in feet and tenths of a foot.

Turning to the extracts for Liverpool we note that on **May 3** the first high water was predicted to occur at **11 hrs. 18 mins.** when the height would be **29·6 ft.** The first low water for the day, found in the next column, occurs at **05 hrs. 52 mins.** when the height is **0·9 ft.** The second high water is given to occur at **23 hrs. 40 mins.**—*i.e.*, 12 hours 22 minutes after the first high water—and the height is **29·2 ft.** The second low water falls between the two high waters for the day and is given at **18 hrs. 17 mins.** with a height of **0·9 ft.,** the same as for the previous low water. From **11 hrs. 18 mins.** to **18 hrs. 17 mins.** there is an interval of 6 hours 59 minutes, during which time the tide is falling. This interval is known as the **duration of fall.** From **18 hrs. 17 mins.** to **23 hrs. 40 mins.** there is an interval of 5 hours 23 minutes, during which time the tide is rising. This interval is known as the **duration of rise.** Thus, in this case, the tide occupies much less time in rising than in falling, a common tidal characteristic in estuaries. On **May 4** high water is scheduled to occur at **12 hrs. 00 mins.** Only one high water is therefore available because the interval between it and the preceding high water is 12 hours 20 minutes, and the interval between it and the following high water is 12 hours 21 minutes. The high water occurring at **11 hrs. 18 mins.** on **May 3** is the highest for the half month, and must be regarded as the spring tide following new moon which occurs on **May 2.** Both the preceding and following low waters give heights of **0·9 ft.,** so that the range of both the rising and the falling tide is **28·7 ft.**

On **June 10,** at Liverpool, one high water occurs at **18 hrs. 00 mins.** with a height of **7·3 ft.** The duration of rise is just 6 hours 00 minutes, and the range **14·7 ft.** This is a neap tide, first quarter occurring on **June 9.**

At Immingham on **September 14** the first low water occurs at **00 hrs. 19 mins.,** with a height of **0·4 ft.,** and the following high water occurs at **06 hrs. 23 mins.,** with a height of **23·8 ft.** The duration of rise is thus 6 hours 04 minutes, and the range **23·4 ft.** The next low water occurs at **12 hrs. 46 mins.,** with a height of ***1·1 ft.**—*i.e.*, — 1·1 ft., or 1·1 ft. below chart datum. The range, still found by subtracting the heights algebraically, becomes 23·8 — (— 1·1), or 23·8 + 1·1—*i.e.*, 24·9 ft. It will be seen from a diagram that this must be so. This is a spring tide as full moon occurs on **September 12.**

Exercise 1 (*a*)

Find the time and height of high water on each of the following dates, and calculate the duration of the subsequent falling tide with its range:

	Place	*Date*
1.	Liverpool	July 5 P.M.
2.	Liverpool	August 9 A.M.

	Place	*Date*
3.	Immingham	December 18 P.M.
4.	Immingham	October 12 A.M.
5.	Greenock	January 5
6.	Greenock	March 4

Exercise 1 (*b*)

Find the time and height of low water on each of the following dates, and calculate the duration of the subsequent rising tide with its range:

	Place	*Date*
1.	Liverpool	June 20 A.M.
2.	Liverpool	May 3 P.M.
3.	Immingham	September 13
4.	Immingham	November 11
5.	Greenock	February 16 A.M.
6.	Greenock	April 3 P.M.

How to Find the Height of Tide at any Time. Values similar to those required for predicting the time and height of high water by the Admiralty method are entered on a special form B in order to calculate the height of tide at any other time. A detailed explanation is not called for here, but it may be found, together with a worked example, in Part III of the Admiralty Tide Tables.

Subsidiary tables with instructions for use are provided in Part I of the Admiralty Tide Tables for finding the height of the tide between the times of high and low water at standard ports. When the tide is perfectly harmonic in form a graphical method of finding the height of tide and the depth of water may be applied to standard ports. The problem is then referred to as a **reduction to soundings.**

Example. Find the depth of water on November 4 at 20.00 hrs. G.M.T. off Immingham where the chart indicates 5 fathoms.

As the tide is assumed to rise or fall harmonically, its progress may be represented by a semi-circle. The interval from high water may then be represented by an angle whose amount will be in the same proportion to 180° as the interval from high water is to the total time occupied in rising or falling. To state the proportion briefly:

$$\frac{\text{Reqd. angle}}{180^\circ} = \frac{\text{interval from high water}}{\text{duration of rise or fall}}.$$

Because the tide oscillates about a mean level the radius of the semi-circle will be the difference between the height of that mean level and the height of the tide's high water or low water—*i.e.*, the amplitude, or half-range.

Before proceeding with the graphical solution of the problem we must ascertain the above values by use of the Admiralty Tide Tables as follows:

	hrs.	*mins.*
Time of nearest high water	23	29
Time of nearest low water	17	20
Duration of rise . .	6	09
Time of high water .	23	29
Given time . . .	20	00
Interval from high water .	3	29

Height of nearest high water	17·4 ft.
Height of nearest low water	6·5 ft.
Sum	23·9 ft.
Mean tide level (half sum)	11·95 ft.
Diff.	10·9 ft.
Half-range (half diff.) .	5·45 ft.

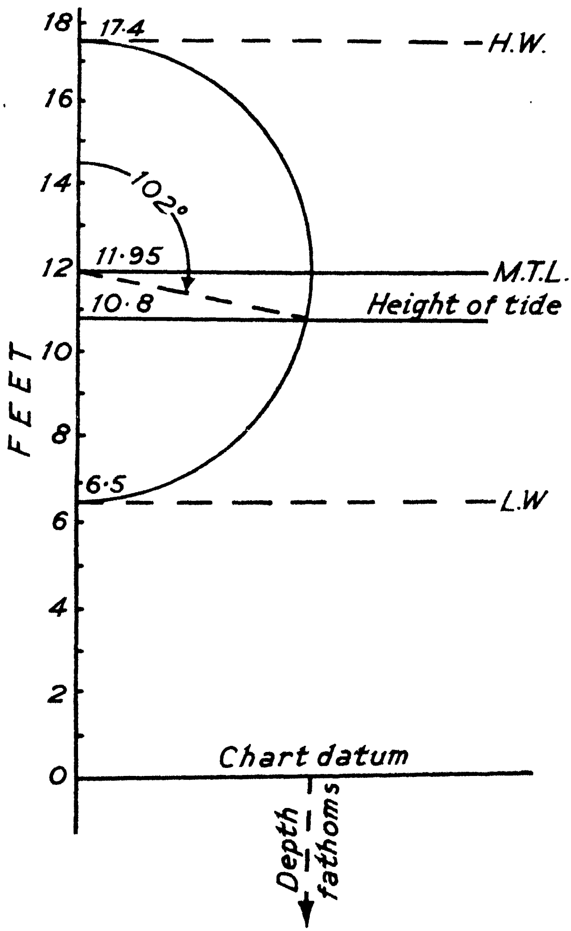


Fig. 33. Tidal Diagram (i)

Fig. 33 shows how the mean tide level of 11·95 ft. is marked on a tide gauge and used as the centre about which the semi-circle with radius equal to the half-range of 5·45 ft. is constructed. To find the height of tide at the given time we need now only calculate the angle corresponding to the interval and enter it on the diagram:

$$\frac{\text{Angle}}{180^\circ} = \frac{\text{interval of 209 mins.}}{\text{duration of 369 mins.}}$$

$$\therefore \text{Angle} = \frac{209 \times 180}{369} = 102^\circ.$$

This angle is now measured from high water and at the point of intersection with the semi-circle a horizontal line is drawn to meet the tide gauge, which then gives the height of tide seen from the diagram to be 10·8 ft. The depth of water is thus 30 + 10·8 = 40·8 ft. or 6¾ fathoms approx.

It should be carefully noted that had the level of low water been below chart datum, M.T.L. would have been ½ (high water height − low water height), while the half-range would have been ½ (high water height + low water height).

Exercise 2

Find the height of tide and depth of water where required in each of the following cases:

	Place	*Date*	*Standard time* *hrs.*	*mins.*	*Charted depth*
1.	Immingham	September 23	04	40	20 feet
2.	Immingham	December 10	20	15	6 fathoms
3.	Liverpool	May 13	03	42	30 feet
4.	Greenock	January 16	05	50	25 feet
5.	Greenock	March 29	04	30	4 fathoms
6.	Liverpool	July 31	22	20	5½ fathoms
7.	Liverpool	August 17	10	10	7 fathoms
8.	Immingham	October 13	08	20	8 fathoms

How to Find the Time at which there will be a Given Depth of Water. Provided the tide is harmonic, a similar method to that described above may be used to find the time at which there will be a given height of tide and depth of water.

Example. Find the P.M. time (G.M.T.) at which there will be 18 ft. of water above chart datum on a falling tide at Liverpool on August 20.

The same details as before are required from the Tide Tables, but this time we are unable to calculate the interval and hence the angle corresponding to it. Instead, the given height of tide is marked on the gauge and a horizontal line drawn from it to meet the semi-circle circumscribed about the mean tide level with the half-range as a radius as shown in Fig. 34. The point of intersection is then joined to the centre, so providing an angle which in its turn is used in the calculation to find its corresponding Interval as follows:

$$\frac{\text{Interval}}{\text{Duration}} = \frac{\text{angle from high water}}{180^\circ}$$

$$\text{Interval} = \frac{\text{angle} \times \text{duration}}{180^\circ}.$$

* From the Tide Tables we have:

	hrs.	mins.
Time of high water . .	15	20
Time of low water . .	22	04
Duration of fall . .	6	44

Height of high water. .	26·2 ft.
Height of low water . .	4·4 ft.
Sum	30·6 ft.
Mean tide level (half sum) .	15·3 ft.
Diff.	21·8 ft.
Half-range (half diff.) .	10·9 ft.

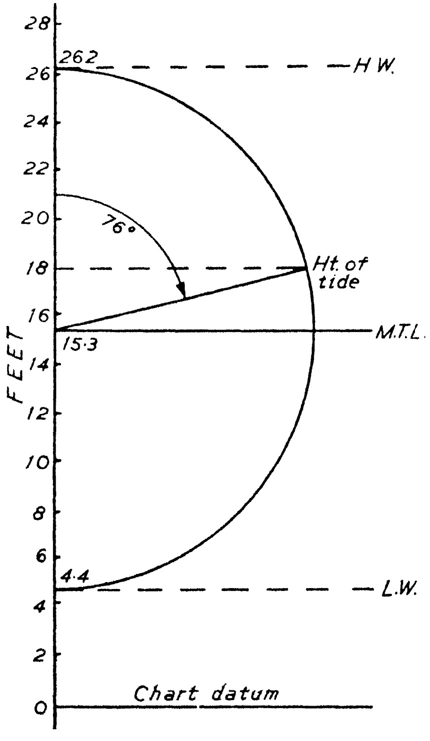


Fig. 34. Tidal Diagram (ii)

Fig. 34 shows that by using the above values in a tidal diagram a height of 18 feet gives an angle of 76°, so

$$\text{Interval} = \frac{76 \times 404}{180} = 171 \text{ mins.}$$
$$= 2 \text{ hrs. } 51 \text{ mins.}$$

The P.M. time (G.M.T.) at which there will be 18 ft. of water above chart datum on a falling tide will, therefore, be 15 hrs. 20 mins. + 2 hrs. 51 mins. = 18 hrs. 11 mins.

The selection of the correct high water and low water for use in the calculation depends upon whether the tide is rising or falling and is largely a matter of judgment which will be facilitated by practice. When a certain depth of water is required part of it will be given by the charted depth and part provided by the tide.

Exercise 3

Find the time at which the given depths of water will occur in each of the following cases:

	Place	*Date*	*Required depth (feet)*	*Charted depth*
1.	Liverpool .	May 20 A.M. falling	40	25 feet
2.	Liverpool .	July 17 P.M. rising	48	5 fathoms
3.	Immingham	December 26 A.M. rising	32	20 feet
4.	Immingham	October 13 A.M. falling	29	4 fathoms
5.	Greenock .	March 6 P.M. falling	20	16 feet
6.	Greenock .	April 17 P.M. rising	25	3 fathoms

Tidal Streams. In addition to the vertical movement of water produced in a tidal wave there is also a certain amount of horizontal displacement. This is known as a tidal stream. In the open sea and ocean such movement is comparatively small from the navigational point of view. The water above mean sea level moves in the direction of wave progression, while that below it moves in the reverse direction. There is thus no horizontal movement when the sea is at its mean level, and the water is then said to be **slack.** Under these conditions it is found that off-shore the tidal stream turns at half tide when the rate of rise or fall is at its greatest.

As the tidal wave from the ocean approaches shallow water it loses its harmonic form and tends to become steeper at the face than at the rear, while decreasing in wave length but increasing in amplitude.

In partially land-locked stretches of water the tidal stream of such a wave tends to flow towards the place at which the level is rising but away from the place at which the level is falling. Slack water thus occurs approximately at both high water and low water. A stream which turns within an hour of high or low water is known as a **flood stream** when the tide is rising, and as an **ebb stream** when the tide is falling. Otherwise the stream must be referred to as, north-going, south-going, ingoing, outgoing, etc.

The range of a tide will generally determine the rate of its stream, which, however, may be subject to the modifying influences of coast line and sea bottom. The stream is usually strongest when following the coast, and especially in narrow channels where it may reach a speed of eight knots. It enters estuaries and causes an indraught into wide bays, but may strike across small bays. Headlands tend to accentuate the rate of flow, while islands divide the stream into sub-streams which may develop turbulent eddies on reuniting. **Eddies** may be produced whenever two tidal streams meet. Large irregularities on the sea bottom may deflect the stream upward and so increase its rate of flow, thereby producing **overfalls** and **tide rips.** Where this action is acute, as off Portland Bill, a **tide race** may be the result. The flow of a river usually increases the rate and duration of the outgoing at the expense of the ingoing stream. A wind blowing with the stream may for a time prolong its duration and increase its speed, while a wind blowing against the stream may have the opposite effect. Other circumstances may, however, intervene to modify this. Away from the land in shallow seas the tidal stream tends to change its direction, frequently moving round through 360° during the course of a tide, clockwise in the northern and anticlockwise in the southern hemisphere.

The method used for predicting the direction and rate of tidal streams follows closely on the Admiralty method of tide prediction discussed earlier. The same values are used. It is fully explained in Part III of the Admiralty Tide Tables.

Tidal Peculiarities. Places inside the Isle of Wight often have four high waters instead of the usual two. At Southampton, for example, there are two high waters of about equal height with about two hours between them and then two more about twelve hours later. At Portsmouth instead of a double high water there is a prolonged stand of tide at high water. At Portland, farther west, there is a double low water. Part I, A.T.T., provides special tables for finding the height of tide at places inside the Isle of Wight. At Rosyth there is a prolonged stand of tide.

When a tidal wave enters a narrowing channel such as an estuary where the depth of water decreases it undergoes a change in shape, becoming steeper on the front slope than on the rear. The duration of rise is then much less than the duration of fall. If such a wave breaks it produces a **bore,** and more than half the total rise of tide may take place in a few minutes. Examples occur in the rivers Severn, Seine, and Hooghly.

Changes in atmospheric pressure affect the level of the tide. High pressure tends to reduce the height, whereas low pressure tends to increase it. Strong winds tend to affect the level of the tide as well as influence the tidal stream. In the North Sea westerly winds tend to raise the level and easterly winds to lower it. Seasonal weather also has an effect.

When the sun and moon are both over or near the equator, their declinations being about 0°, their tide-generating force is at its maximum and spring tides become greater than usual. Such tides are referred to as **equinoctial tides** or extraordinary spring tides. When at solstices the sun and moon tend to reduce spring tides.

Practical and Commercial Importance of Tides. Most industrial areas and world markets are located inland so that the farther a ship can penetrate into a country by river the easier becomes the problem of assembling or distributing her cargo. Without the aid of tides, however, many of our chief ports, centres of assembly and distribution, would be inaccessible to the large ocean-going ships of to-day. Tides thus enable overseas goods to be carried farther by ship, and thereby reduce the cost of transport. The raising of the water above its mean level often permits a vessel to avoid long and tortuous channels and so expedites her voyage. Lock gates often hold back the tide to enable the docks of our ports to maintain a sufficient depth of water and keep ships afloat at all states of the tide when loading or discharging cargo. This allows more ships to be accommodated in a port than would be the case if only riverside quays were available. The trapping of water in docks to keep ships afloat has the disadvantage that vessels cannot enter or leave at will, they must be passed through a lock or wait until the water level is the same outside as inside. The traffic of some ports is thus entirely dependent on the tides.

Neap tides bring their own peculiar problem. Ports which are normally accessible to deep-draughted vessels at or about the time of spring tides become unapproachable to these vessels at neap tides, owing to the reduced height of high-water level. Similarly ships which can enter a port in ballast on any tide may not be able to sail as desired when loaded. The intervention of neap tides may reduce for a time the maximum depth of water obtainable to less than that required to float the vessel safely from the port. Ships under these conditions are said to be **neaped.** Neap tides may be turned to advantage by shallow-draughted vessels when navigating a channel or coastal sea-route. Banks which could not be crossed at or near the time of low-water springs owing to the excessive low level of the tide may be safely negotiated at low-water neaps when the height of water level is greater.

Tidal streams may assist or obstruct a vessel on the move. If the directions of ship and stream coincide or nearly so, the ship's rate of progress will be increased, but should their directions be opposed the ship's progress will be hindered. With a little care the streams may, by arrangement, be made to assist more often than they obstruct. When moving in the direction of places at which the tide is rising it is possible to have the assistance of the stream for a much longer period of time than that taken by the stream to flow past a stationary object. This happens when south bound in the North Sea.

Tidal streams often act as erosive agents taking material from one part of the coast and depositing it elsewhere—usually where it is not wanted—with a complete disregard of all boundaries. Isolated rocky islands off the coast are usually signs of erosion, while sand-banks indicate deposition with its consequent obstruction to navigation. The inclination of a stream to carve out its own future channel is often so strong that it has to be kept within bounds by sea-walls and break-waters.

The tide is often used as a lifting agent during salvage operations when it is desired to raise a vessel from the sea bottom. Pontoons with an air capacity sufficient to support the sunken craft are attached to it by vertical wires at low water. When the tide rises it lifts the pontoons, which in turn raise the wreck off the bottom and allow it to be drawn into shallow water where the operation is repeated until the craft is above water at lower states of the tide.

Some use has already been made of tides to drive water-turbines for the generation of electricity. The time is probably not far distant when this natural source of power will, in the interests of economy, be tapped more extensively.

CHAPTER V

MAGNETISM AND THE COMPASS

Magnetic materials—Laws of magnetism—The earth as a magnet—Magnetic dip—Horizontal and vertical force—Magnetic variation—Application of variation—Magnetic compass—Pelorus—Deviation—Deviation card—Correction of courses and bearings—Swinging an aircraft—Adjustment of compasses—Turning and acceleration errors—Gyro compass—Distant-reading Gyro-magnetic Compass.

THE geographical pole has an attractive competitor, known as the magnetic pole, to which all compass needles owe allegiance. With the exception of minor distractions they point towards it with unremitting constancy. This is no new discovery; the Chinese are credited with a knowledge of it long before the Middle Ages, and used it to guide their junks across open sea. They had no needles as we know them but merely a piece of ore possessing magnetic qualities, which was floated on water. Some centuries elapsed before the properties of this magnetite, as the ore is called (after Magnesia in Asia Minor), became known and appreciated in the West. The information may have been carried to the Mediterranean by Arab navigators who traded with the East—as a sort of compass was used by them in the Mediterranean at the time of the Crusades—although it is more likely that the Vikings, who are credited with the use of a compass at an earlier date, introduced it.

Magnetic Materials. It is now generally known that the attractive quality of magnetite is possessed, in at least a latent form, by certain other metals, notably those of the ferrous group. This quality, said to be due to electronic activity, may be permanent, sub-permanent, or merely transient in iron. If you put a piece of iron in contact with magnetite you will cause it to exhibit qualities of attraction similar to those possessed by the magnetite, and those qualities will remain as long as contact is maintained. On removing the iron the qualities will disappear from it unless in the meantime it has been stroked by the magnetite or subjected to vibration such as is produced by hammering. When the quality remains we say that the iron has been magnetized, and two points of maximum intensity diametrically opposite may be found. These are known as the poles of the magnet and are named + and − or 'red' and 'blue' for distinction. Certain types of iron respond readily to magnetization while others are much slower to respond. But, easy come easy go; those which accept magnetism readily are just as ready to part from it. Such iron is known as **soft iron.** When resistance to magnetization has to be overcome the effect is much more durable, and the iron is then referred to as **hard iron.** To produce a permanent magnet, therefore, it is necessary to select a rod of hard iron, usually containing carbon, and subject it to treatment. One method is to place the iron on some insulating material and stroke it with a magnet from end to end. Magnetization will be speeded up, however, if the rod is placed across opposite poles of two magnets and stroked simultaneously from the centre to the ends by the opposite poles of two other magnets as illustrated by Fig. 35. Magnets may also be produced by passing an electric current through a wire wound round the iron rod to be magnetized. The direction of the winding will determine the name of the poles. If the winding be made clockwise the end of the rod reached first by the current will acquire blue polarity, but if the winding be made anti-clockwise the end reached first by the current will acquire red polarity. Poles of opposite name will appear in the other end, they cannot exist independently. Soft iron magnetized temporarily in this way is often used for loading and unloading iron or steel cargoes. While the electric current is running through the wire wound round the soft-iron core the soft iron will act as a magnet and lift a load, but as soon as the current is shut off the load is released.

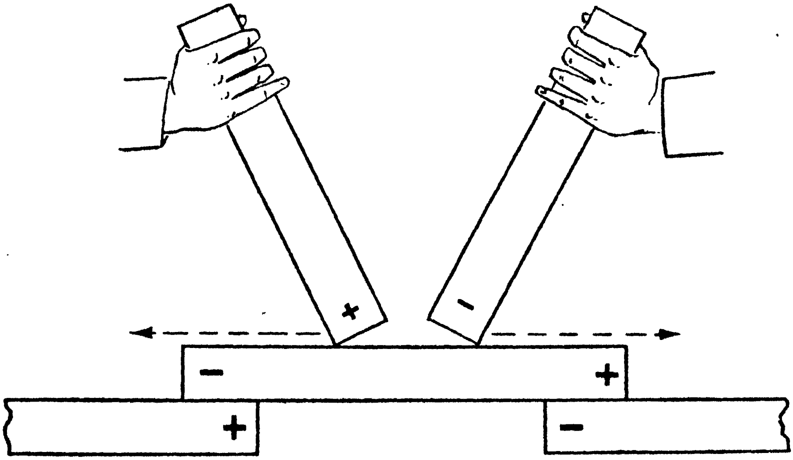


Fig. 35. Making a Bar Magnet

Laws of Magnetism. Magnetism does not exist just in the magnet but pervades the space immediately surrounding it. The extent of this sphere of influence or magnetic field, as it is called, depends upon the composition of the magnet and the intensity to which it has been magnetized. When fully magnetized it is said to be saturated. A piece of iron exposed in the field of a magnet acquires magnetism in the same way as when placed in contact with the magnet. Magnetism is then said to be induced into the iron, and poles will be found in line, but not necessarily in a straight line, with those of the magnet.

Fig. 36 shows how the bar magnet M induces magnetism into the soft-iron rod R. The poles nearest to each other are always of opposite name no matter how the rod is turned and a state of attraction is found to exist between them. The directions followed by the attractive force are indicated in Fig. 36 by pecked lines. These are known as **lines of force,** and are seen to leave the magnet by one pole and, after following a curved path, re-enter it by the other. If the iron rod were replaced by a permanent magnet with poles in the same position as shown for R, the state of attraction between + and — poles would increase in intensity due to the increase in combined pole strength. If, however, the permanent magnet were then reversed so that two positive poles faced each other, a state of repulsion would exist between them because the continuity in direction of the lines of force would be broken. We thus have our first law, that **poles of unlike name attract each other,** whereas **poles of like name repel each other.** The nearer the poles are placed together the greater the attractive force or repulsive force will be. **The force between isolated poles varies in inverse proportion to the square of their distance apart.** This is our second law. It means that if the distance between the poles be halved the force between them will be quadrupled, and if the distance be trebled the force will be reduced to one ninth.

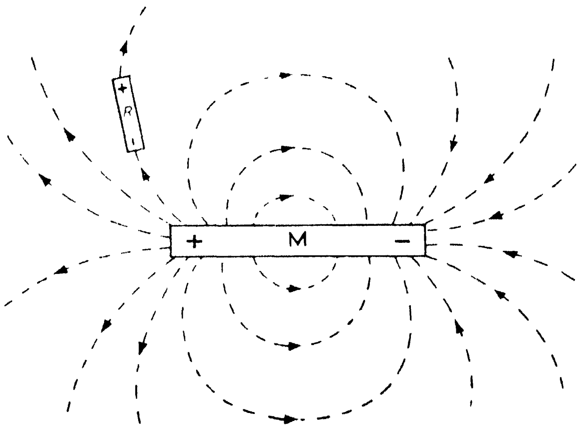


Fig. 36. LINES OF FORCE

The Earth as a Magnet. Without the co-operation of the earth no general principle of magnetic direction could be enunciated. Fortunately, however, the earth possesses magnetic qualities and acts very much as if a magnet were situated well below the surface, as illustrated in Fig. 37. The reason why the earth is a magnet at all is not fully understood, despite careful research. Experiment shows us that the earth's field is not strong, nor is it uniform. Further, it is peculiar in that the points of maximum intensity, or magnetic foci, are not at the poles but at four positions near them, two in each hemisphere. Fig. 37 shows how the lines of force extending beyond the earth cut its surface at angles which vary in size according to position. The line of force which meets the earth's surface vertically indicates the positions of the magnetic poles. The negative, or blue, pole is situated in the northern hemisphere in about latitude 70° 05′ N. longitude 96° 46′ W., N in Fig. 37, and the positive, or red, pole in approximately latitude 72° 25′ S. longitude 155° 16′ E., S in Fig. 37. The earth's magnetic poles are thus not diametrically opposite nor are their positions constant. They may be regarded as rotating round the geographical poles in a westerly direction, the 'blue' pole describing a circle of 17° radius in about 960 years. In 1945 the R.A.F. Lancaster "Aries," on a research flight, ascertained the north magnetic pole to be situated in Sverdrup Island, 400 n.m. north of Boothia Pensinsula, its previously estimated locality. At certain positions on the earth's surface the lines of force will be horizontal, as at A, Fig. 37. A line passing through all such positions is known as the **magnetic equator.** This is shown in Fig. 38 to cross the geographical equator in about longitudes 20° W. and 145° W., reaching its maximum south latitude of about 15° in Brazil and maximum north latitude of about 8° in Africa.

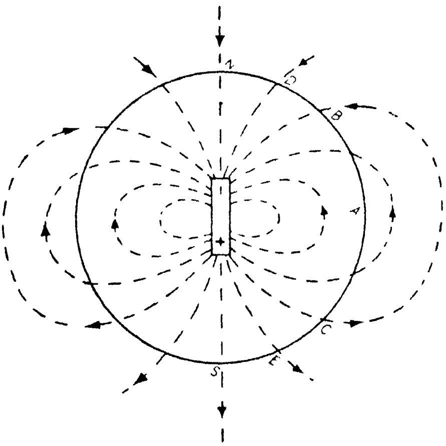


Fig. 37. The Earth's Magnetic Field

Magnetic Dip. If a thin magnetic rod or needle were freely suspended at its centre of gravity, its poles, responding to those of the earth, would take up positions along the earth's line of force passing through the place. Its positive, or red, pole would be directed towards the earth's negative, or blue, pole and would thus be **north seeking,** while its negative, or blue, pole would be directed towards the earth's positive, or red, pole and be **south seeking.** It will be seen from Fig. 37 that at the earth's magnetic poles the needle would be vertical—*i.e.*, at an angle of 90° to the surface—while at A and any other place along the magnetic equator it would be horizontal.

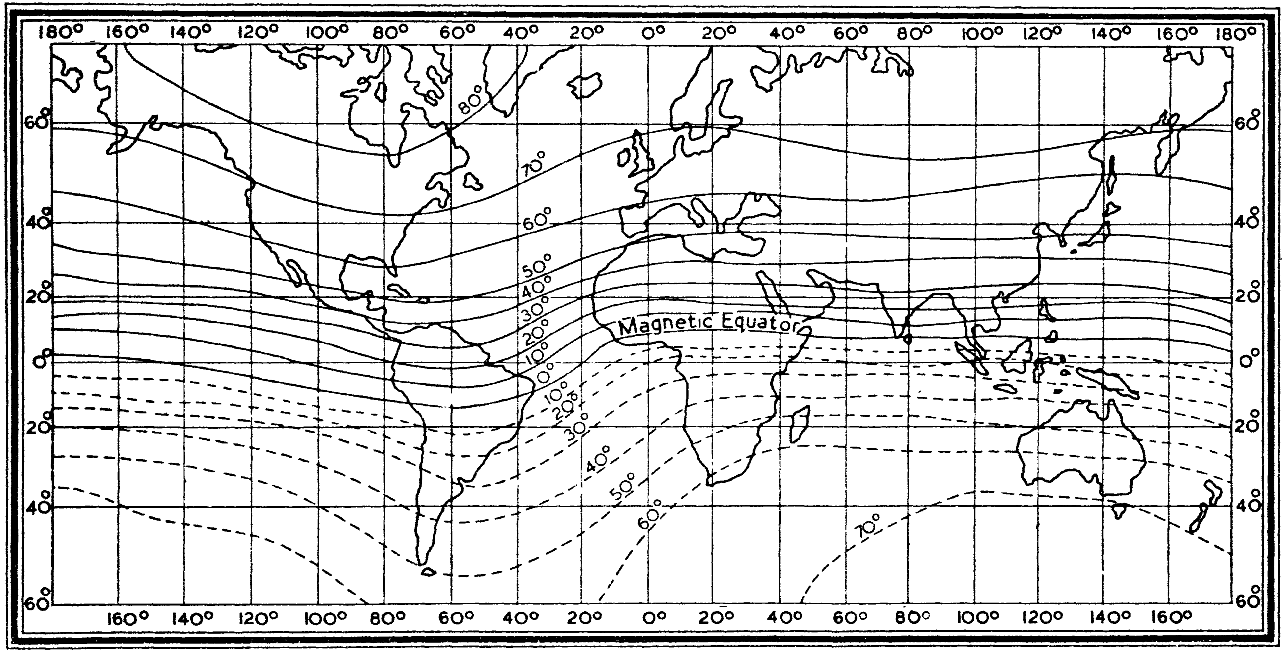


Fig. 38. Lines of Equal Magnetic Dip, 1922

Reproduced from Admiralty Chart No. 2598 with permission of H.M. Stationery Office and the Hydrographer of the Navy.

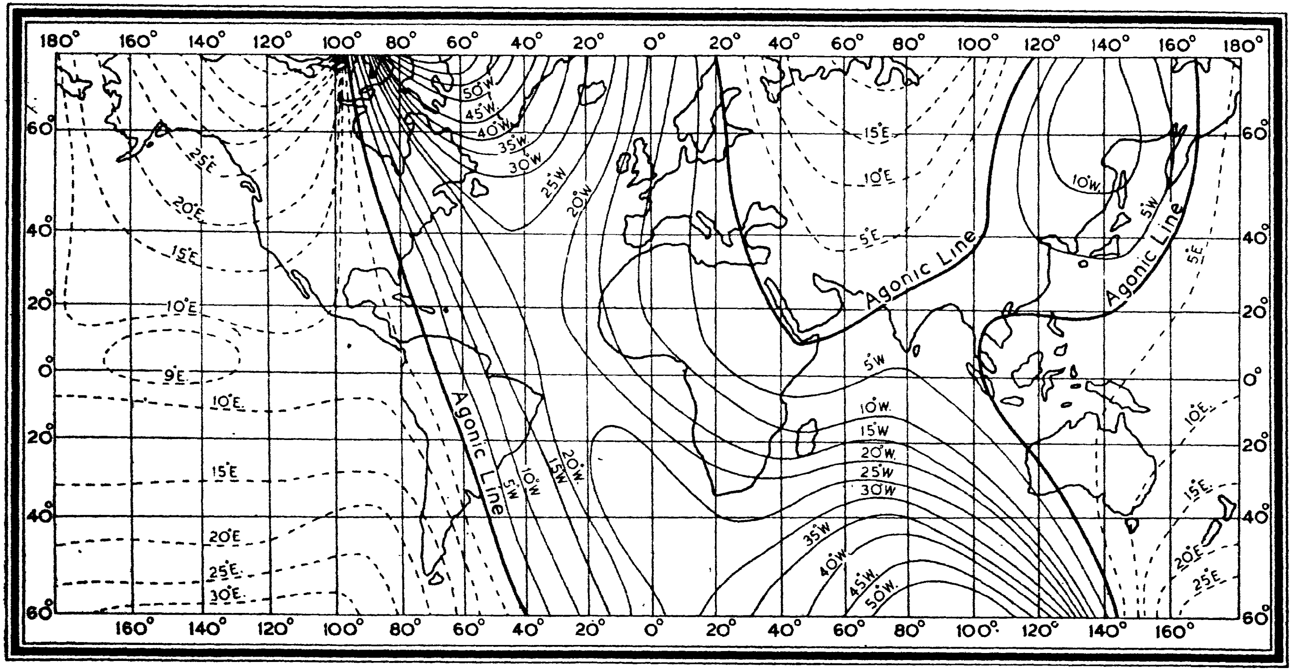


Fig. 39. LINES OF EQUAL MAGNETIC VARIATION, 1937

Reproduced from Admiralty Chart No. 3598 with permission of H.M. Stationery Office and the Hydrographer of the Navy.

Between these two extremes, at B, C, D, or E, for example, the angle would be oblique to the surface. The angle between a freely suspended magnetic needle and the horizontal plane at a place is **magnetic dip.** Lines passing through all places of equal magnetic dip are known as **isoclinal lines,** and Fig. 38 shows these at intervals of 10°. They indicate magnetic latitude, and are drawn plain and named + when north of the magnetic equator and drawn pecked and named — when south of it.

Horizontal and Vertical Force. When the angle of dip is 90° —*i.e.*, at either magnetic pole—the earth's total magnetic force acts vertically on the needle, but when the angle of dip is 0°—*i.e.*, at the magnetic equator—the earth's total magnetic force acts horizontally on the needle. When the angle of dip is more than 0° but less than 90° the earth's total force acts obliquely and may be resolved into horizontal and vertical components H and Z respectively. In Fig. 40 O represents the position of the observer at an intermediate position, and OA the direction taken up by a magnetic needle freely suspended at its centre of gravity. The needle follows the direction of the earth's line of magnetic force at that point. This direction is not in the plane of the geographical meridian OG, nor is it horizontal nor vertical. If, in Fig. 41, OA be drawn to scale to represent the earth's total magnetic force, then the horizontal line OB in the same plane will represent the horizontal force H to scale when it meets the vertical

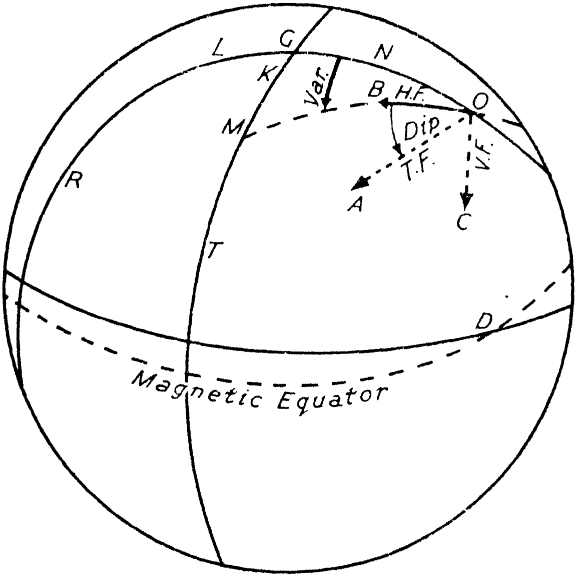


Fig. 40. Elements of Terrestrial Magnetism

line through A. Similarly the vertical line OC will represent the vertical force Z to scale when it meets the horizontal plane through A. It will be understood from Fig. 41 that, if the earth's total magnetic force be assumed to remain constant, the vertical component represented by OC will increase and the horizontal component represented by OB will decrease when the angle of dip increases. Now, because BO and AC are horizontal and OC and BA vertical, the figure OCAB must be a rectangle in which BA = OC. In the right-angled triangle OAB, BA may be taken to represent the vertical component of the earth's total magnetic force at O. Given, then, two values in the triangle, in addition to the right angle, the other values may be found from a scale drawing. Mathematically the vertical component varies as the sine of the dip, while the horizontal component varies as the cosine of the dip. The ratio of vertical component to horizontal component is, therefore, as the tangent of the dip—*i.e.*, $\frac{\text{V.F.}}{\text{H.F.}}$ = tangent dip.

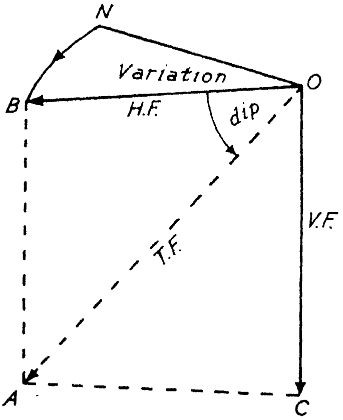


Fig. 41. Elements of Terrestrial Magnetism

Example. Given H.F. = ·34 when magnetic dip = 30°, find H.F. when magnetic dip = 65°.

$$\frac{\text{Reqd. H.F.}}{\text{Given H.F.}} = \frac{\text{cosine } 65^\circ}{\text{cosine } 30^\circ}.$$

I.e., reqd. H.F. = Given H.F. × cos. 65° × sec. 30°.

Given H.F. = ·34 log. = $\bar{1}\cdot531479$
Mag. dip = 65° l.cos. = $\bar{1}\cdot625948$
Mag. dip = 30° l.sec. = $0\cdot062469$

Reqd. H.F. = ·166 log. = $\bar{1}\cdot219896$

Example. Given H.F. = ·26 when magnetic dip = 50°. Find the V.F.

$$\frac{\text{V.F.}}{\text{H.F.}} = \text{tan. dip}$$

I.e., V.F. = H.F. × tan. dip

Given H.F. = ·26 log. = $\bar{1}\cdot414973$
Given dip = 50° l.tan. = $0\cdot076186$

Reqd. V.F. = ·31 log. = $\bar{1}\cdot491159$

The horizontal component of the earth's total magnetic force is that which provides a compass with its directive force and is, therefore, extremely important in navigation. Lines joining places of equal horizontal force are known as isodynamic lines. The vertical component, when effective, acts rather as a disturbing force.

Magnetic Variation. A magnetic needle lying along the earth's line of force passing through a place indicates the direction of the **magnetic meridian,** which we may define as **a horizontal line in the same vertical plane as a magnetic needle under the influence of the earth's magnetic field only.** In Fig. 40 the triangle OAB lies in the plane of the magnetic meridian passing through O. The horizontal line ON, however, lies in the plane of the geographical meridian. The horizontal angle between the two planes is known as **magnetic variation,** which is **the horizontal angle between the geographical meridian and the magnetic meridian.** Angle NOB, therefore, represents the magnetic variation at O. Because the direction of magnetic north from O lies to the left, or westward, of true north indicated by the geographical meridian, the variation is named westerly. If the direction of magnetic north lay to the right of true north, the variation would be named easterly. As direction is usually measured clockwise from true north westerly variation is sometimes named negative and easterly variation positive. Variation in Great Britain is westerly because the magnetic pole situated in northern Canada, lies to the westward of our true north. Its amount at Greenwich is now about 9½° W. (1946), although in 1580, the earliest record, it was 11° 15′ E., since when it became zero in 1659, and then increased to 24½° W., its maximum, in 1823. The present rate of decrease at Greenwich is estimated at 10′ annually. The change may be regarded as largely due to the rotation of the magnetic pole westward about the geographical pole, although the reason for such rotation is not yet understood. Cape Agulhas (South Africa) derives its name from the fact that when discovered by Bartholomew Diaz in 1486 the variation there was nil (Portuguese, *agulha*, needle).

Lines joining places of equal variation are known as **isogonal lines** (Greek, *isos*, equal; *gonia*, angle). Fig. 39 shows their arrangement for the year 1937 on a Mercator chart of the world between latitudes 70° N. and 60° S., westerly variation being indicated by continuous lines and easterly variation by pecked lines. It will be noticed that the isogonals have little regularity, and in two instances even enclose an oval, although the tendency of many is to run northward and southward. When the variation is required for a certain position we take it from the chart according to the latitude and longitude of the place. On the variation chart of the world it will often be necessary to interpolate between the values given by the isogonals whenever the position does not lie on one of them. For instance the variation for

Land's End would fall between 10° W. and 15° W. and be 13° W. to the nearest degree. On larger scale charts, such as ocean charts, the variation is given for each degree and may be taken off to the nearest half degree. Isogonals are not usually employed to indicate variation

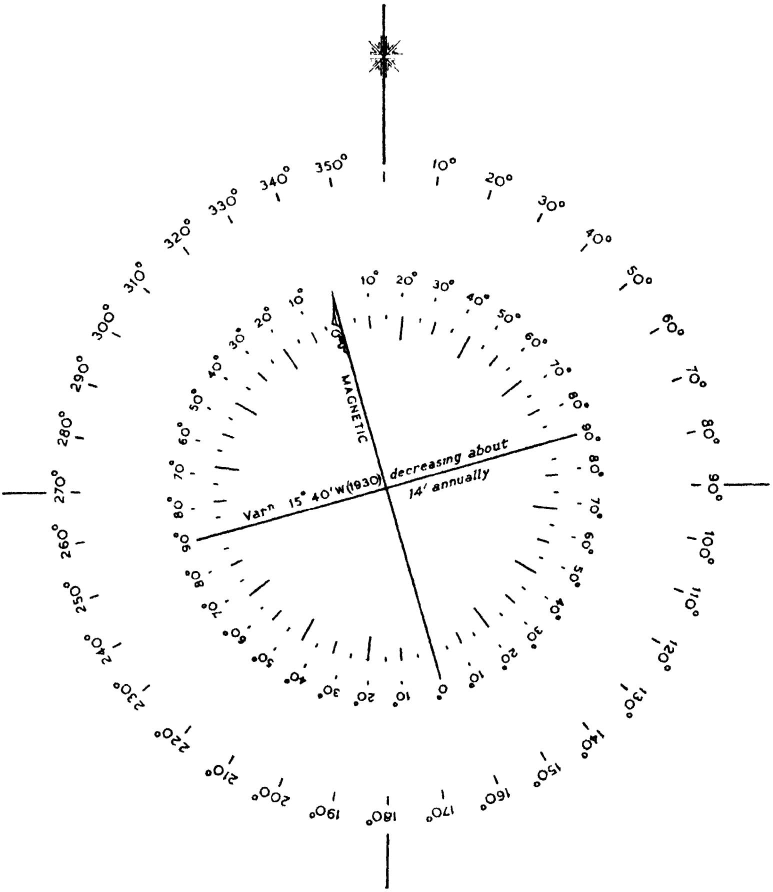


Fig. 42. The Compass Rose

on the largest scale marine charts. Instead, compass roses, similar to that shown by Fig. 42, are engraved at intervals. The outer ring of figures gives the direction in degrees measured clockwise from true north. The inner ring of figures gives the direction in degrees measured clockwise and anti-clockwise from north and south magnetic to east and west magnetic—*i.e.*, in quadrants. The innermost markings indicate the points and half points of the compass. A comparison of these with degrees is given by Fig. 47, below which their relative values are tabulated.

The value of the variation—*i.e.*, angle between the directions of true north and magnetic north—is given in Fig. 42 as 15° 40′ W. in 1930, since when it has been decreasing at the rate of 14′ annually. To find the value of the variation in this particular position at a later date, the annual decrease would have to be multiplied by the number of elapsed years since 1930. For example, 14 years elapsed up to 1944 so that the accumulated decrease would be 14 × 14′ = 196′ = 3° 16′, which when subtracted from 15° 40′ W. gives 12° 24′ W. as the variation in 1944.

Exercise 1

Bring the variation up to date for the year 1944 in the following cases:

	Year	*Variation*
1.	1920	20° 30′ W. decreasing 12′ annually.
2.	1914	17° 45′ E. increasing 8′ annually.
3.	1928	1° 20′ W. decreasing 10′ annually.
4.	1931	9° 40′ W. increasing 6′ annually.
5.	1937	12° 55′ E. increasing 14′ annually.
6.	1925	2° 00′ E. decreasing 11′ annually.

The value of variation on plotting sheets and maps used in the air is always given by means of isogonal lines at intervals of 1° on the former, and of 30′ on the latter. The rate of annual change is given in the margin. Owing to the limitations of the Mercator projection Fig. 39 does not indicate a variation greater than 50°. Variation charts of polar regions, however, show values greatly in excess of this; in fact, between the geographical and magnetic poles the variation reaches its maximum of 180° E. or W. Lines joining places having no magnetic variation are known as **agonic lines.** If you were to start from the agonic line passing through a place T in the U.S.A. (Fig. 40) and travel westward round the north magnetic pole, your variation would be easterly at R and increase as you proceeded towards L, where it would be about 90° E., until at some position K between the poles it would reach its maximum, and change to westerly variation, which would decrease until you reached your starting point at which it would again be nil.

Application of Variation. Having found the variation for a place it is a simple matter to use it in converting a magnetic direction to a true direction, and vice versa. If the magnetic direction is given draw a line to represent the magnetic meridian, OM in Fig. 43, and a second line OX to show the given direction relative to magnetic north. Suppose the variation to be 14° W., the direction of true north, OT, can be drawn in by making an angle of 14° with OM so that OM lies to the westward of OT, the variation being westerly. All that remains is to measure the angle TOX, and the direction of X from O relative to true north is found.

Using the three-figure method of graduation, MOX, the magnetic direction measured by the rotation of OM clockwise, will be 300° M., TOX, the true direction measured clockwise, will obviously be 14° less—*i.e.*, 286° T. When converting magnetic directions to true directions we have, therefore, the equation:

$$\text{true} = \text{magnetic} \begin{matrix}(-\text{ W.})\\(+\text{ E.})\end{matrix} \text{ variation, or briefly } T = M \begin{matrix}(-\text{ W.})\\(+\text{ E.})\end{matrix} V,$$

or, in couplet form:

Variation west, Magnetic best.
Variation east, Magnetic least.

If both true and magnetic directions are given, the difference in angular value between them is, of course, the variation, whose name may be determined by asking yourself whether it is necessary to add the variation to or subtract it from the true direction in order to obtain the magnetic direction. When additive to true it must be westerly, and when subtractive, easterly.

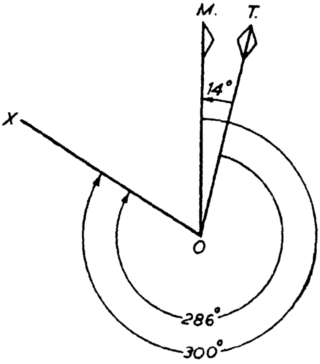


Fig. 43. MAGNETIC VARIATION

Exercise 2

Complete the following table:

True direction	*Variation*	*Magnetic direction*
144°	12° W.	
266°	8° E.	
352°	14° W.	
005°	10° E.	
	11° E.	084°
	17° W.	183°
	10° W.	007°
	19° E.	355°
210°		195°
236°		242°
004°		348°
359°		008°

Magnetic Compass. The modern magnetic compass consists of a non-magnetic metal bowl in the centre of which is a rigid vertical support for balancing the compass system. The point of the support is sharp and generally tipped with some very durable metal such as iridium. To provide a bearing as frictionless as possible, the compass system is fitted with a jewel, for example, agate or sapphire, and the weight of the system reduced to a minimum.

An even number of needles is used so that they may be arranged

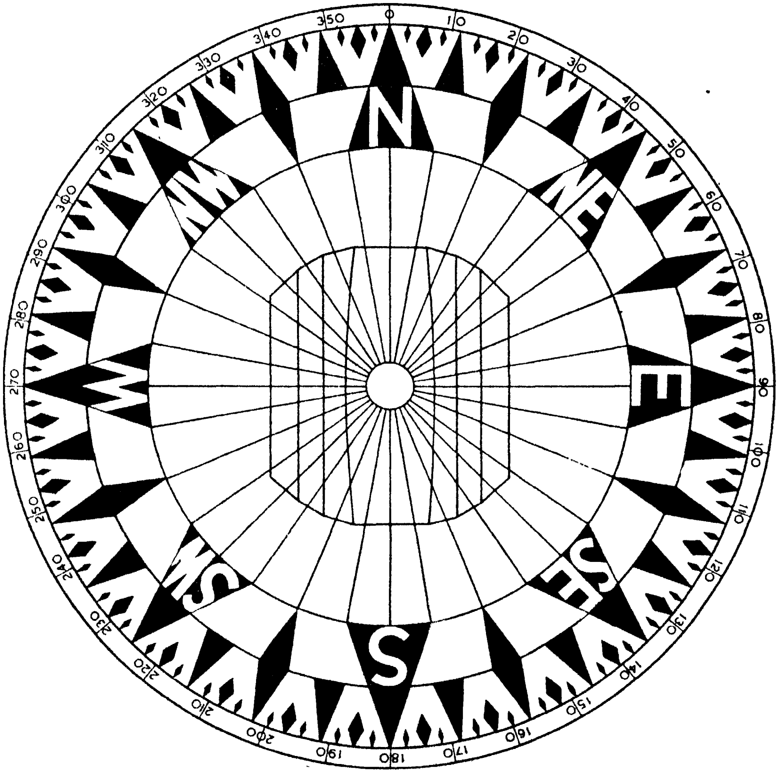


Fig. 44. Compass Card: Kelvin Pattern

symmetrically on either side of the pivot. One large needle would be unwieldy and produce an undesirable magnetic field around the compass. In marine compasses and the aircraft observer's compass the needles are attached to a compass card. Fig. 44 illustrates the method of attachment in the marine dry compass, where the card consists of rice paper and silk threads surrounded by an aluminium rim. The weight of the whole, including needles, is about 200 grains.

Because horizontal direction is desired the compass card is constrained to remain horizontal by having its point of support well above its centre of gravity—the point through which its total weight is assumed to act. Fig. 45 shows how the turning power, or magnetic couple about the pivot P produced by the interaction of the earth's vertical component and a magnetic needle is balanced by the weight of the card acting vertically downward through G, its centre of gravity, about the same pivot. G is constantly endeavouring to take up a position vertically below P and thus acts like the ball of a pendulum suspended from P. Fig. 45 (i) shows the position of the needle when at rest, balanced by the magnetic forces acting at R and B and the mechanical force acting at G. The angle formed by the needle and the horizontal plane is then not likely to exceed 3° even in high latitudes. Fig. 45 (ii) shows the needle tilted downward and the mechanical force through G in action to right it. As the compass depends for direction on the earth's horizontal force it will be more reliable when in low magnetic latitudes where the earth's H.F. is greater, and less reliable in high magnetic latitudes where the H.F. is smaller.

When assembled the marine dry compass appears as shown in Fig. 46. It is 'shipped' in a binnacle, a stand in the fore-and-aft line of the ship, and rests on gimbals to keep it horizontal when the ship rolls or pitches. The card has a diameter of about 10 inches and is graduated along the edge into degrees and points. The degrees may be numbered clockwise from 000° at north, through 090° at east, 180° at south, 270° at west back to 360° at north, or in quadrants from 0° at north and south to 90° at east and west. The three-figure method is now almost universally used in the R.N. and R.A.F., but in the M.N. direction is still often reckoned in quadrant measurement or in points. A comparison of all three methods is provided by Fig. 47, below which the values are tabulated. In order to steer a ship in a prescribed direction by compass the ship's head must be brought into line with the centre of the compass and the degree or point required. For this purpose a line is drawn on the inside of the fixed compass bowl to represent the ship's head. This line, called the **lubber's line,** is indicated in Fig. 46. It marks the ship's **course by compass, the angle between the compass needles and the fore-and-aft line of the ship.** The compass direction of any object outside the ship is obtained by means of an azimuth mirror, a prism mounted horizontally on a frame, Fig. 48. By rotating the prism the reflection of an object above the horizontal is cast on to the compass card from which the **compass bearing—horizontal angle between the compass needles and the direction of the object**—is read. To obtain the compass bearing of an object near the horizontal plane a reflection of the compass-card graduations is brought up into line with the object by manipulation of the prism.

Craft liable to excessive vibration or gyratory movement are

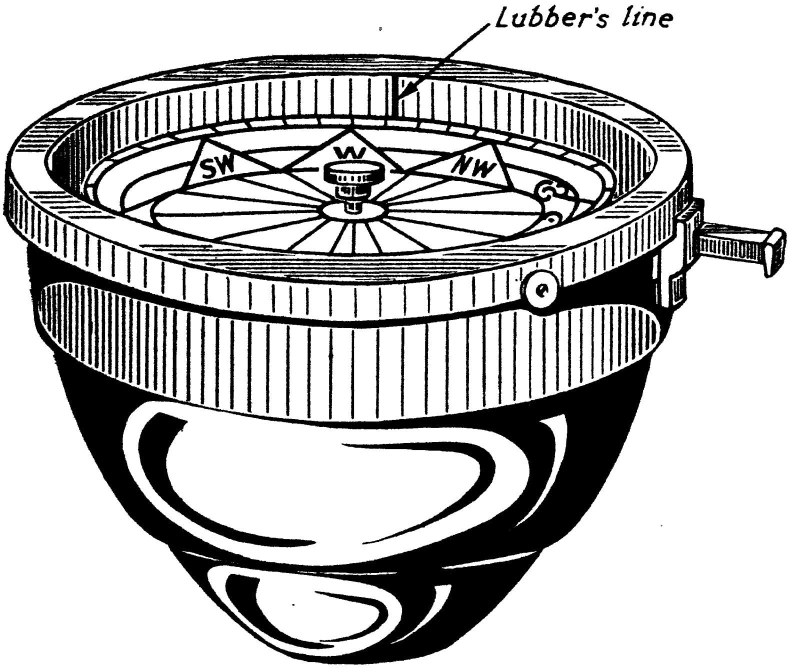


Fig. 46. MARINE DRY COMPASS

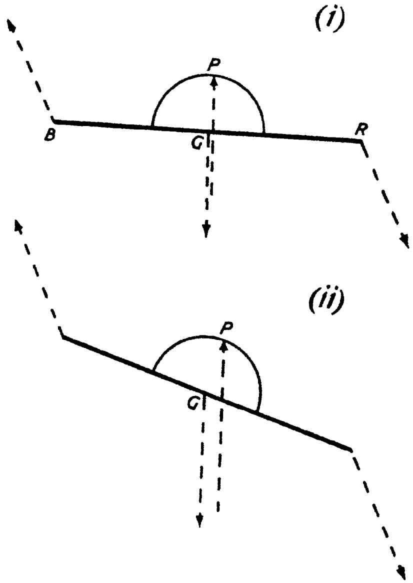


Fig. 45. BALANCE OF COMPASS NEEDLE

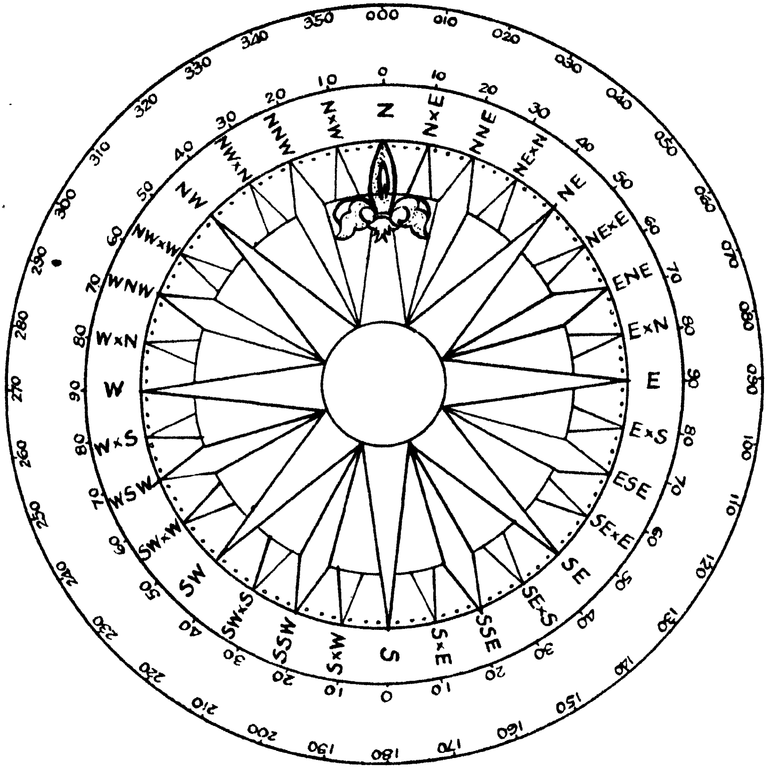


Fig. 47. Compass Card Graduations

Points of the Compass with their Equivalents in Degrees

North	N. 0° E. or W.	000° 00′	S. by W.	S. 11¼° W.	191° 15′
N. by E.	N. 11¼° E.	011° 15′	S.S.W.	S. 22½° W.	202° 30′
N.N.E.	N. 22½° E.	022° 30′	S.W. by S.	S. 33¾° W.	213° 45′
N.E. by N.	N. 33¾° E.	033° 45′	S.W.	S. 45° W.	225° 00′
N.E.	N. 45° E.	045° 00′	S.W. by W.	S. 56¼° W.	236° 15′
N.E. by E.	N. 56¼° E.	056° 15′	W.S.W.	S. 67½° W.	247° 30′
E.N.E.	N. 67½° E.	067° 30′	W. by S.	S. 78¾° W.	258° 45′
E. by N.	N. 78¾° E.	078° 45′	West	N. or S. 90° W.	270° 00′
East	N. or S. 90° E.	090° 00′	W. by N.	N. 78¾° W.	281° 15′
E. by S.	S. 78¾° E.	101° 15′	W.N.W.	N. 67½° W.	292° 30′
E.S.E.	S. 67½° E.	112° 30′	N.W. by W.	N. 56¼° W.	303° 45′
S.E. by E.	S. 56¼° E.	123° 45′	N.W.	N. 45° W.	315° 00′
S.E.	S. 45° E.	135° 00′	N.W. by N.	N. 33¾° W.	326° 15′
S.E. by S.	S. 33¾° E.	146° 15′	N.N.W.	N. 22½° W.	337° 30′
S.S.E.	S. 22½° E.	157° 30′	N. by W.	N. 11¼° W.	348° 45′
S. by E.	S. 11¼° E.	168° 45′	North	N. 0° E. or W.	360° 00′
South	S. 0° E. or W.	180° 00′			

supplied with liquid-filled compasses. The liquid acts as a brake by exerting a damping effect on the compass system and thus increases the period of vibration. Liquid also reduces the weight of the system because it provides an immersed body with partial support in just the same way as it provides a floating body with full support. Friction is thereby reduced at the pivot. The liquid must fill the bowl completely, there must be no bubbles. It must be transparent so that the immersed compass card or pointers are plainly visible; it must be as non-viscous as possible so that the compass system is not carried round with the bowl; it must have a low freezing point, as a frost-bound compass would not function. Expansion of the liquid is allowed for by the incorporation with the bowl of a bellows-shaped expansion chamber near the bottom. A cheap liquid satisfying these conditions is obtained by mixing alcohol and distilled water in the ratio of one to two respectively. Aircraft compasses usually contain absolute alcohol to

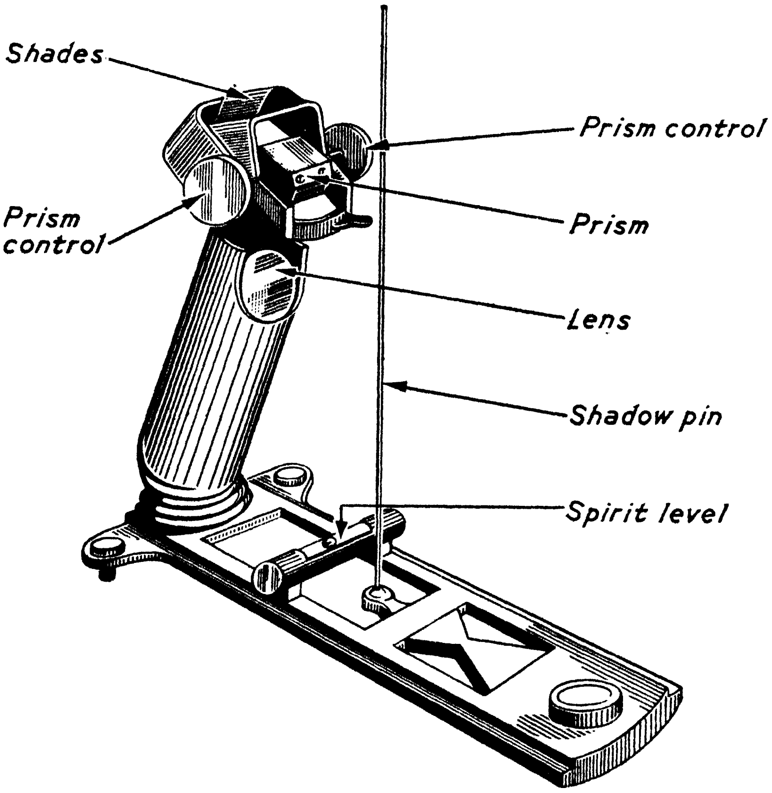


Fig. 48. Azimuth Mirror

resist the effect of excessively low temperatures. In order to reduce the effect of eddies at the edges of the bowl the diameter of the card is reduced. An aircraft pilot's type of compass has no card in the liquid, although pointers indicating the cardinal and quadrantal points are attached to the compass system to assist the damping, the cardinal points being luminous and north marked by a cross-piece.

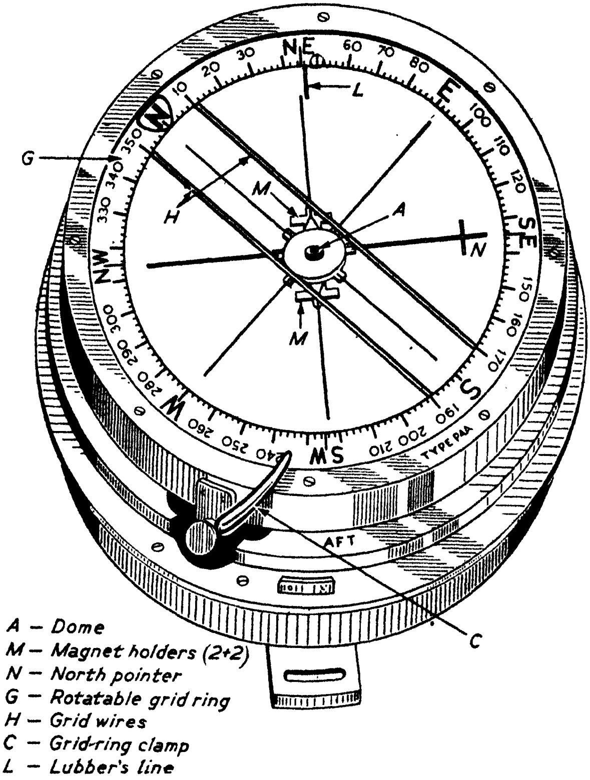


Fig. 49. Pilot's Type Compass

The degrees are graduated on a grid ring, which is rotatable and turned until the desired course is opposite the lubber's line. The aircraft is then turned to bring north by grid ring on to north by compass, whereupon the aircraft will be found to be heading in the required direction. Liquid compasses are now so efficient that when the needles are deflected they return to their original direction without vibrating. They are then said to be **aperiodic,** or 'dead beat.'

Fig. 49 illustrates the appearance of a pilot's type compass used in aircraft. A indicates the dome to which the small permanent magnets M and damping wires are attached; N is the north pointer, G the

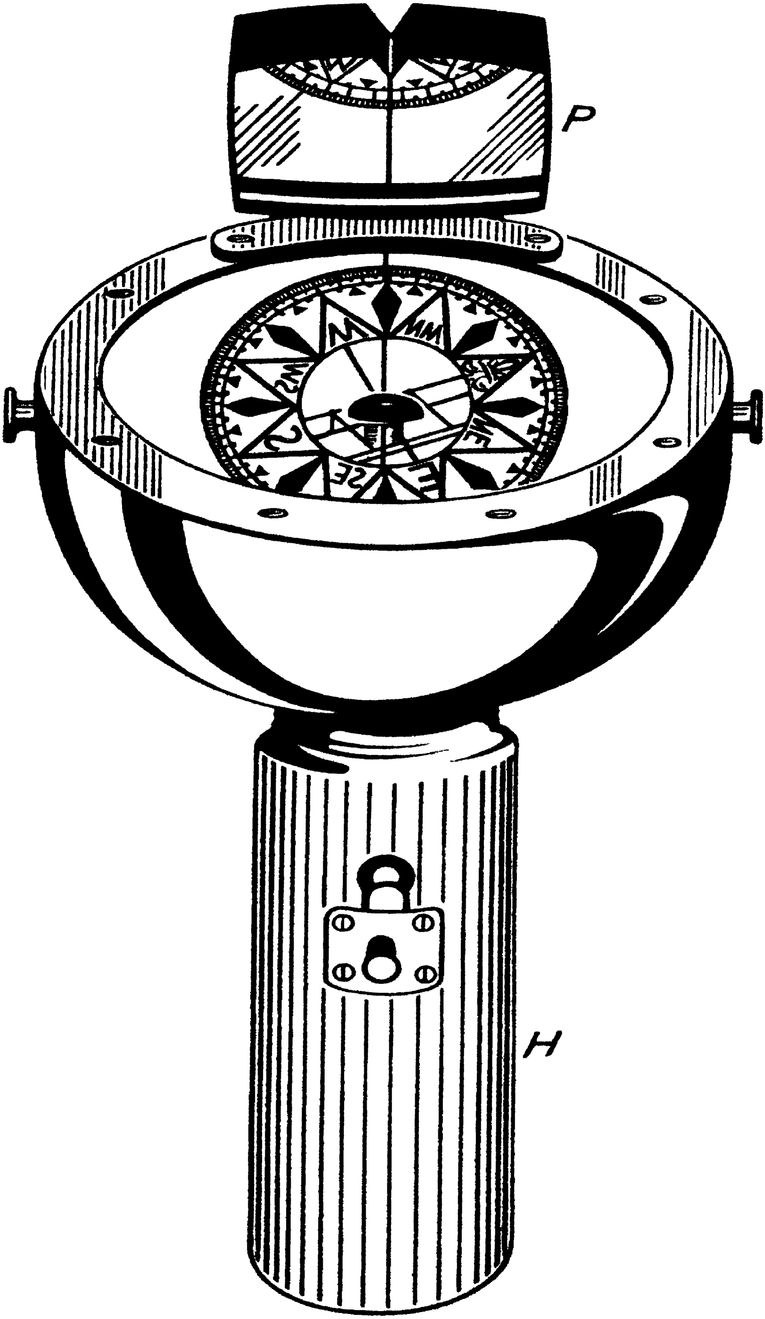


Fig. 50. Hand-bearing Compass

rotatable grid ring graduated for every two degrees, H two parallel grid wires rotatable with the grid ring, C the grid-ring clamp, and L the lubber's line. The bowl rests on helical springs inside the container to reduce vibration. A floating ring prevents its turning inside the container, and a lid ring prevents its falling out when reversed. The container is attached to the aircraft by three lugs. The graduations on the grid ring, the grid wires, the damping wires, and the lubber's line are all luminous to assist reading the compass in the dark. The grid wires enable the pilot to maintain course even when not seated directly behind the compass as it is only necessary for him to keep the north pointer parallel with the grid wires.

The bearing or observer's compass used in aircraft also contains liquid and may be portable or fixed to the aircraft. Fig. 50 illustrates the former type. To the dome is attached the compass card under which the needles are arranged; P is a fixed prism fitted with open sights. The graduation numerals and letters on the card are reversed so that when reflected and read through the prism they appear correct. The handle H contains a battery and electric bulb by means of which the graduations are illuminated at night.

Pelorus. When an all-round view is not possible from a compass, bearings may be obtained by means of a reference plate or dumb compass card which is portable and fitted with sights. This is known as a pelorus, and when required for use must be placed so that a line joining the centre of its card to its lubber's line is parallel with the ship's fore-and-aft line. Fig. 51 is an illustration of this instrument. The compass card is graduated in the usual way, and both it and the sights are rotatable about its centre and capable of being clamped.

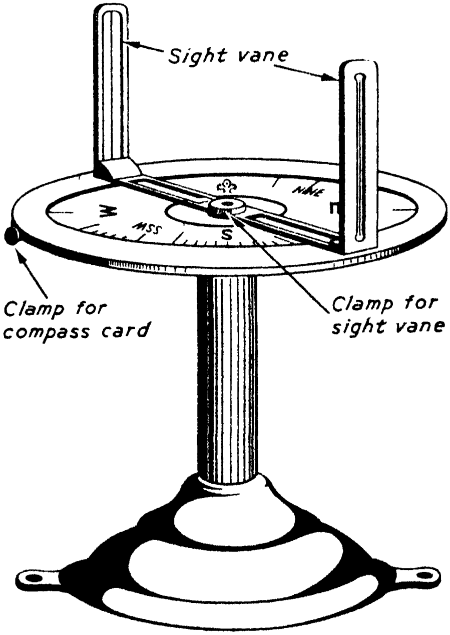


Fig. 51. Pelorus

To obtain a compass bearing the course being steered by compass is set at the lubber's line of the pelorus. The sights are then rotated until in line with the object, when the bearing can be read direct from the card.

To place the ship's head on a particular magnetic course by means of an object whose magnetic bearing is already known, the required course should be set on the pelorus and the sights clamped to the known bearing. The ship should then be turned until the object comes into line with the sights, when the craft will be heading in the required direction.

Deviation. If an aircraft or ship were built entirely of wood or non-magnetic metal we could expect the 'red' end of the compass needle to point to magnetic north, but owing to the presence of some form of iron or steel in almost all craft (research craft being a notable exception), the needle is usually deflected to the right or left of magnetic north.

The angle at the compass between the direction of the magnetic meridian and that of the compass needle is magnetic deviation. It is named easterly when the north-seeking end of the needle is drawn to the right, or eastward, of the magnetic meridian and westerly when the north-seeking end of the needle is drawn to the left, or westward, of the magnetic meridian. The poles causing this deflection are the resultant poles of numerous magnetic fields made up in two main divisions, namely, the magnetism existing more or less permanently in hard iron, and the temporary magnetism due to induction from the earth's field. A ship acquires its permanent magnetism while building. An aircraft acquires it on being assembled. It is caused by the vibration produced in the iron, and, in the case of the ship, is as if the earth's lines of magnetic force were fastened down to it by the clenching of each successive rivet. To show how vibration can produce and change polarity, hold an iron poker in line with the magnetic meridian and at the approximate angle of dip for the locality. Beat it with a piece of heavy wood and then hold the lower end near the red, or north-seeking, end of a compass needle. You will find that it will repel the needle, proving that it must possess red polarity. Approach the needle with the other end of the poker and you will find the red end of the needle attracted, proving the upper end of the poker to have blue polarity—as you would expect, knowing that poles, like puffins, work in pairs. Now align the poker in the magnetic meridian as before but with ends reversed and beat it again for a somewhat longer time. On applying the poker to the needle this time, you will notice that the polarity in it has been reversed.

When a ship is built in the northern hemisphere it acquires a preponderance of red sub-permanent polarity in its lower half. Fig. 52 shows that if it were possible to build a ship at the north magnetic pole, position A, the lower half would be entirely red while the upper half would be entirely blue, the axis of the sub-permanent poles being vertical. If built on the magnetic equator, position D, the axis of the sub-permanent poles would be horizontal. Between these extremes the poles would be aligned according to the magnetic latitude of the building yard as shown in Fig. 52, position C. Corresponding conditions exist in the southern hemisphere. The direction of building would determine whether the poles lay in the fore-and-aft line, in the athwartship line, or otherwise. On returning to the northern hemisphere after a few years' trading in the southern hemisphere it is often found that polarity has been reversed, due, no doubt, to vibrational causes while nearer the red magnetic pole than the blue. Hence we speak of such magnetism being more or less permanent—*i.e.*, sub-permanent, in ships.

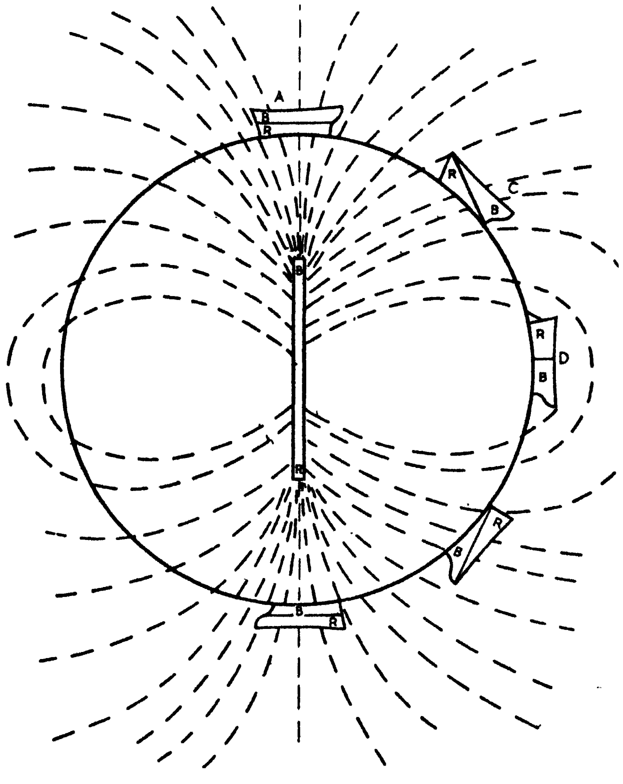


Fig. 52. Acquiring Magnetism

Sub-permanent magnetism may be resolved into three components —*viz.*, P longitudinally, Q transversely, and R vertically.

To demonstrate the changes which take place in the direction of the compass needle due to the effect of horizontal sub-permanent magnetism in the ship or aircraft, a plane cardboard model should be constructed to rotate about a cardboard compass needle with the north-seeking end marked red. Blue and red poles should then be marked in colour or fitted at diametrically opposite positions on the model, which should then be placed centrally on a magnetic-compass board. The compass needle, if undisturbed by the craft's magnetism, would point rigidly to magnetic north, but, because like poles repel and unlike poles attract, the red pole in the ship or aircraft will repel the red end of the compass needle, and the blue pole of the ship or aircraft will attract the red end of the compass needle. The resulting deflections of the compass needle will be found to be to the right, or eastward, when the course is in one semicircle but to the left, or westward, when the course is in the other semicircle. When the poles of the craft are in line with magnetic north they will not deflect the needle but will assist the action of the earth's north magnetic pole in attracting the compass needle whenever the blue pole of the ship or aircraft lies on the polar side of it. They will oppose the action of the north magnetic pole whenever the red pole of the ship or aircraft lies on the polar side. Consequently the directive force on the compass needle varies according to the course. Fig. 53 is intended to illustrate how this deflection of the compass needle, the deviation, varies as the ship changes her magnetic course. Position 1 shows a ship with a

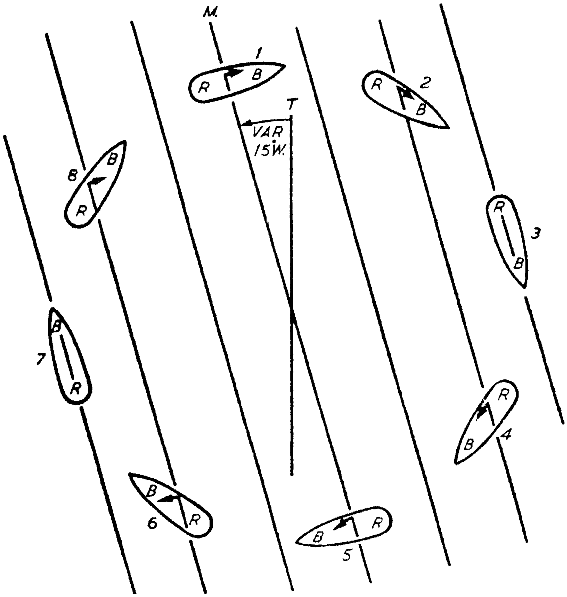


Fig. 53. Deviation due to Sub-permanent Magnetism

blue pole at the bow heading east (Magnetic); the polarity having been acquired while being built on the stocks and heading in a southerly direction. The red pole of the compass needle will be attracted by the ship's blue pole and the blue pole of the compass needle by the ship's red pole. This action on each end of the needle, known as a magnetic couple, deflects the north-seeking end of the compass needle to the right, or eastward, and easterly deviation results. The maximum deflection is reached when the couple is acting at right angles to the compass needle.

When the course is altered to S.E. (M.), position 2, the deflection is still to the right and the deviation easterly but the amount will be less. When the ship is heading south (M.), position 3, the red pole at the stern will be situated between the earth's blue pole and the red pole of the compass and, although there is no deviation, the directive force of the needle will be reduced due to the opposition of the ship's red pole. On S.W. (M.) the blue pole in the bow will repel the needle's blue pole and the north-seeking end of the needle will be deflected to the left, producing westerly deviation. Heading west (M.) the blue pole at the bow will attract the red pole of the needle and westerly deviation will result again, and this time be near its maximum. Westerly deviation is produced again on N.W. (M.), but on north (M.) there will be no deflection; and because the ship's blue pole is now assisting the earth's blue pole the directive force will be strengthened. Turning on to N.E. (M.) the deviation will again be easterly, so it can be seen that easterly deviation is continuous in one semicircle and westerly deviation in the other. This happens no matter where the permanent poles might be situated, although, naturally, the semicircle will vary. If permanent magnetism were the only factor causing deviation, it would be sufficient to put the ship's head on to four equidistant directions in order to construct a deviation curve for a table of deviations. Induced magnetism in horizontal iron, however, causes deviation varying in alternate quadrants, and so it is necessary to steady the ship's head on eight equidistant compass headings in order to construct a curve and compile a table of deviation. Further, induced magnetism in horizontal iron always has a repelling pole between the earth's magnetic pole and the compass needle, so reducing the attractive force on all headings. The effect of vertical soft iron is always semicircular, the same as hard iron, but the semicircle does not vary. The effect is always nil on north and south (M.) and maximum on east and west. When a ship is not upright the general effect of its magnetism on the compass is altered.

Deviation Card. We now proceed to construct a table of deviations.

Example. While heading by compass on the cardinal and intercardinal points these compass bearings of a distant object were found:

Compass course	*Compass bearing*	*Magnetic bearing*	*Deviation*	*Magnetic course*
N.	063°	060°	3° W.	357°
N.E.	060°	060°	0°	045°
E.	057°	060°	3° E.	093°
S.E.	054°	060°	6° E.	141°
S.	056°	060°	4° E.	184°
S.W.	061°	060°	1° W.	224°
W.	064°	060°	4° W.	266°
N.W.	065°	060°	5° W.	310°

8)480°
060° = Magnetic bearing

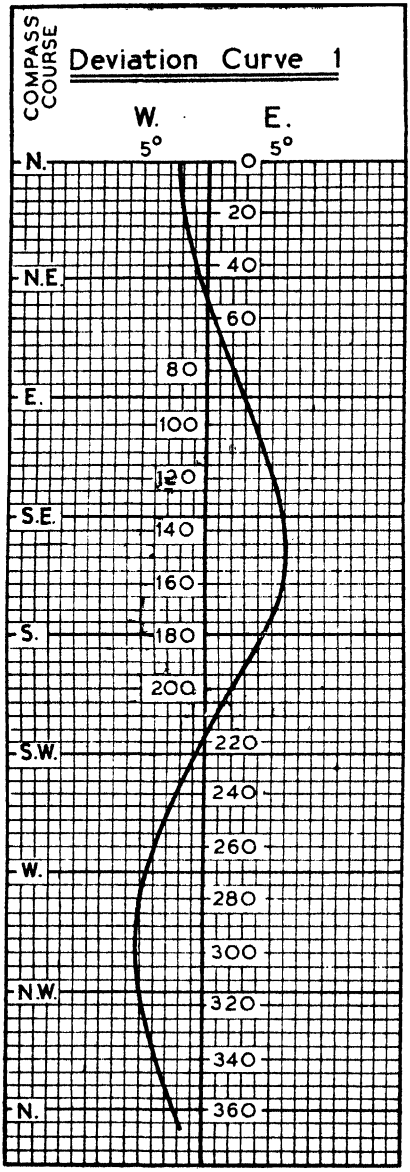
The mean of the compass bearings will give the magnetic bearing of the object, if not already known, and the difference between this and each compass bearing will give the deviation for the compass course on which it was taken. The name of the deviation, whether E. or W., is easily ascertained from a simple figure consisting of the magnetic meridian and magnetic bearing, supplemented by the direction of compass north deduced from the compass bearing. If compass north lies to the west of magnetic north the deviation is westerly, but if to the east then the deviation is easterly. Fig. 54 (p. 113) illustrates how the name of the deviation was deduced for the compass course N.W. in the above table. OM represents the magnetic meridian, OX the direction of the distant object from the observer, and MOX the magnetic bearing N. 60° E. As the compass bearing is N. 65° E. the direction of the compass needle must lie 65° to the left of OX—*i.e.*, 5° to the left of OM. Therefore the deviation must be 5° W. If the compass bearing were N. 55° E. the direction of the compass needle would be 5° to the right of the magnetic meridian and produce a deviation of 5° E.

Exercise 3

Find the magnetic bearing, the deviations, and the magnetic courses from each of the following sets of equidistant compass bearings.

	Compass course	*Compass bearings*			
		(*a*)	(*b*)	(*c*)	(*d*)
1.	N.	142°	266°	358°	S. 86° E.
2.	N.E.	139°	263°	359°	S. 88° E.
3.	E.	137°	262°	002°	N. 89° E.
4.	S.E.	136°	264°	005°	N. 86° E.
5.	S.	138°	269°	007°	N. 87° E.
6.	S.W.	141°	272°	006°	East
7.	W.	144°	271°	001°	S. 86° E.
8.	N.W.	143°	269°	358°	S. 84° E.

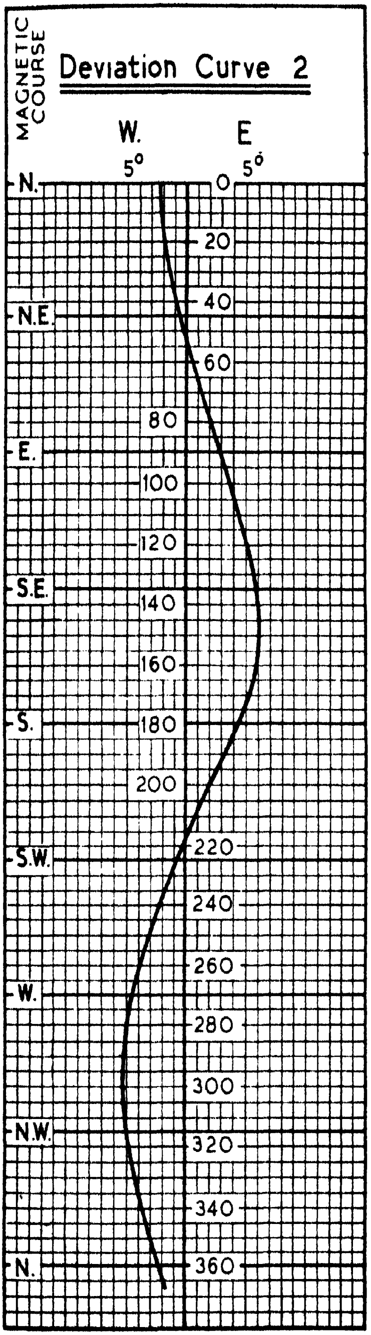
We can now plot on graph paper the deviations obtained in the above example, easterly to the right and westerly to the left of a central

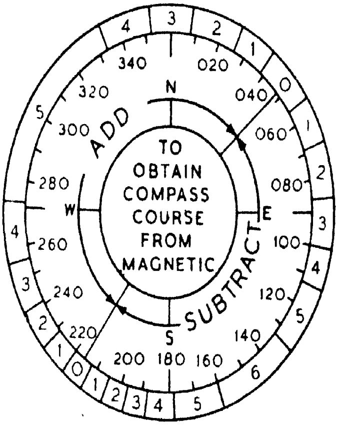


MARINE DEVIATION CARD 1

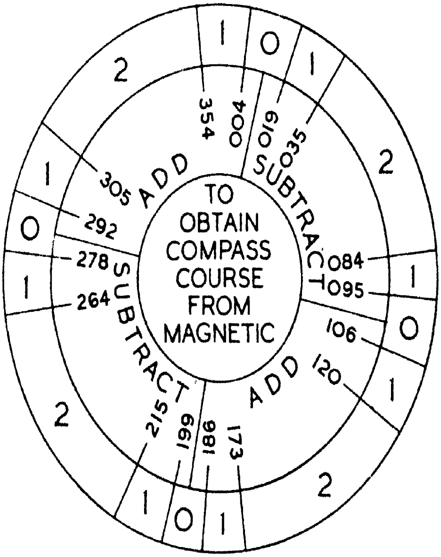
Ship's head by compass	*Deviation*
N.	3° W.
N. by E.	2½° W.
N.N.E.	1¾° W.
N.E. by N.	1° W.
N.E.	0°
N.E. by E.	¾° E.
E.N.E.	1½° E.
E. by N.	2¼° E.
E.	3° E.
E. by S.	4° E.
E.S.E.	5° E.
S.E. by E.	5¾° E.
S.E.	6° E.
S.E. by S.	5¾° E.
S.S.E.	5½° E.
S. by E.	4½° E.
S.	4° E.
S.	4° E.
S. by W.	3° E.
S.S.W.	2° E.
S.W. by S.	½° E.
S.W.	1° W.
S.W. by W.	2° W.
W.S.W.	2¾° W.
W. by S.	3½° W.
W.	4° W.
W. by N.	4½° W.
W.N.W.	4¾° W.
N.W. by W.	4¾° W.
N.W.	5° W.
N.W. by N.	4¾° W.
N.N.W.	4½° W.
N. by W.	3¾° W.
N.	3° W.

line representing the rim of the compass card, and, connecting them by a curve (Curve 1), discover the deviations on other courses. Deviation card 1, for use on ships, has been compiled directly from Curve 1, the method being merely to read off the deviation for each point of





Form 316*a*



Form 316*b*

11¼°. It would, of course, be simpler to tabulate the deviation for every 10°, and this may become the practice when the 360° compass card is in universal use. Deviations on a nautical card may be given for either ship's head by compass or ship's head magnetic, and care must be taken to notice which is supplied. If a ship's head by compass is given and you require the deviation for a compass course, you can take it direct from the card, even if interpolation is necessary. For example, the deviation for a course of 100° by compass would fall about midway between 3° E. and 4° E., so we should use 3½° E. If the change in deviation is large (a condition not tolerated in practice), greater precision is, of course, necessary when interpolating. Should the deviation be required for a magnetic course, as is often the case, it can be obtained from the same card by first turning the neighbouring compass courses into magnetic courses, and then selecting the deviation as before. Suppose the deviation were required for a course of 200° M., the corresponding magnetic course to

S.S.W. (C.) would be 202½° + 2° = 204½° M.
and to S. by W. (C.) ,, ,, 191¼° + 3° = 194¼° M.

The course 200° M. lies between these two, so that the deviation for it will be between 2° E. and 3° E., and may be found by proportion as follows:

$$\text{Deviation} = 2 + \frac{204\frac{1}{2} - 200}{204\frac{1}{2} - 194\frac{1}{4}} \times 1 = 2 + \frac{4\frac{1}{2}}{10\frac{1}{4}} = 2\tfrac{1}{2}^\circ \text{ E. approx.}$$

The deviation card used by the R.A.F. has undergone several modifications. The latest type, illustrated as Form 316*a*, is a critical table designed for simplicity of application. It supplies deviations without sign, to the nearest degree for magnetic headings and may be constructed as follows.

A curve of deviations for the magnetic courses obtained in the above example is drawn (Curve 2), and from it the limits of all deviations to the nearest degree are taken off and marked on the circle comprising Form 316*a*. The limits are found by working from the mid-point between each degree of deviation—*i.e.*, from one half to the next half, so that the limits for deviation 5° W. are the magnetic courses for which the deviation is 4½° W. and 5½° W. respectively. These are seen to be 272° and 333°. Westerly deviations are added to magnetic courses to obtain compass courses, but instead of prefixing signs + and —, which are really reserved for correcting courses from compass to magnetic, the words ADD and SUBTRACT are used to avoid confusion.

The direction OH in which the craft was heading at the time of taking the bearing has been added to Fig. 54, as it must never be forgotten that *the name and amount of the deviation will vary with the course,*

not *with the bearing.* The true meridian, OT, is also added to show what the true bearing and true course would be if a variation of 12° W were allowed. TOM represents the variation, TOX the true bearing 048°, and TOH the true course 298° measured clockwise, corresponding to the compass course COH 315° measured clockwise, N. 45° W. measured anti-clockwise. The angular distance between these two courses represents the compass error, 17° W., the algebraical sum of variation and deviation, and in order to convert a true course to a compass course or vice versa it will be necessary to apply the corrections either together or separately, as may be deduced from the Figure.

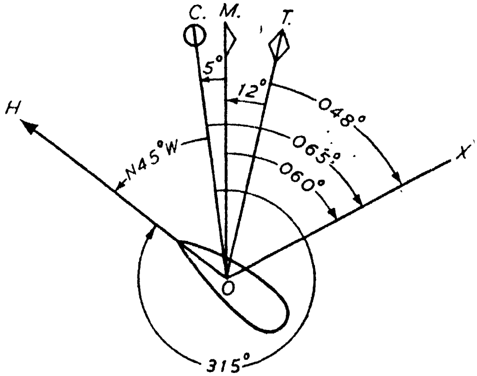


Fig. 54. MAGNETIC DEVIATION

The effect of a craft's magnetism will be different on different compasses and will vary according to the position of a compass. A separate deviation card will be required for each fixed compass and for each position in which a portable compass is likely to be used.

Correction of Courses and Bearings. It can be seen from Fig. 54 that westerly corrections applied to a true direction must be added, because the angle increases, and easterly corrections must, therefore, be subtracted. Conversely, westerly corrections applied to a compass direction must be subtracted, and easterly corrections added. The rule is best illustrated as follows:

West add, East subtract
——————————————————→
TRUE variation MAGNETIC deviation COMPASS
←——————————————————
West subtract, East add

from which it will be noted that the variation must be applied first when converting directions from true to compass, but deviation first when converting from compass to true. Each change must take place via magnetic. Converting a true direction to a compass direction does not mean that the observer changes the direction of his gaze, but merely that a given direction is related successively to different datum lines. You can regard the direction of true north as scratch line where variation and deviation constitute handicaps, or, if you prefer literature, you may recall the couplet mentioned in connexion with variation and adapt it to deviation thus:

Deviation west, Compass best.
Deviation east, Compass least.

These rules cannot be followed when using the quadrant compass, as the degrees increase clockwise—*i.e.*, to the right in two quadrants only (see Fig. 47), but if a direction given according to a particular quadrant is converted into the three-figure notation no further rules are necessary. There seems to be no sound reason why the three-figure notation should not be used to the exclusion of all other methods of compass graduation. Rules for the correction of courses and bearings in the quadrant notation are still in use at sea, however, and will be found in other text-books, so they are given below, although regarded by the author as clumsy. Courses given in points may be converted into degrees by reference to the compass card (Fig. 47) and table appended.

When using the quadrant compass card for converting directions 'right' must be substituted for 'add' and 'left' for 'subtract,' so that in correcting a compass course or bearing for deviation you imagine yourself standing at the centre of the compass card facing the direction of the given course or bearing, and turn right or left for the amount of deviation according to whether the deviation is easterly or westerly. The direction in which you then face will give the magnetic course, or bearing. By applying the variation in the same manner the true direction will be obtained. The converse holds good when converting from a true direction to a compass direction. A rule-of-thumb method is provided by naming the quadrants Right or Left when the degrees are numbered to the right or left. Corrections applied to the right are then added to a given direction in the R quadrant and subtracted from a given direction in the L quadrant, so the values of like name are added and those of unlike name subtracted.

Example. Find the true course when the compass course is S.40° E. (140°), the deviation 3° E., and the variation 11° W.

Three-figure method		*Quadrant method*	
Course	140° (C.)	Course (C.)	S. 40° E.L.
Deviation	3° E.	Deviation	3° E.R.
Course	143° (M.)	Course (M.)	S. 37° E.L.
Variation	11° W.	Variation	11° W.L.
Course	132° (T.)	Course (T.)	S. 48° E.L.

In air navigation + is sometimes prefixed to the correction of a compass which reads low, and — to the correction of a compass which reads high. This can be confusing unless it is remembered that these signs refer to corrections applied one way only—*i.e.*, from compass to true, and must be reversed to be valid the other way. Thus + = E. and — = W., because adding to a compass course must mean applying easterly deviation.

Exercise 4

Complete the following table:

	Course (C.)	*Deviation*	*Course (M.)*	*Variation*	*Course (T.)*
1.	300°	4° E.		12° W.	
2.	140°	3° W.		17° E.	
3.		5° E.		9° E.	060°
4.		2° E.		15° W.	284°
5.		7° W.	175°	5° E.	
6.		2° W.	310°	11° W.	
7.	042°	3° E.			055°
8.	290°	4° W.			274°
9.	138°			10° E.	150°
10.	360°			18° W.	335°
11.	242°		246°	22° E.	
12.	095°		092°	14° W.	
13.	200°			25° W.	177°
14.		3° E.		12° W.	355°
15.	005°	8° W.		5° E.	
16.	120°		124°		108°
17.	358°	5° E.			352°

'Swinging' an Aircraft. In order to ascertain the deviation of an aircraft's compass the aircraft should be 'swung' in the air under normal flying conditions. To obtain satisfactory results the aircraft must fly straight and level at constant speed. It is not advisable, therefore, to swing the aircraft continuously in a circle but rather follow a plan similar to that illustrated in Fig. 55. The aircraft is steered by compass successively on the cardinal and intercardinal points. These courses are then compared with the magnetic courses shown by the astro-compass, an instrument specially designed to give true or magnetic bearings of celestial objects and hence the courses. The difference between the magnetic course shown by astro-compass and the compass course shown simultaneously by the magnetic compass, is the deviation on that course. If the astro-compass is not available, the aircraft may be steered in turn directly towards distant objects whose magnetic bearings from the aerodrome or focal-point of operation are known. The difference between each magnetic bearing and the compass course steered on it will be the deviation in each case. Some aircraft may be swung on to the cardinal points only, as the effect of induction is often very small; but it is more satisfactory to swing the aircraft on to the intercardinal points too, so that the deviation card might be completed more adequately.

An alternative but less satisfactory method of ascertaining the deviation of a pilot's compass is by swinging the aircraft on the ground. The aircraft is pulled or turned round and placed so as to be heading successively as nearly as possible on the cardinal and intercardinal points by compass. Bearings of the longitude axis are then taken by portable bearing compass (Fig. 50) from aft, well clear of the aircraft. As the bearing compass is unaffected by magnetic material the directions obtained from it are magnetic, and, on being compared with simultaneous readings of the pilot's compass, will give the nature and amount of the deviation on each course. A deviation card can then be compiled.

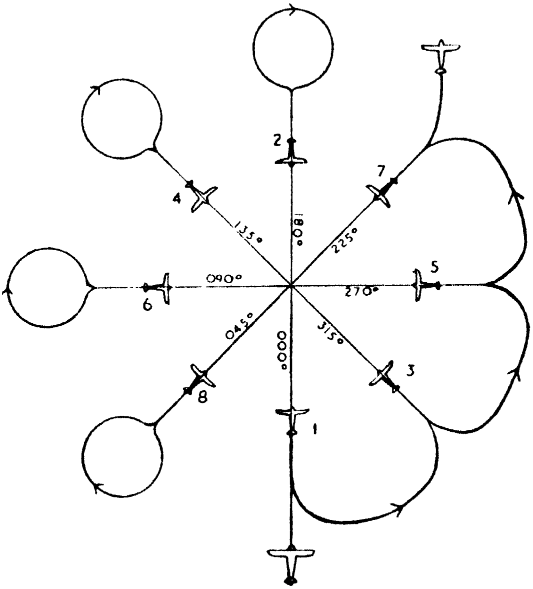


Fig. 55. Swinging an Aircraft

Adjustment of Compasses. It is naturally undesirable to have large deviations, in fact it should be the pilot's aim to reduce them to a minimum by manipulating the correctors effectively. These in an aircraft consist of two pairs of permanent magnets arranged in a circular metal box under the compass with opposite poles together, like two pairs of scissors, which when closed lie one in the longitudinal axis and the other transversely. By manipulating a key the scissors are opened and a magnetic field created which neutralizes the effect of the aircraft's magnetism on the compass needles when heading on the cardinal points.

Figs. 56 (i) and (ii) illustrate how opposing fields are introduced to counteract the craft's sub-permanent magnetism. Fig. 56 (i) shows a craft heading north magnetic. The compass needle, indicated by the arrow, is deflected to the right by a blue-component pole to starboard and a red-component pole to port, causing easterly deviation. A pair of compensating magnets lying together neutralized in the longitudinal axis is opened out until the poles are far enough apart to persuade the needle to return to the magnetic meridian. The wider apart the poles become the greater becomes their combined turning power.

Fig. 56 (ii) shows a craft heading east magnetic. The needle is this time deflected to the left by a red-component pole ahead and a blue-component pole aft, causing westerly deviation. A pair of compensating magnets lying together transversely is opened out until the poles are sufficiently wide apart to bring the needle back to the magnetic meridian. After carrying through both these operations the sub-permanent magnetism of the craft should be almost completely counteracted.

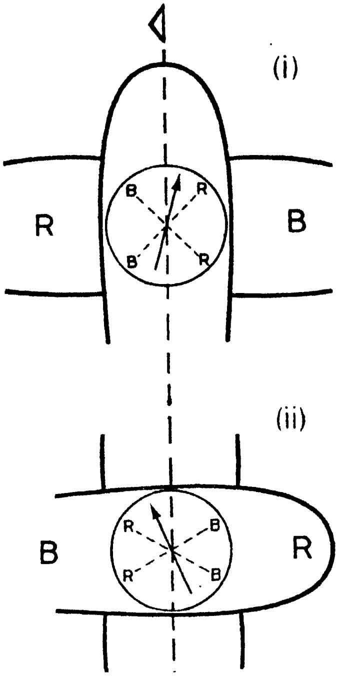


Fig. 56. Action of Micro-adjuster

Turning and Acceleration Errors. The magnetic-compass system of an aircraft in flight is thrown out of its horizontal position whenever the aircraft turns, accelerates, or decelerates, due to the fact that the system having its centre of gravity below its point of support acts like a pendulum. While horizontal, as in normal flight it is constrained to be, the horizontal component of the earth's magnetic force is the only effective component. When, however, the system becomes inclined due to centrifugal force the vertical component of terrestrial magnetism comes into action and in the northern hemisphere pulls the north-seeking end downward. If the inclination of the horizontal plane is not in the direction of the magnetic meridian, deviation will occur. A simple demonstration with an exercise book as the 'plane and a pencil as the compass needle will show you whether the north-seeking end of the needle is drawn downward to the right, causing easterly deviation, or downward to the left, causing westerly deviation. Easterly deviation is added to the compass reading, so in the first case the compass must under-read. Westerly deviation is subtracted from compass, so that in the second case the compass must over-read. Deviation due to turning, whether banked or flat, is nil on east and west magnetic, and maximum on north and south. Deviation due to acceleration and deceleration is nil on north and south magnetic, but maximum on east and west.

Gyro Compass. The use of a gyro compass avoids the ill effects of magnetic attraction. This practical application of the gyroscopic principle was for a long time unappreciated. The instrument was first devised by Professor W. R. Johnson, in 1832, and used as a scientific toy. Later it was adapted by M. Foucault to demonstrate the rotation of the earth. Now it has many uses. It consists of a fly-wheel, or rotor, so balanced in gimbals that it has three degrees of freedom, and when rotated at high speed maintains the direction of its axle constant in space. If the axle is set in the direction of a particular star it will continue to point in that direction unless disturbed by external forces. As the earth rotates the direction of the axle will appear to change—it will follow the path of the star, so making a circle round the pole. The desirability of maintaining the axle horizontal for directional purposes on the earth's surface is obvious. Suppose, then, that the axle be set to point to a star on the horizon in the direction of true north. Its immediate tendency would be to follow the star eastward, but if this tendency—precession, it is called—could be overcome, the axle would continue to point horizontally to true north. A mechanical device is therefore introduced to produce a precession in the opposite direction, westward. This is known as gravity control, and in some cases consists of an adjustable weight called the latitude rider. If the selected star were the Pole Star the direction of true north would be obtained immediately, but to bring the north end of the axle down to the horizon would cause it to precess eastward, and so act in the same way as if following any other star. The treatment in both cases is therefore the same. It can be seen, however, that as the pole is approached and coercion increases the gyro compass is bound to lose its directive power and become useless in high latitudes. In this respect it can be compared with the magnetic compass. In addition to this defect there are two errors which have to be counteracted. Firstly, there is the error due to the rate at which the compass is being carried northward or southward; this is known as speed-error, and depends on course, speed, and latitude. It is balanced mechanically sometimes by moving a lubber-ring through an angle equivalent to the error. Secondly, there is the error due to rapid change of course or speed, this is known as ballistic precession, and is counteracted by mercury or oil pressure. The total result of these actions is to cause the north end of the gyro axle to follow a very flat spiral path. When disturbed the axis will come to rest in the centre of this spiral in about 90 minutes. In ships the motive force for the rotor is provided by electricity. A master gyro is situated near the ship's centre of gravity, and repeater compasses are installed where required for navigational purposes and operated from the master by electrical contact. In submarines a gyro compass is essential owing to the screening effect of the hull on the magnetic compass.

In aircraft the motive force is provided by two parallel air-jets which, operated by a venturi-tube, strike cups made in the rotor. They assist in controlling precession by striking the sides of the cups whenever the rotor tends to tilt. The card is divided into 5° intervals, each 30° being numbered. Owing to the uncertain amount of precessional 'wander,' the course by gyro compass must be compared frequently with the course by magnetic compass.

The Distant-reading Gyro-magnetic Compass. The principles of both the gyro and the magnetic compass are combined in an instrument known as the Distant-reading Gyro-magnetic Compass which embodies the merits of each while eliminating the faults. Each type of compass contributes its own peculiar advantages and tends to offset the weaknesses of the other.

The master unit consists of two main elements, a small, accurately constructed gyroscope and a north-seeking magnetic needle, together with an electrical transmitter system. As already explained, a gyroscope when spinning at a very high speed possesses great rigidity in space, and when mounted in a gimbal-ring system will remain pointing to a pre-set position despite any relative movement of its outer case. Such a gyroscope provides a fixed datum direction, not necessarily north, from which any changes of course by an aircraft can be measured.

The datum direction adopted by the distant-reading compass is that of magnetic north, and is obtained from a magnetic needle corrected for deviation. This needle exercises a slow precessional control over the gyro by maintaining the latter's axis in alignment with the needle's own mean direction. The gyro thus registers the magnetic course of the aircraft, and, because of its high rotational speed of 12,000 R.P.M., does not suffer from the sluggishness of the needle but acts as a stabilizer, and indicates immediately any change in direction of the aircraft's nose. In addition, the needle is prevented from registering its customary response to the earth's vertical force during accelerations. The needle, for its part, ignores the precessional tendency of the gyro. To obtain complete co-operation the two elements are co-related automatically every three seconds.

The instrument is placed in the tail of the aircraft, as far as possible from magnetic material, so that the directional force acting on the needle will be at its maximum and deviational disturbance at a minimum. The readings of the master unit are transmitted electrically to repeater dials placed as required in various parts of the aircraft—*e.g.*, one for the pilot to steer by, and one for the navigator to take bearings from. For this latter purpose the repeater must be fixed in a suitable stand and fitted with an azimuth circle. The master unit also controls the automatic pilot.

A variation corrector installed between the master unit and the repeaters allows the magnetic readings of the master unit to be converted into true readings on the repeaters. This facilitates rapid reference between the repeaters and a map or plotting sheet. The master unit functions correctly with angles of bank or pitch up to 75°, but will be deranged temporarily if this value is exceeded, and must then be reset.

General Exercises

1. What are the essentials of an efficient magnetic compass?

2. Enumerate the elements of terrestrial magnetism, and draw figures to illustrate their relationship. Where is the earth's magnetic force most intense?

3. When is a magnetic compass most reliable, and when useless?

4. When are horizontal force and vertical force at their maximum and minimum values? When are they equal?

5. What is meant by damping, and how is it effected?

6. What is it that maintains the compass card or system horizontal?

7. When does vertical force play an important part?

8. What is meant by directive force?

9. Define magnetic variation. When would you expect it to be nil, and when at its maximum?

10. What are isogonal lines, and what is an agonic line?

11. What do you understand by the terms hard iron and soft iron?

12. When is a compass said to be aperiodic, or 'dead beat'?

13. How does a ship or aircraft acquire its magnetism? Is it permanent?

14. What is induction?

15. Define deviation, and account for the ways in which changes arise in it.

16. Would you expect the deviation on two compasses to agree or not?

17. What deviation would result if the lubber's line were displaced 3° to port?

18. In manœuvring a ship or aircraft would the arc of turn be the same as the change of course? Give reasons.

19. The following eight bearings of a distant object were taken on the cardinal and intercardinal points. Find the magnetic bearing and

thereby the deviation on each heading for use in the construction of a deviation card.

Compass course	*Compass bearing*	*Compass course*	*Compass bearing*
N.	S. 46° W. (226°)	S.	S. 44° W. (224°)
N.E.	S. 48° W. (228°)	S.W.	S. 42° W. (222°)
E.	S. 50° W. (230°)	W.	S. 40° W. (220°)
S.E.	S. 49° W. (229°)	N.W.	S. 41° W. (221°)

20. Employing the method used in Question 19 construct another deviation card from the following compass bearings:

Compass course	*Compass bearing*	*Compass course*	*Compass bearing*
N.	S. 87° E. (093°)	S.	N. 89° E. (089°)
N.E.	East (090°)	S.W.	S. 88° E. (092°)
E.	N. 88° E. (088°)	W.	S. 86° E. (094°)
S.E.	N. 87° E. (087°)	N.W.	S. 85° E. (095°)

21. How do you convert a compass bearing to a true bearing for laying off on a chart?

22. Given:

Course by steering compass 085° Deviation 2° E., variation 15° W.
Course by bearing compass 080°
Bearing by bearing compass 215°

Find:

(*a*) True bearing of object.
(*b*) Deviation of bearing compass.

23. Draw figures to illustrate the following values and complete the table:

			Course			*Bearing*		
	Variation	*Deviation*	*True*	*Magnetic*	*Compass*	*True*	*Magnetic*	*Compass*
(*a*)	14° W.	5° E.	050°					290°
(*b*)	10° E.	3° W.	170°					044°
(*c*)	17° W.	6° E.	345°					005°
(*d*)	16° E.		240°		220°	010°		
(*e*)		4° W.			300°	225°		242°
(*f*)		2° E.		200°		075°		060°
(*g*)			095°	084°			164°	168°
(*h*)			356°		012°	020°	038°	
(*i*)	20° W.		004°				039°	035°
(*j*)		5° E.		186°		077°	065°	
(*k*)			144°	130°	133°	210°		
(*l*)					010°	152°	168°	164°
(*m*)	17° E.	6° W.		145°			222°	
(*n*)	15° W.	7° E.			048°			082°
(*o*)	5° W.			096°	091°	318°		
(*p*)		10° W.	116°	122°			033°	
(*q*)	0°	4° E.			006°	344°		
(*r*)	25° W.		350°		012°			236°
(*s*)			230°	234°			086°	082°
(*t*)				100°	106°	254°	240°	
(*u*)	22° E.			012°	016°	012°		
(*v*)		5° W.	066°		060°			168°

24. Bring the following variations up to the year 1945 by applying the annual increase or decrease:

	Year	*Variation*	*Annual change*
(*a*)	1924	17° 30′ W.	− 10′
(*b*)	1930	14° 20′ E.	+ 12′
(*c*)	1932	7° 45′ W.	+ 11′
(*d*)	1918	3° 00′ E.	− 14′
(*e*)	1935	10° 10′ E.	+ 6′
(*f*)	1922	20° 00′ W.	− 8′

25. Draw figures to illustrate the following values, and complete the table using variation from the chart (Fig. 39, p. 90) and deviation to the nearest degree from Marine Deviation Card 1 or Form 316*a*.

	Position		*Course*			*Bearing*		
	Latitude	*Longitude*	*True*	*Magnetic*	*Compass*	*True*	*Magnetic*	*Compass*
(*a*)	20° N.	20° W.	160°					300°
(*b*)	40° S.	80° W.	042°				352°	
(*c*)	00°	160° E.			208°	005°		
(*d*)	10° S.	80° E.		172°				054°
(*e*)	30° N.	140° W.		350°			012°	
(*f*)	50° N.	30° W.	020°			340°		
(*g*)	45° S.	110° E.			150°		074°	
(*h*)	50° S.	120° W.		095°				210°

26. Two leading lights in line (transit) on a bearing of 346° T. bore by compass 002°. The variation was 12° W. What was the deviation?

27. The sun bore at noon 190° C. The variation was 14° W. What was the deviation?

28. From a craft steering 270° C. the relative bearing of the Pole Star was 080°. The variation was 12° E. What were the deviation, the true course, and the compass bearing of an object whose relative bearing was 300°?

29. A craft is heading 222° by steering (P4) compass and 217° by standard (bearing) compass. The deviation on the steering (P4) compass is 2° W. and the variation is 11° W. The standard (bearing) compass gives the bearing of a lighthouse as 152°. What is the true bearing of the lighthouse?

30. From a craft steering 180° by compass the sun at noon bore 10° to starboard, and at the same time an object appeared 40° to port. If the variation is 12° W., what are the deviation of the compass, the true course, and the magnetic bearing of the object?

31. From a craft on a course of 030° by steering (P4) compass the Pole Star bore by standard (bearing) compass 020°. The deviation on the steering (P4) compass was 2° E. and the variation was 14° W. What was the bearing by standard (bearing) compass of an object 25° to port?

32. A craft was on a course of 250° by steering compass when two objects came into line (transit) 25° to starboard, bearing by standard

(bearing) compass 280°. The true bearing of the objects was found from the chart to be 265° and the variation was 13° W. Find the deviation on both compasses.

33. The bearing of an object by steering (P4) compass was 331° and by standard (bearing) compass 327°. If the deviation on the steering (P4) compass was 3° W., and the variation 12° W., what was the deviation on the standard (bearing) compass and the true bearing of the object?

34. The course of an aircraft by P4 compass was 075° and the drift was 6° to starboard, the variation being 15° W. and the deviation on the P4 compass 2° E. If the back-bearing of an object were 250° by bearing compass, what were the track and the deviation on the bearing compass?

CHAPTER VI

THE TRIANGLE OF VELOCITIES

Track—Course—Drift—Air speed and ground speed—Wind—Current—Vectors—The addition of vectors—Vectors applied to navigation—To find track and ground speed—To find course and air speed—To find course and ground speed—Wind finding—Wind by track and ground speed—Multiple-drift method—Drift and wind-lane method—Drift and bearing method—Reverse-course method.

Track. If you had to make a journey of several hours' duration by foot across featureless moorland, you would try to follow the direct route so as not to lose your way. Before setting out you would consult a map, and by joining your point of departure to your destination ascertain the magnetic direction in which you ought to travel. Then you would rely on a compass to guide you. This is just what the navigator of a ship or aircraft does when his destination and all other fixed points en route are out of sight, except that instead of using a map he uses a Mercator chart, which, as we saw in Chapter III, is specially designed to enable straight lines to represent constant direction. The datum, or reference direction, from which all other directions are measured on a Mercator chart, is that of true north. **The angle measured clockwise from true north between any meridian and a straight line joining two places on a Mercator chart is the track between the two places.**

Referring to the example chart (end of book) we see that position X when joined to IJMUIDEN gives a track of 064°. From IJMUIDEN to X the track is 244°, the reciprocal of 064°. The projected track is not always realized—*i.e.*, the navigator of an aircraft may at some stage of a flight find by observation of terrestrial objects that he is not on the straight line joining his point of departure to his destination. The direction relative to true north in which the aircraft has actually moved is then called the track made good (T.M.G.). In a ship it is known as the course made good.

Course. Track and Course are closely connected. Both are angles measured clockwise from the direction of true north, but, whereas track is the direction of movement over the ground, course is the direction of movement through the air in the case of an aircraft, or through the water in the case of a ship. In still air or still water track and course are identical, but when the medium through which the craft is moving is itself in motion track and course cannot agree, unless the direction of movement of the air or water is along the track line—*i.e.*, in either the direction of the track or its reciprocal. In all other cases an angle exists between the direction of movement relative to the ground and relative to the air or water. We have all, at some time, seen an aircraft heading in one direction but moving crabwise in another. The direction in which the aircraft is heading is its course. **The true course is the angle measured clockwise from true north between the meridian and the longitudinal axis of the ship or aircraft.** The angle between the direction in which the aircraft is heading and the direction in which it is moving over the ground—*i.e.*, the difference in angular value between course and track—is the **angle of drift,** named starboard when movement over the ground is to the right of the course, and port when to the left of the course. In Fig. 57 the true course is given as 060°. When the track is greater—*e.g.*, 075°, the drift is to starboard, when the track is less—*e.g.*, 045°, the drift is to port.

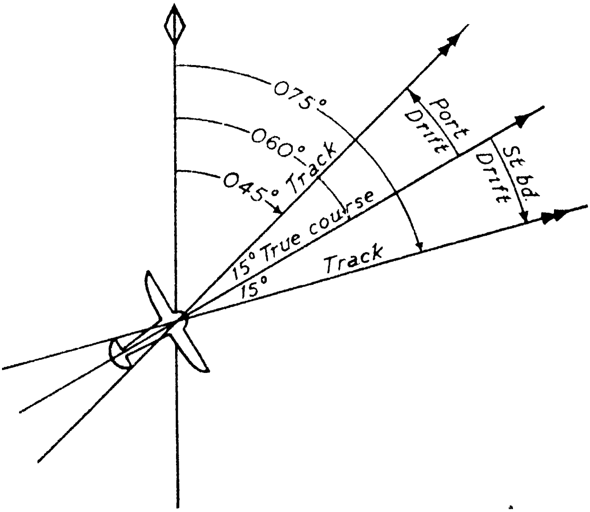


Fig. 57. Track, Course, and Drift

Air Speed and Ground Speed. Air speed is the rate in miles per hour or knots at which an aircraft is moving through the air. It depends for its amount on the propulsive power or thrust of the propellers or jets, modified by constructional features of the aircraft, and is read from an instrument known as an air-speed indicator, which measures the difference between atmospheric pressure and pressure due to the movement of the aircraft through the air. The indicated reading is subject to corrections which, when applied, determine the true air speed (T.A.S.). Provided the instrument remains in working order we may say that the true air speed is, generally speaking, always obtainable mechanically. The rate of change of position over the ground, or **the speed at which a craft moves along a track is the ground speed (G.S.).** It depends for its amount on the true air speed, but usually differs from it because of the effect of moving air

—*i.e.*, wind. When air moves in the direction of the track, the ground speed will be the sum of true air speed and wind speed, but when it moves in the reciprocal direction, ground speed will be the difference between true air speed and wind speed. Ground speed is ascertained by observation of objects on the ground. If an aircraft steering a steady course flies from one point to another in a given time, the ground speed may be ascertained by measuring on a map or chart the distance between the points. This, divided by the time in hours, will give the ground speed in miles per hour or knots. Referring to the chart (at end of book) the distance between point X and IJMUIDEN is found to be 84 nautical miles. If this were covered in 28 minutes—*i.e.*, $\frac{7}{15}$ of an hour—the speed would be $\frac{84 \times 15}{7}$ *i.e.*, 180 knots.

The nautical equivalent of air speed is speed through the water. This is obtained by means of a rotator towed behind a ship, the number of rotations being registered by a mechanism whose dial is calibrated in nautical miles. The speed is obtained by noting the number of nautical miles registered in one hour or in any fraction or multiple of an hour. Effective speed is the nautical equivalent of ground speed. It also is obtained by noting the time taken to travel from one known position to another whose distance apart is known or obtainable. The difference between speed through the water, or speed by log, and effective speed is caused by the effect of the horizontal movement of the water—*i.e.*, current.

Vectors. If an aircraft covered a distance of 180 nautical miles along a track in one hour, we should say that the ground speed was 180 knots. This could be represented by 180 units of any constant length measured along a straight line in the appropriate direction. Such a line is known as a **Vector,** and, as constant speed in a fixed direction is velocity, a **vector is a straight line drawn to scale to represent a velocity.**

When drawing vectors on a Mercator chart it is customary to use the constant longitude scale for speed measurements and not the latitude scale, which, as is explained in Chapter III, is variable and should be confined to the measurement of distance in nautical miles only.

The Addition of Vectors. Single vectors are of little use, but when two or more are linked together they provide a diagram to represent what happens to a body when two or more velocities act on it, either successively or simultaneously. By linking up vectors for this purpose we are said to add them.

Suppose A in Fig. 58 to be the position of the body moving at any instant with a velocity represented by AB. One minute later the first velocity ceases and a second velocity represented by BC begins to operate. At the end of another minute the position of the body may be represented by C whose direction from A is given by AC, which will also represent the rate of change of position in two minutes. If, however, the forces had been applied together and the second force represented by AD the body would have reached C in one minute, that is, AC is the resultant of AB and AD. Because AD and BC are equal and parallel, ABCD must be a parallelogram of which AC is the diagonal from the original point of application of the velocities, and so we have the following condition.

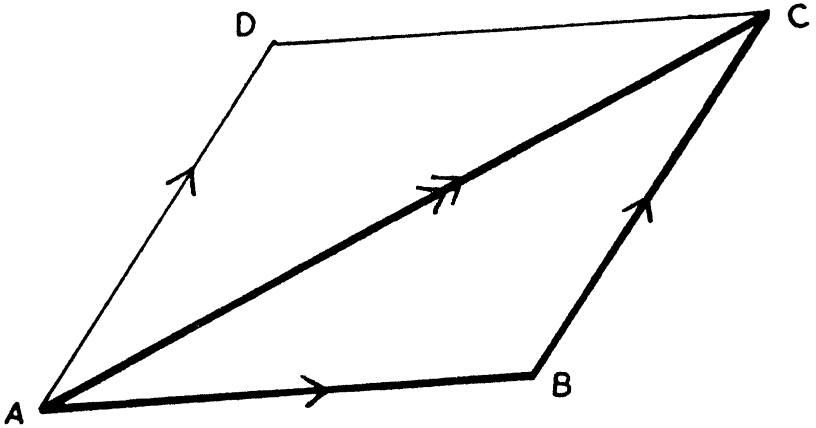


Fig. 58. The Triangle of Velocities

If two component velocities acting at a point be represented in magnitude and direction by two straight lines drawn from the point, and a parallelogram be constructed on the two straight lines, then the resultant velocity is completely represented by the diagonal of the parallelogram drawn from the point.

The complete parallelogram need not be drawn, as the three velocities are represented in direction and magnitude, if not in position, by the three sides of the triangle ABC. This we call a triangle of velocities.

If a moving point possesses simultaneous velocities represented by two sides of a triangle taken in order, these are equivalent to a velocity represented by the third side of the triangle.

Vectors Applied to Navigation. Navigation is largely concerned with moving craft, and as a ship or aeroplane is generally subject to at least two separate and distinct horizontal velocities at the same time these may be compounded to give a resultant. We thus have three velocities each of which balances the other two, so that we may construct a triangle of velocities to illustrate the inter-action of one on the others. When, therefore, two velocities are known the third may be found by construction. This has an important bearing on navigation, as one velocity, the course and air speed or speed through the water, may be regarded as always known and one of the others may from time to time be obtained by observation—*e.g.*, track and ground speed—or, from external sources, wind or current velocity. In Fig. 58 we could let AB act as the course and speed vector, while BC represented the wind or current vector for the same period. The resultant of these two is AC, which would then represent the track and ground speed. To avoid confusion the vectors in such a triangle of velocities are marked by arrows to indicate what they actually represent. In the R.A.F. one arrow indicates the course and air-speed vector, two arrows the track and ground-speed vector, while three arrows indicate the wind vector.

To Find Track and Ground Speed. Different methods of constructing the triangle of velocities must be employed according to which navigational values are given and which are required. The following example illustrates the procedure when course, speed, and wind are known and track and ground speed are required.

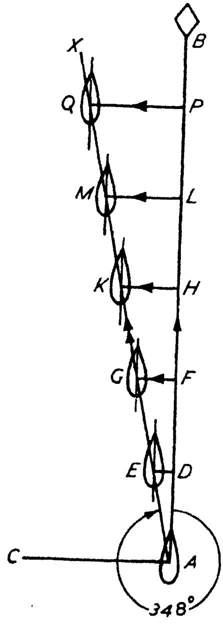


Fig. 59

Example. A vessel is steaming on a course 000° T. at 10 knots through a current setting 270° at 2 knots. An aircraft is flying on a course 000° T. at 200 knots the wind velocity being 090°/40 knots. What is the resultant direction and speed relative to the ground?

Fig. 59 illustrates the progress of either craft over the period of an hour. Starting from A the steamer would have covered 2 miles, represented by AD, in 12 minutes in a direction AB through the water, but the water is moving in the direction AC and in 12 minutes will have moved ·4 miles, represented by DE, so that relative to the earth the vessel will be at E after steaming for 12 minutes. Similarly after 24 minutes it will be at G, after 36 minutes at K, and after one hour at Q, having moved in a direction AX over the ground for a distance of 10·2 miles, represented by AQ, in one hour. Substitute air for water, multiply the above values by 20, and the Figure will represent the successive relative positions of the aircraft over the period of one hour. The reflex angle BAX then represents the track 348° T., AQ the ground speed 204 knots, and the acute angle BAX the drift angle of 12° to port. So that to find the track and ground speed (G.S.) of either ship or aircraft, draw a line making an angle with the meridian equal to the true course and long enough to represent the speed through the water or air to a convenient scale, say 10 centimetres = 10 knots for a steamer or 200 knots for an aircraft. At the end of the course vector join the current or wind vector—a line representing the current or wind in direction and force to the same scale—in the direction in which the water or air is moving. A line joining the beginning of the course vector to the end of the current or wind vector will then represent the track and ground speed in the same scale and for the same period.

Exercise 1

By completing a triangle of velocities, find the track and ground speed in each of the following cases.

	Course (T.)	*Speed (knots)*	*Wind Velocity*	*Current*
1.	220°	120 (12)	150°/20 knots	330°/2 knots
2.	040°	200 (20)	090°/30 knots	270°/3 knots
3.	310°	145 (14·5)	170°/25 knots	350°/2·5 knots
4.	145°	170 (17)	045°/27 knots	225°/2·7 knots
5.	286°	180 (18)	225°/15 knots	045°/1·5 knots
6.	005°	164 (16·4)	050°/36 knots	230°/3·6 knots
7.	000°	155 (15·5)	180°/25 knots	000°/2·5 knots
8.	357°	192 (19·2)	300°/32 knots	120°/3·2 knots

To Find Course and Speed. When a ship or aircraft is working to a time-table a given distance may have to be covered within a certain time-limit, which means that the desired track is known and the ground speed fixed. If the wind blows or a current sets across the track, the course must be set so as to counteract the effect of either. To find the true course an hour's ground speed is marked along the track and the wind or current vector linked at the beginning in the direction taken by them. The two vectors of the triangle being complete, the third may be found by joining the free ends of the other two. The direction of the course vector may then be obtained by comparison, and the speed found by measuring the length of the vector to scale. The following example will illustrate this.

Example. An aircraft (steamship) had to cover a distance of 500 (50) nautical miles along a track of 210° T. in 3 hours 20 minutes. The wind velocity (W/V) was from 160° T. at 25 knots. The current velocity was towards 340° T. at 2·5 knots. Find the true course to be steered and the speed necessary in order to reach the destination in the required time.

Draw a straight line to represent a meridian, as in Fig. 60, and lay in the track from a point A on it. In this problem 500 n.m. must be covered by the aircraft or 50 n.m. by the steamship in 3 hours 20 minutes. Dividing distance by time we find the required ground speeds to be 150 knots and 15 knots respectively. Measure this distance AB to scale along the track. Put in the wind vector for one hour at A, the beginning of the track, in the direction AC, the reciprocal of that given—*i.e.*, 340° T. downwind—or put in the current vector for one hour from A in the same direction as that in which the current is setting. Join the free ends C to B and you have the third side of the triangle, the vector representing course and speed. The angle which this makes with the meridian is the course. It is found to be 203°. The length of the side to scale gives a speed of 167 knots for the aircraft or 16·7 knots for the steamer.

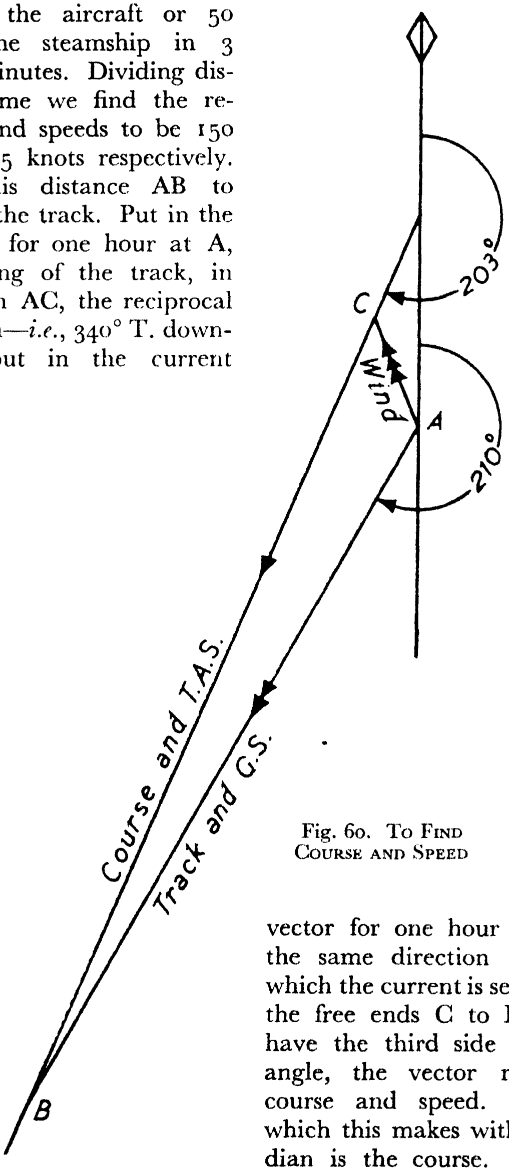


Fig. 60. To Find Course and Speed

Exercise 2

Using the given wind or current velocity, find the course and speed required to make good the following tracks and distances in the given time.

	Track	*Distance* n. m.	*Time*	*Wind Velocity*	*Current*
1.	030°	200 (20)	1 hour	240°/30 knots	060°/3 knots
2.	240°	145 (14·5)	1 hour	000°/20 knots	180°/2 knots
3.	155°	280 (28)	1¾ hours	270°/24 knots	090°/2·4 knots
4.	318°	375 (37·5)	2½ hours	110°/18 knots	290°/1·8 knots
5.	075°	560 (56)	4 hours	300°/27 knots	120°/2·7 knots
6.	357°	65 (6·5)	26 mins.	055°/35 knots	235°/3·5 knots
7.	268°	1350 (135)	7½ hours	200°/32 knots	020°/3·2 knots
8.	134°	2000 (200)	17½ hours	170°/12 knots	350°/1·2 knots

To Find Course and Ground Speed. Two complete vectors are not always given, but the triangle may still be constructed from a knowledge of any four parts, each vector consisting of two parts, a direction and a speed. The most common problem of this type is that of finding the course and ground speed from a knowledge of the other values. When the track is predetermined and the current or wind known, the problem of finding the course which the craft must steer

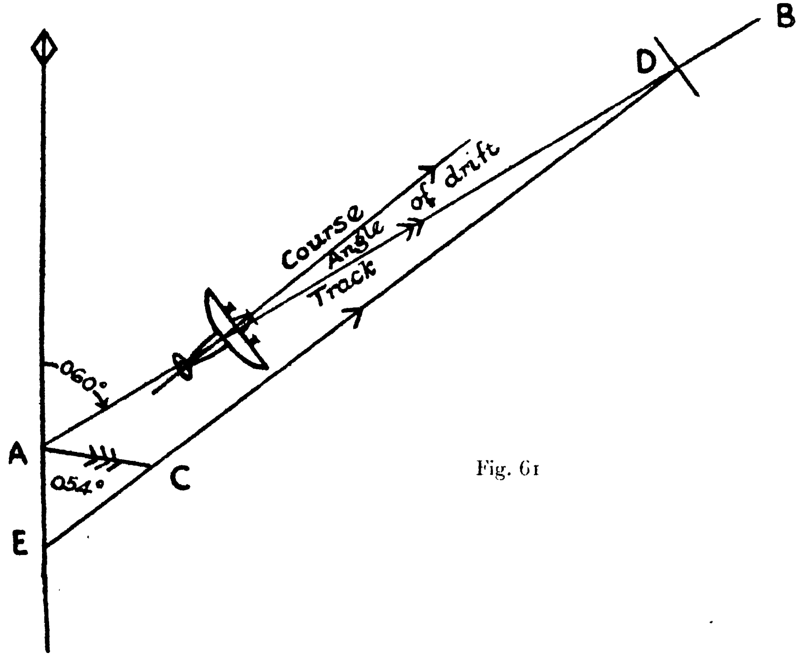


Fig. 61

in order to move at an unknown ground speed along the track is best solved by plotting the current or wind vector at the beginning of the track, in the direction in which the water or air is moving, as in the last example, and from the end of it use the steaming speed or air speed as a radius to intersect the track by an arc. The line joining the end of the wind vector to this point of intersection will represent the course and can be measured by noting the angle at which it, produced if necessary, crosses the existing meridian, or which it makes with a second meridian added for that purpose. The ground speed is then found by measuring the length of the track up to the point of intersection. Fig. 61 illustrates this from the following example.

Example. A ship, speed 18 knots (aircraft's speed 180 knots) has to make good a direction 060° T., the set of the current being in a direction 100° T. at a rate of 3 knots (the direction of wind being from 280° T. at 30 knots). Find the true course to steer to do this, and the speed over the ground (G.S.).

Draw a line AB to represent the track, making an angle of 060° with the meridian, plot AC to represent the current or wind to scale, then from C, with steaming speed or air speed as a radius, make an arc intersecting AB at D. CD will then represent the course in direction and when produced backward will cut the meridian at an angle of 054°. AD represents the speed over the ground 20·2 knots for the steamer, or 202 knots for the aircraft.

Exercise 3

Complete a triangle of velocities and find the course and ground speed in each of the following cases.

	Track (*T.*)	*Air speed* (*knots*)	*Steaming speed* (*knots*)	*Wind velocity*	*Current*
1.	090°	150	15	120°/30 knots	300°/3·0 knots
2.	075°	200	20	180°/36 knots	000°/3·6 knots
3.	340°	160	16	100°/28 knots	280°/2·8 knots
4.	252°	220	22	320°/25 knots	140°/2·5 knots
5.	166°	180	18	060°/30 knots	240°/3·0 knots
6.	295°	240	24	240°/32 knots	060°/3·2 knots
7.	020°	195	19·5	140°/20 knots	320°/2·0 knots
8.	308°	172	17·2	200°/15 knots	020°/1·5 knots
9.	355°	120	12·0	090°/40 knots	270°/4·0 knots
10.	244°	250	25·0	045°/22 knots	225°/2·2 knots
11.	006°	135	13·5	270°/42 knots	090°/4·2 knots

Wind Finding. A constantly varying factor of prime importance to the air navigator is wind, and many methods have been devised for discovering its velocity. All are not of equal importance, and some are more complicated than others, so a selection of what are considered most useful is given here.

Wind by Track and Ground Speed. The course and speed of an aircraft are generally known to the navigator. If, then, the track and ground speed are obtained by observation of terrestrial objects, a triangle of velocities may be completed and the wind velocity found from it. Both course and track are drawn from the same point on a meridian. The true air speed is then measured to scale along the course and the ground speed along the track. By joining the free ends of these vectors by a straight line the triangle is completed and a vector obtained which represents the wind velocity. This can be seen from Fig. 62, which illustrates the following example.

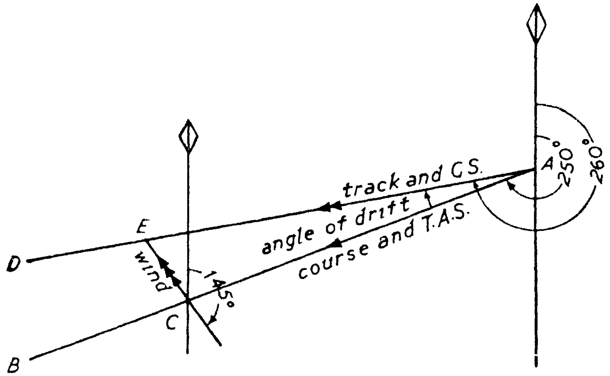


Fig. 62

Example. The navigator of an aircraft flying on a course of 250° T. at a true air speed (T.A.S.) of 150 knots finds that a distance of 40 nautical miles is covered in 15 minutes along a track of 260° T. What was the direction and speed of the wind?

AB is drawn to represent the course making an angle of 250° with the direction of true north, and a distance AC, to scale corresponding to 37½ n.m., is measured along it to represent 15 minutes' air speed. AD is then drawn in a direction 260° T. to represent the track, and a distance AE representing 40 n.m. is measured along it. Point C is the air position, the position in which the aircraft would have been had there been no wind, and point E is the aircraft's actual position. The obvious conclusion is that the atmosphere must have carried the aircraft in an average direction of 325° T. for a distance of 7·5 n.m. in 15 minutes, or, as we generally say, the wind has blown from 145° T. at a speed of 30 knots. Wind always blows from course to track. The angle of drift is 10° starboard.

Exercise 4

Complete the triangle of velocities from the following values, and give the wind velocity in each case.

	Course (*T.*)	*Air speed* (*knots*)	*Track* (*T.*)	*G.S.* (*knots*)
1.	330°	180	320°	200
2.	300°	200	309°	196
3.	240°	180	244°	200
4.	150°	240	158°	257
5.	060°	150	071°	142
6.	325°	125	317°	160
7.	180°	210	170°	200
8.	290°	165	300°	177
9.	012°	130	358°	158
10.	270°	250	265°	273
11.	350°	140	005°	131

Multiple-drift Method. The angle of drift is the result of wind effect, so that a knowledge of the drift angle will help in discovering the wind velocity. The amount of drift is most easily ascertained by passing directly over a prominent object and noting its direction relative to the longitudinal axis about two minutes later. Various instruments such as the drift recorder or course-setting bomb-sight may be used for the purpose, or the angle obtained merely by drift markings on the tail. Assuming, then, the course and air speed to be known and the drift observed, we have three parts of the triangle of velocities, as the drift, when applied to the course, will give the track. Without the fourth part, the ground speed, we are unable to complete the triangle, but we can, by altering course appreciably, find a second angle of drift. This, when combined with the first, will help to solve our problem, and will complete not one but two triangles of velocities, in which three parts—*viz.*, air speed, wind direction, and wind speed are common. Every additional new course with consequent drift would, when combined with others, serve to increase the number of triangles, and so clinch the matter, as the same three parts would be common to all.

Because the air speed is common, we draw a circle, known as an air-speed circle, with it to scale as the radius. A diameter is added to act as meridian, and the various courses are then all drawn inward from circumference to centre. Their respective angles of drift are each marked off at the circumference and the resulting tracks produced until they intersect. A straight line joining the centre of the circle to this point of intersection will complete and be common to all triangles. It represents the wind velocity. The following example will illustrate the method.

Example. An aircraft is flying over land on a course of 080° T. at T.A.S. of 200 M.P.H., and the navigator requires to find the wind velocity. He observes the drift by back bearing (B.B.) to be 5° to starboard and the pilot then alters course 60° to port on to 020° T. The drift on this course is found to be 12° to starboard and three minutes after the first alteration of course the pilot alters course 120° to starboard on to 140° T. and the drift is found to be 4½° to port. After three minutes on this course an alteration of 60° port is made and the original course of 080° T. thereby resumed. What is the wind velocity?

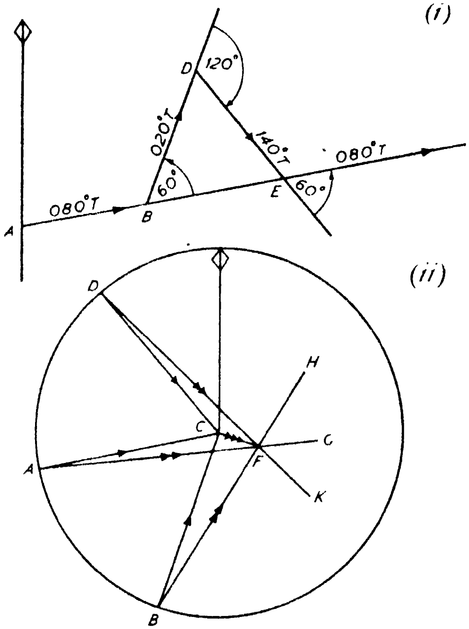


Fig. 63. Air-speed Circle

Fig. 63 (i) is a plot of the courses steered, together with an indication of the alterations made. The first drift angle was measured while on the course from A to B, the second from B to D, and the third from D to E. BD and DE were each flown in three minutes, so that BD = DE, and because the included angle BDE = 60° the triangle BDE must be equilateral and BE = BD = DE. The aircraft therefore arrives at E after six minutes' flying instead of after three minutes' flying. The manœuvre has thus been completed with a minimum of inconvenience to navigation, and the aircraft is back on its original course.

Fig. 63 (ii) shows how the wind velocity is found. The circle is drawn with the air speed 200 m.p.h. to scale as radius. True north is marked and each course is drawn inward from the circumference to the centre. Taken in turn they are represented by AC, BC, and DC.

An angle representing the drift found on each course is then constructed at the point concerned, so that AG, BH, and DK represent the respective tracks which intersect at F. If the tracks do not all intersect at the same point, F may be placed at the centre of any triangle so formed. The line joining C to F then represents the wind velocity and is found to be from 290° T. at a speed of 40 M.P.H. Any navigational problem involving the triangle or polygon of velocities may be solved by means of the air-speed circle.

Exercise 5

Find the wind velocity by multiple-drift method from the following data:

	T.A.S. (knots)	*First course (T.)*	*Drift*	*Second course (T.)*	*Drift*	*Third course (T.)*	*Drift*
1.	150	050°	5° S.	350°	12° S.	110°	4° P.
2.	180	240°	10° P.	300°	9° P.	180°	3° P.
3.	120	150°	12° P.	210°	6° P.	090°	6° P.
4.	200	330°	8° S.	020°	3° S.	260°	6° S.
5.	240	105°	5° S.	180°	7° S.	045°	4° P.
6.	220	027°	6° P.	330°	3° S.	090°	11° P.
7.	160	200°	9° S.	260°	0°	150°	11° S.

Drift and Wind-lane Method. When over the sea the direction of the wind may often be obtained from an observation of the waves. This, of course, is surface direction, so as the wind veers—*i.e.*, changes direction clockwise, with altitude an allowance must be added to the surface direction. This is reckoned as 10° at 1500 ft. and proportionately less at lower altitudes, although the amount often varies with strength of wind. Above 1500 ft. the amount is less certain. If shortly before or afterwards the drift angle is observed from some fixed object on the coast, four parts of the triangle of velocities will be known, and the triangle may be completed, so discovering the other two—*i.e.*, wind speed and ground speed.

The method of construction is first to draw the course, then, by applying the drift angle at the beginning, put in the track. Mark the true air speed along the course vector, and from the free end draw in the wind direction, allowing for veer, to cross the track. This distance represents wind speed. As the observed wind direction may be the correct one or its reciprocal both directions are usually given, but no mistake need be made so long as it is remembered that wind blows from course to track. Here is a problem of this type.

Example. Flying at an altitude of 1500 ft. and steering a course of 330° T. at a T.A.S. of 210 knots, the angle of drift on crossing the coast was observed to be 8° to port and a few minutes later the wind was estimated by wave direction to be from 035° or 215°. What was the wind speed and G.S.?

Fig. 64 is drawn to illustrate the problem.

KL is drawn in a direction of 330° T. and an angle LKM constructed to represent port drift so that KM lies 8° to the left of KL. The true air speed is 210 knots, and we may measure a full hour's air speed from K towards L as a consideration of time does not enter the question. The wind vector will then represent the wind velocity per hour. KP is measured to scale to represent the air speed and a line drawn through P to represent the wind direction, allowance being made for 10° veer. As the drift is being made to port the wind must be blowing from 045°, so that the wind vector is drawn down-wind (*i.e.*, in the direction 225°) and cuts the track KM in Q. PQ then represents the wind velocity per hour to scale and KQ the ground speed. These are found to be 28 knots and 205 knots respectively.

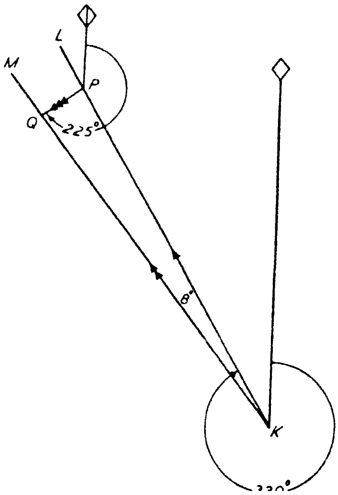


Fig. 64. Wind-lane Method

Exercise 6

Using the drift and wind-lane method, find the wind velocity and ground speed (ignoring wind veer) from the following data.

	T.A.S. (knots)	*Course (T.)*	*Drift*	*Wind-lanes*
1.	150	120°	6° P.	000°—180°
2.	200	045°	7° S.	120°—300°
3.	120	300°	10° S.	050°—230°
4.	180	240°	9° P.	170°—350°
5.	220	165°	6° P.	060°—240°
6.	160	010°	9° S.	090°—270°
7.	250	315°	8° P.	010°—190°

Drift and Bearing Method. As in the drift and wind-lane method the angle of drift is measured after the aircraft has passed over a prominent object, or, in other words, the back bearing of an object is taken. The aircraft then alters course appreciably and a second bearing of the object is taken after a further short interval. The point at which the second bearing intersects the back bearing is then joined to the position of the prominent object by a straight line, which then represents the wind velocity for the period of time from passing over the object to taking the second bearing.

Example. An aircraft steering 300° T. at T.A.S. 180 knots passes over a prominent object. Three minutes later the back bearing of the object is 112° T., and course is altered to 050° T. After a further two minutes' flying the object is found to bear 145° T. What is the wind velocity?

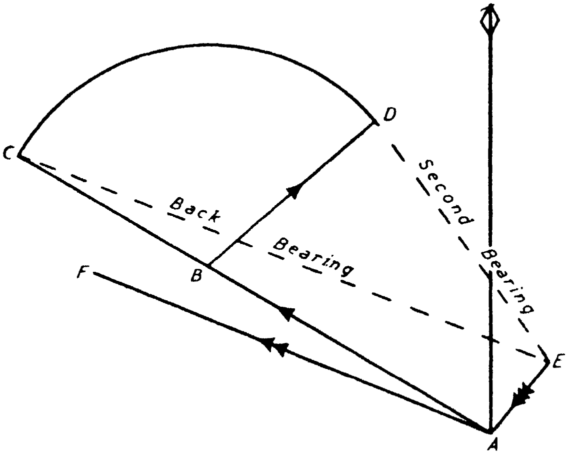


Fig. 65. Drift and Bearing Method

Fig. 65 shows how the problem is solved. AC is drawn in the direction 300° T. to represent the first course over the full period of five minutes. The back bearing 112° T. is then laid in, which shows that the track was really 292° T. and the aircraft must have been somewhere along AF up to the time of altering course. After three minutes' flying, corresponding to 9 nautical miles along the first course, an alteration of course was made to 050° T. for two minutes, when a second bearing of A was taken. This, being 145° T., cuts the back bearing at a point E, which when joined to A gives the wind velocity as from 040° T. at 2·4 n.m. in five minutes, a speed of 28·8 knots.

Exercise 7

Find the wind velocity by the drift and bearing method from the following data.

	T.A.S. (*knots*)	*First course* (*T.*)	*Interval* (*mins.*)	*Back bearing* (*T.*)	*Second course* (*T.*)	*Interval* (*mins.*)	*Second bearing* (*T.*)
1.	180	300°	2	125°	360°	3	160°
2.	200	020°	2	193°	090°	2½	225°
3.	150	240°	3	070°	180°	3	030°
4.	220	180°	1	352°	250°	2	041°
5.	120	045°	3	235°	340°	4	182°
6.	250	150°	1	324°	080°	2	282°

Reverse-course Method. When a course is set while over a prominent object and continued for a given number of minutes, the reciprocal course would bring the aircraft back over the object again in the same time if there were no wind. If, after steering a certain course from an object for, say, four minutes, and then the reciprocal course for four minutes, the aircraft is not over the object, the effect of the atmosphere in eight minutes has been to carry the aircraft from the object to the actual position at the end of eight minutes' flying. By steering towards the object the aircraft will be flying up-wind and the course will be the same as the wind direction. The difference between air speed and ground speed will give the wind speed.

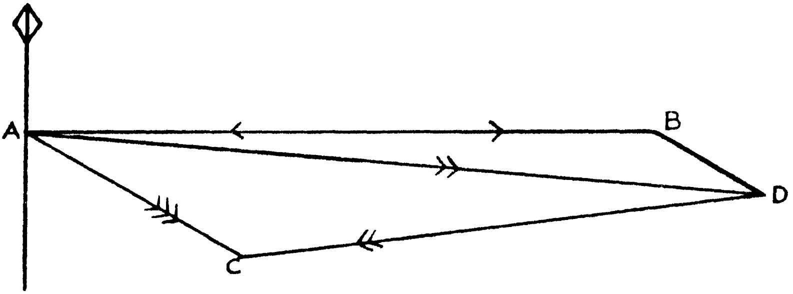


Fig. 66. Reverse-course Method

Example. An aircraft flies over a prominent object steering 090° T. at a T.A.S. of 120 knots. Four minutes later the course is reversed, and after flying for a further four minutes the aircraft has to be steered on a course of 300° T. for two minutes in order to reach the object. What is the wind velocity?

In Fig. 66 AB represents the course out for four minutes and BA the course back. At the end of eight minutes the aircraft finds the object A to be in a direction 300° T. which is, therefore, the direction of the wind. As the distance from A is not given the wind speed is found from the formula:

$$\text{Wind speed} = \frac{\text{T.A.S.} \times \text{time (secs.) on third course}}{\text{Total time (secs.)} + \text{time (secs.) on third course}}$$

$$= \frac{120 \times 120}{480 + 120}$$

$$= \frac{144\cancel{0}\cancel{0}}{6\cancel{0}\cancel{0}} = \underline{24 \text{ knots}}$$

At the end of eight minutes' flying, therefore, the aircraft must have been at a position C, 120° from A, distant $\frac{8}{60} \times 24 = 3{\cdot}2$ nautical miles. After four minutes' flying the aircraft must have been at D where BD = ½ AC. AD was, therefore, the track out and DC the track back.

Exercise 8

Using the reverse-course method find the wind velocity from the following data.

	Course out (T.)	*T.A.S. (knots)*	*Time out*	*Bearing of object (T.)*	*Time to reach object*
1.	030°	150	3 mins.	100°	1 min.
2.	240°	200	5 mins.	300°	45 secs.
3.	160°	180	6 mins.	080°	90 secs.
4.	320°	160	4½ mins.	270°	100 secs.
5.	045°	120	7½ mins.	225°	3 mins.

The following problem, including examples of the four simple cases of the triangle of velocities, has been worked out on the example chart (in pocket at end of book) to which the student should now turn.

Example. An aircraft sets out at 10.10 hrs. from an aerodrome A 51° 20′ N. 0° 50′ W. to fly to LINCOLN via NORTH FORELAND and YARMOUTH at a T.A.S. of 150 knots, the wind being regarded as negligible. Give distance, track, and true course.

10.38 hrs. DEAL. Find track made good, wind velocity, and ground speed. Followed coast to NORTH FORELAND.

10.42 hrs. NORTH FORELAND. Set course for YARMOUTH using wind velocity 300°/30 knots.

10.44 hrs. Back bearing of NORTH FORELAND 183° T.
What is the drift angle?

10.48 hrs. Direction of wind by observation of waves 260°–080°.
Give wind velocity and ground speed. Altered course 4° to starboard.

11.11 hrs. YARMOUTH. Set course for LINCOLN, steering 290° T.

11.25 hrs. Give the expected position, assuming the wind to be from 270° at 25 knots, also ground speed and estimated time of arrival (E.T.A.) at LINCOLN.

There are one or two points to be noted before an explanation of the plotting can be given. Firstly, as already explained, nautical miles are minutes of latitude, and distance must be measured along the latitude scale whenever nautical miles are being used, care being taken to measure the minutes of latitude opposite the line whose length is required. A knot is a nautical mile per hour, but in constructing triangles of velocities a constant scale is required and minutes of longitude may be used if desired for this purpose, provided that they are never used for measuring distance, but only to represent speeds.

The first step in the problem is to join the aerodrome A to North Foreland lighthouse and measure the distance in nautical miles by taking the distance off with dividers and putting it alongside the latitude scale opposite. This is found to give 87 nautical miles. The track is the direction which the line joining the two places makes with any meridian, and measures 088° T., which must be the same as the course, no wind being assumed. If that had actually been the case the aircraft would have been over position B 28 minutes after leaving A, but we know that it is over DEAL. Joining A to Deal we find the track made good (T.M.G.) to be 095° T., and joining B to Deal gives the wind vector 305°/37 knots. The distance from A to Deal is 85 n.m., so that ground speed is $\frac{85}{28} \times 60 = 182$ knots. After reaching North Foreland, course is set to Yarmouth by laying in the wind vector at the beginning, using constant longitude scale (1′ = 2 knots). Taking the T.A.S. as radius to the same scale on a pair of compasses, and using G as centre, the arc will intersect the track at H. GH is then the course vector and is found to make an angle of 355° with the true meridian. As the aircraft has passed over North Foreland lighthouse the reversed back bearing must represent the track, which is then drawn in from North Foreland in the direction 003° T. making an angle of 8° with the transferred course line. This is the angle of drift to starboard. The angle between the observed track and the estimated track is known as the **track error** (T.E.). It is 3° to port. The air speed for 15 minutes is now measured from North Foreland along the course to K. Any interval could be used, but 15 minutes is convenient, and the wind direction, as observed, put at the end. This must be from 260° as the track lies to starboard. KM then represents the wind velocity for 15 minutes, giving 20 knots, the ground speed NM being found to give 154 knots. From Yarmouth a true course of 290° is steered. After 14 minutes a distance corresponding to the air speed in latitude units is measured along it, reaching the point V. The wind vector VX is then added and is found to give a position of 52° 48′ N. 00° 56′ E. The distance YX is 29½ n.m., and this gives a ground speed of 127 knots. As the distance from Y. to L. is 90 n.m., 42 minutes must be occupied on the flight at the speed, making the estimated time of arrival (E.T.A.) 11.11 + 42 = 11.53 hrs.

Supplementary Exercises

As revision the following exercises should be worked on a tracing of the Mercator chart provided for plotting purposes, using Form 316*a* or Nautical Deviation Card 1 as required (pp. 110–111).

Exercise 9

Find the aircraft's track, G.S., and position after one hour's flying.

	Departure point	*Compass course*	*T.A.S. (knots)*	*Wind velocity*
1.	53° 44′ N. 00° 21′ W.	080°	120	315°/20 knots
2.	53° 13′ N. 00° 33′ W.	100°	150	040°/30 knots
3.	52° 12′ N. 00° 08′ E.	035°	140	150°/24 knots
4.	52° 45′ N. 00° 24′ E.	120°	175	220°/36 knots
5.	52° 05′ N. 05° 08′ E.	270°	125	170°/15 knots
6.	51° 13′ N. 04° 25′ E.	300°	160	010°/28 knots

Use the variation as found for the mean position.

Exercise 10

Find the ship's course made good, effective speed, and position after five hours' steaming.

	Departure point	*Compass course*	*Steaming speed (knots)*	*Current*
1.	51° 32′ N. 03° 26′ E.	348°	10·4	240°/1·4 knots
2.	52° 36′ N. 01° 47′ E.	086°	12·0	180°/1·6 knots
3.	54° 29′ N. 00° 30′ W.	139°	20·8	150°/1·8 knots
4.	51° 22′ N. 01° 27′ E.	054°	13·2	020°/2·4 knots
5.	52° 58′ N. 04° 42′ E.	251°	16·6	000°/1·5 knots
6.	52° 56′ N. 01° 20′ E.	017°	15·0	300°/2·0 knots

Use the variation found locally on the chart to the nearest degree.

Exercise 11

Find the course (C.) and T.A.S. required to fly from A to B in the given time.

	A	*B*	*Time hrs.*	*mins.*	*Wind velocity*
1.	54° 10′ N. 0° 30′ W.	52° 10′ N. 1° 20′ E.	1	00	250°/24 knots
2.	52° 35′ N. 1° 00′ E.	52° 20′ N. 4° 50′ E.	1	00	340°/30 knots
3.	53° 10′ N. 5° 30′ E.	51° 40′ N. 0° 20′ W.	2	00	300°/26 knots
4.	51° 50′ N. 0° 40′ E.	52° 55′ N. 0° 35′ W.	0	30	270°/20 knots
5.	52° 20′ N. 0° 10′ W.	53° 50′ N. 0° 15′ W.	0	40	070°/15 knots
6.	53° 20′ N. 0° 10′ E.	52° 05′ N. 4° 20′ E.	1	30	020°/28 knots

Exercise 12

Find the compass course and steaming speed required by a ship to steam from A to B in the given time.

	A	*B*	*Time hrs.*	*mins.*	*Average current*
1.	54° 10′ N. 0° 10′ E.	53° 35′ N. 0° 20′ E.	3	00	330°/2·0 knots
2.	52° 40′ N. 2° 20′ E.	52° 00′ N. 3° 55′ E.	5	00	200°/1·5 knots
3.	51° 40′ N. 1° 45′ E.	52° 35′ N. 2° 00′ E.	4	00	040°/1·8 knots
4.	53° 00′ N. 4° 10′ E.	51° 40′ N. 2° 30′ E.	6	00	180°/2·2 knots
5.	51° 50′ N. 3° 20′ E.	52° 45′ N. 1° 55′ E.	5	30	020°/1·6 knots
6.	53° 30′ N. 4° 40′ E.	53° 50′ N. 6° 05′ E.	3	40	270°/1·9 knots

Exercise 13

Find the course (C.) an aircraft must steer in order to proceed along a track from A to B, and give the G.S.

	A	*B*	*T.A.S. (knots)*	*Wind velocity*
1.	52° 38′ N. 01° 17′ E.	51° 55′ N. 04° 27′ E.	120	180°/30 knots
2.	53° 45′ N. 00° 20′ W.	52° 03′ N. 01° 09′ E.	150	270°/24 knots
3.	52° 05′ N. 04° 20′ E.	52° 15′ N. 00° 10′ W.	140	060°/20 knots
4.	51° 40′ N. 00° 30′ W.	53° 20′ N. 00° 10′ E.	110	330°/25 knots
5.	52° 45′ N. 00° 25′ E.	52° 20′ N. 04° 50′ E.	180	030°/35 knots
6.	51° 15′ N. 04° 25′ E.	51° 45′ N. 00° 15′ W.	160	220°/28 knots

Exercise 14

Find the compass course a ship must steam in order to proceed along a route from A to B, and give the effective speed.

	A	*B*	*Steaming speed (knots)*	*Current*
1.	54° 10′ N. 00° 10′ E.	53° 30′ N. 00° 55′ E.	15·0	180°/2·0 knots
2.	51° 35′ N. 01° 45′ E.	52° 58′ N. 04° 19′ E.	20·0	000°/2·5 knots
3.	52° 55′ N. 04° 20′ E.	52° 40′ N. 02° 20′ E.	12·0	210°/1·5 knots
4.	53° 35′ N. 00° 30′ E.	53° 55′ N. 02° 40′ E.	13·5	330°/1·8 knots
5.	53° 45′ N. 01° 40′ E.	54° 30′ N. 00° 15′ W.	16·2	150°/1·6 knots
6.	52° 40′ N. 02° 20′ E.	52° 00′ N. 03° 50′ E.	14·4	350°/1·4 knots

Exercise 15

Find the T.M.G., G.S., and W/V experienced by an aircraft flying from A under the following conditions.

	A	*Compass course*	*T.A.S. (knots)*	*Flying time hrs.*	*mins.*	*Final position*
1.	53° 08′ N. 00° 20′ E.	105°	150	1	00	52° 28′ N. 04° 35′ E.
2.	53° 45′ N. 00° 20′ W.	185°	120	1	00	52° 00′ N. 00° 12′ E.
3.	52° 13′ N. 00° 08′ E.	050°	120	0	30	52° 55′ N. 01° 20′ E.
4.	51° 56′ N. 04° 27′ E.	314°	135	1	30	53° 13′ N. 00° 33′ W.
5.	52° 03′ N. 01° 09′ E.	133°	140	0	36	51° 12′ N. 03° 14′ E.
6.	51° 20′ N. 03° 30′ E.	334°	160	1	15	53° 08′ N. 00° 20′ W.

Exercise 16

Find the course made good by a ship, her effective speed, and the current experienced in steaming from A under the following conditions.

	A	*Compass course*	*Steaming speed (knots)*	*Steaming time hrs.*	*mins.*	*Final position*
1.	53° 34′ N. 00° 22′ E.	090°	10·0	5	00	53° 35′ N. 01° 45′ E.
2.	52° 00′ N. 03° 50′ E.	273°	12·0	5	00	51° 55′ N. 02° 10′ E.
3.	52° 55′ N. 04° 20′ E.	241°	20·0	4	00	52° 10′ N. 02° 40′ E.
4.	53° 30′ N. 00° 50′ E.	348°	15·0	4	20	54° 30′ N. 00° 10′ W.
5.	52° 40′ N. 02° 00′ E.	159°	14·5	3	45	51° 40′ N. 02° 30′ E.
6.	51° 50′ N. 01° 45′ E.	021°	16·8	3	15	52° 35′ N. 02° 00′ E.

Exercise 17

The following practical problems provide exercise in the combined use of triangles of velocities on the chart, of which a tracing should be taken. Use the mean variation for each course, and Form 316*b* (p .iii) as required. Consult the list of abbreviations (p. 9) where necessary.

1. Give the tracks and distances on the round flight from 53° 10′ N. 0° 10′ E. via 52° 50′ N. 6° 15′ E., Antwerp, and Reading (51° 27′ N. 0° 59′ W.) back to 53° 10′ N. 0° 10′ E.
   10.15 hours S/C. T.A.S. 160 knots. W/V 320°/24 knots.
   Give first compass course (dev. 3° E.), G.S., and E.T.A.
   11.12 hours P.P. 53° 11′ N. 4° 51′ E.
   Find T.M.G., G.S., and W/V experienced.
   Using W/V 275°/23 knots, find also
   Compass course (dev. 3° E.) along new track to 52° 50′ N. 6° 15′ E. and along the remaining tracks (devs. 1° W., 4° W., and 2° E. resp.). Give each G.S. and time occupied in flying from one turning point to the next.
2. Leaving 53° 20′ N. 0° 10′ E. at 10.20 hrs., S/C for Rotterdam. T.A.S. 160 knots, W/V 240°/25 knots.
   Give: track, distance, course, T., M., and C., G.S., and E.T.A.
   10.45. hrs. Newarp light-vessel.
   Give: T.M.G., G.S. and W/V experienced, adjusted course to Rotterdam using W/V 229°/29 knots, new G.S. and E.T.A.
3. 19.15 hrs. S/C 070° T. from Cambridge.
   T.A.S. 150 knots.
   19.35 hrs. Yarmouth
   Give: air position
   W/V experienced.
   19.40 hrs. A/C for Helder (52° 57′ N. 4° 45′ E.) using W/V 242°/40 knots
   Give: Track
   Course T.
   E.T.A.
   20.15 hrs. Helder
   Give: W/V experienced.
4. 10.00 hrs. S/C 080° C. from Cambridge
   T.A.S. 150 knots
   10.30 hrs. Newarp light-vessel
   Give: Air position
   W/V experienced.
   10.35 hrs. S/C for 53° 30′ N. 4° 10′ E. using new W/V
   Give: track, course, T., M., and C., G.S. and E.T.A.
   11.08 hrs. A/C 220° C.
   11.45 hrs. Give: expected position.
5. 11.45 hrs. S/C 070° C. from 51° 50′ N. 0° 10′ W.
   T.A.S. 150 knots.
   12.02 hrs. Ipswich (52° 03′ N. 1° 09′ E.)
   Required: Course T.
   T.M.G.
   G.S.
   W/V experienced.
   Give time and position at which you expect to cross the coast.
   Follow the coast to Lowestoft.
   12.20 hrs. Lowestoft S/C for 53° 10′ N. 5° 50′ E.
   W/V 310°/35 knots.

12.45 hrs. Give: expected position.
E.T.A. at destination.
6. 10.45 hrs. S/C 070° C. from 52° 10′ N. 0° 10′ E.
T.A.S. 140 knots.
11.10 hrs. Yarmouth
Give: T.M.G.
Drift
G.S.
W/V experienced
S/C for Ijmuiden using W/V 270°/20 knots.
Give: Track
Distance
Course, T., M., and C.
G.S.
E.T.A.
11.50 hrs. Ijmuiden S/C 245° C.
W/V 270°/20 knots.
Give: Track
G.S.
12.15 hrs. Give: Position
Track to starting-point.
Distance to starting-point.
Course, T., M., and C.
G.S.
E.T.A.
7. 09.12 hrs. Norwich S/C 080° C. T.A.S. 160 knots.
09.22 hrs. Newarp light-vessel
Give: T.M.G.
G.S.
W/V experienced
09.36 hrs. Give: expected position
Using W/V 095°/20 knots reset course so as to arrive at 54° 00′ N. 5° 00′ E. at 10.30 hours, and give T.A.S. necessary to do this.
Circle this position for 10 mins. and reset course to 53° 20′ N. 0° 10′ E., T.A.S. 150 knots.
Give: E.T.A.
8. 14.30 hrs. Whitby S/C 098° T. T.A.S. 150 knots.
15.30 hrs. A/C 200° T.
16.15 hrs. Yarmouth
Give: W/V experienced
A/C for Whitby making no wind allowance
Give: Track and course.
16.45 hrs. Spurn Head lighthouse (53° 34′ N. 00° 07′ E.)
Give: T.M.G.
G.S.
Drift
W/V experienced
New track to Whitby
Course using W/V 080°/30 knots
E.T.A.
9. 07.14 hrs. S/C from 52° 30′ N. 0° 10′ E.
to 51° 30′ N. 4° 50′ E.
Give: Track and distance
Using T.A.S. 140 knots, and W/V 070°/30 knots.
Give: Course, T., M., and C.

G.S.
E.T.A.
07·34 hrs. Norwich bears 010° C.
Southwold (52° 20′ N. 1° 41′ E.) bears 100° C.
Find: Position
T.M.G.
G.S.
W/V experienced
Expected time of crossing coast.

On crossing coast readjust course to destination allowing for W/V 082°/14 knots.

General Exercises

1. Construct a triangle of velocities to find the track and ground speed of an aircraft under the following conditions:

	Course (T.)	*T.A.S. (knots)*	*Wind velocity*	
(*a*)	296°	185	165°/37 knots	
(*b*)	005°	160	130°/26 knots	
(*c*)	047°	210	169°/46 knots	
(*d*)	350°	270	101°/44 knots	Give also the drift
(*e*)	030°	215	157°/40 knots	Give also the drift
(*f*)	356°	125	247°/30 knots	Give also the drift

2. By means of the triangle of velocities find the course to steer to make good the given tracks under the following conditions:

	Track	*T.A.S. (knots)*	*Wind velocity*	
(*a*)	200°	230	330°/24 knots	
(*b*)	240°	200	100°/36 knots	
(*c*)	312°	180	270°/32 knots	Give also G.S. and drift
(*d*)	116°	148	080°/40 knots	Give also G.S. and drift
(*e*)	010°	260	250°/45 knots	Give also G.S. and drift
(*f*)	045°	130	175°/20 knots	Give also G.S. and drift

3. Find the wind velocity experienced under the following conditions:

	Course (T.)	*T.A.S. (knots)*	*Track*	*G.S.*
(*a*)	260°	240	250°	220 knots
(*b*)	075°	190	075°	218 knots
(*c*)	310°	250	318°	262 knots
(*d*)	354°	120	006°	124 knots
(*e*)	144°	220	150°	11·9 n.m. in 3½ mins.
(*f*)	188°	280	197°	11·0 n.m. in 2½ mins.

4. Using the following information find G.S. and wind speed:

	Course (T.)	*T.A.S. (knots)*	*Track*	*Wind direction*
(*a*)	190°	170	182°	310° T.
(*b*)	085°	150	093°	020° T.
(*c*)	330°	200	322°	100° T.

5. If a vessel steered the following courses at the given speed, what course and speed would she make good in each case when the current set as indicated?

	Course steered		*Current*	
	(T.)	*Speed (knots)*	*Set (T.)*	*Rate (knots)*
(*a*)	060°	9·5	190°	1·5
(*b*)	344°	12·7	025°	2·2
(*c*)	255°	16·2	210°	3·0
(*d*)	172°	13·4	305°	1·8

6. Find the course a vessel must steer under the following circumstances. Give her effective speed.

	Course to make good		*Current*	
	(T.)	*Speed (knots)*	*Set (T.)*	*Rate (knots)*
(*a*)	040°	10	000°	2
(*b*)	324°	12	250°	1
(*c*)	112°	18	060°	1½
(*d*)	286°	9	150°	2
(*e*)	357°	11	232°	2½
(*f*)	004°	14	300°	3

7. What would be the set of the current and the rate of drift under the following circumstances?

	Course steered (T.)	*Speed by log (knots)*	*Course made good (T.)*	*Effective speed (knots)*
(*a*)	090°	11·2	085°	10·6
(*b*)	244°	9·5	240°	9·8
(*c*)	312°	15·0	315°	14·4
(*d*)	152°	12·7	159°	12·2

8. Find the speed at which it would be necessary to steam, and the course to steer, in order to comply with the following conditions:

				Current	
	Course to make good	*Distance*	*Time*	*Set*	*Rate*
(*a*)	300° (T.)	200 n.m.	24 hrs.	180° (T.)	2 knots
(*b*)	045° (T.)	350 n.m.	28 hrs.	340° (T.)	1½ knots

9. Using the air-speed circle, find W/V by multiple-drift method.

T.A.S.	*Course (T.)*	*Drift*	*Course (T.)*	*Drift*
(*a*) 200 knots	020°	8° S.	090°	6° P.
(*b*) 160 knots	300°	5° P.	200°	7° S.
(*c*) 240 knots	140°	10° P.	060°	3° P.

CHAPTER VII

DEAD RECKONING

The air plot—Plotting without a chart—Departure into d. long—Parallel sailing—Distance along a parallel—Mean latitude—Middle latitude—Traverse table—Plane, or traverse sailing—Continued traverse or day's work—Mercator sailing—Great-circle navigation—Comparison of nautical and aeronautical terms.

The Air Plot. When we require to find the position of an aircraft without reference to fixed objects our track or course must be plotted on a Mercator chart, and the distance flown marked off along it. Where the course only is plotted the positions along it at any time are known as air positions—that is, they are positions arrived at without consideration of wind effect. When wind effect is allowed for by the addition of a line representing it drawn at the end of the course, the position so deduced is known as a position by dead reckoning (D.R.). If several alterations of course are made before a D.R. position is required, it is unnecessary to draw in the wind at the end of each course but merely apply the total wind effect at the end of the last course. The following worked example will help to make this clear.

Example. An aircraft left A (latitude 49° 50′ N. longitude 5° 20′ W.) at 09.00 hrs. steering 250° T. at a speed of 150 knots through a wind blowing from 120° at 20 knots, and the following alterations of course were subsequently made.

Hrs.	(*T.*)
09.10	A/C 210°
09.16	A/C 180°
09.24	A/C 130°
09.31	A/C 040°
09.37½	Aircraft recalled.

What would be the air position and D.R. position of the aircraft at 09.10 hrs. and 09.37½ hrs.? Give also the track and true course to steer from the D.R. position at 09.37½ hrs. to reach the starting-point A.

Fig. 67 shows a section of a Mercator plotting-chart, on which the problem has been worked out. A marks the starting-point and B, C, D, and E the respective air positions at the times of altering course. The distances along the course lines are worked out according to time-interval and speed, and measured in units of minutes of latitude taken from the side of the chart. The first air position at B is obtained by measuring a distance equivalent to ten minutes' air speed, *i.e.*, $\frac{10 \times 150}{60} = 25$ nautical miles (minutes of latitude) in a direction 250° T. from A. The latitude, on reference to the side of the chart, is found to be 49° 41½′ N., and the longitude, on reference to the bottom of the chart, is found to be 5° 55′ W. The corresponding D.R. position at R is obtained by applying wind effect—*i.e.*, drawing a line from B in the direction towards which the wind is blowing (300° T.)—and measuring along it a distance equivalent to 10 minutes' wind speed, $\frac{10 \times 20}{60} = 3\cdot3$ nautical miles. The first D.R. position is then found to be 49° 43′ N. 5° 59′ W.

The final air position is at F, whose latitude and longitude are found to be 49° 09′ N. and 5° 30′ W. respectively. The effect of the wind from 120° T. at 20 knots for 37½ minutes since leaving A is represented by FG in the direction 300° T. for 12½ nautical miles. Position G is then the D.R. position, and found to be 49° 15′ N. 5° 47′ W. To find the track back to A, G is joined to A by a straight line and the angle this makes with the direction of true north is the track. Measured by Douglas protractor, it gives 026°. To find the course we construct a triangle of velocities, as explained in the last chapter, by drawing a wind vector downwind from G, GL in the Figure, and from the free end L use the T.A.S. as a radius to intersect the track at a point M. Any scale will do for constructing the sides of the triangle, provided it is constant and applied to all three sides. The longitude scale is useful here, but when measuring distance and not speed—*e.g.*, G to A—the latitude scale must invariably be used and always that scale in the same latitude as the distance being measured. As explained in Chapter III, the latitude scale is not constant on a Mercator chart but increases towards the poles. On measuring by protractor the angle which LM makes with the direction of true north we find the course to be 033½° T. The ground speed GM divided into the distance GA will give the time taken from G to A. This, added to 09.37½ hrs., will give the estimated time of arrival (E.T.A.) at A.

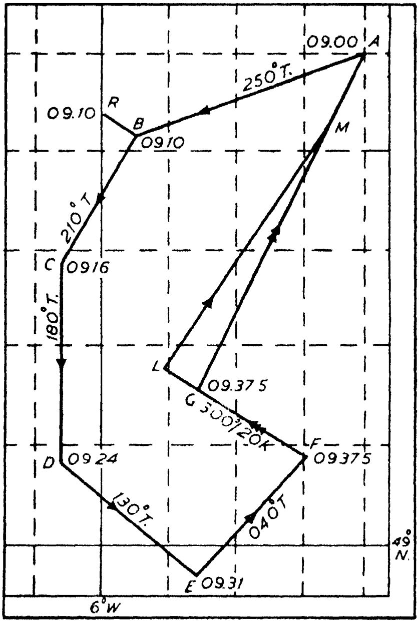


Fig. 67. The Air Plot

Wind Finding by Air Plot. Suppose that instead of using a forecast wind we assume there is no wind at all, we should imagine ourselves to be actually at F (Fig. 67) at 09.37½ hrs., whereas, in fact, we may by observation have found our position to be over some point G. This would show that we were wrong in our assumption that there was no wind, because we were in a position which was not the air position. The conclusion we must come to is that the wind has been blowing in a direction from the air position to the observed position at a speed corresponding to the distance between them covered in the time since the air plot began. In our example the distance is 12½ n.m. in 37½ mins., corresponding to a speed of $\frac{12\frac{1}{2} \times 60}{37\frac{1}{2}} = 20$ knots.

Exercise 1

From each of the following sets of details construct an air plot on a tracing of the Practice Chart supplied. Find the final air position, the D.R. position, and also the track and course to return to the starting point, together with the estimated time of arrival (E.T.A.) there. Where the position by observation or radio is given, find the wind velocity.

1. Point of departure A 53° 10′ N. 0° 20′ W.

T.A.S. 120 knots. W/V { First hour 225°/30 knots.
Remainder 280°/20 knots.

Time (*hrs.*)		(*T.*)
12.00	S/C	075°
12.40	A/C	020°
13.10	A/C	110°
14.00	A/C	260°
14.30	Recalled to A.	

2. Point of departure A 52° 30′ N. 1° 20′ E.

T.A.S. 100 knots. W/V { 190°/30 knots for 1¼ hours.
220°/20 knots for remainder.

Time (*hrs.*)		(*T.*)
06.00	S/C	350°
06.36	A/C	038°
07.00	A/C	115°
07.30	A/C	170°
08.06	A/C	237°
08.36	Recalled to A.	

3. Point of departure A 53° 08′ N. 0° 40′ W.

T.A.S. 120 knots.

Time (*hrs.*)		(*T.*)
14.00	S/C	038°
14.30	A/C	076°
14.55	A/C	200°
15.20	A/C	130°
16.00	A/C	042°
16.40	Position by observation	53° 38′ N. 6° 18′ E.
16.40	S/C to A.	

4. Point of departure A 53° 25′ N. 00° 20′ W.

T.A.S. 120 knots. W/V 220°/30 knots for first hour.
260°/27 knots for remainder.

Time (*hrs.*)		(*T.*)
07.00	S/C	048°
07.20	A/C	333°
07.45	A/C	100°
08.15	A/C	110°
08.35	A/C	190°
09.20	position by radio	53° 19′ N. 3° 50′ E.
09.20	S/C to A.	

Plotting without a Chart. To find the position of an aircraft or a ship after several alterations of course have been made, it is not imperative to plot the courses on a chart, they may be plotted on squared or plain paper (see Fig. 75). A scale for distance must then be selected and, as a nautical mile and a minute of latitude are synonymous, the scale for latitude also is thereby fixed. **Distance in nautical miles made good east or west is termed departure,** but when a position is required in latitude and longitude, departure must be converted into difference of longitude (d. long.). A definite relationship between departure and d. long. must, therefore, be established before a position can be ascertained fully.

Departure into D. Long. This is a question of reckoning a given distance in terms of different units, for although nautical miles in which departure is reckoned may be regarded as constant all over the world, no such claim can be made for minutes of longitude. We know that minutes of longitude become smaller as we move towards the pole but are constant in size along any parallel of latitude. The gauge we must use in comparing the respective lengths of a nautical mile and a minute of longitude is then the latitude. Minutes of longitude at the equator are for practical purposes the same length as a nautical mile. So in comparing one minute of longitude at the equator with one minute of longitude in any other latitude we are, in effect, finding the ratio between departure and d. long. for that particular latitude. Fig. 20 (Chapter III) shows how the ratio between the lengths of corresponding arcs on the equator and on any parallel of latitude may be found. Two meridians intercept equal arcs of any two circles of latitude, so that EF represents the same number of minutes of longitude as AB but the length of EF in nautical miles is less than that of AB. So if we know the length of EF in nautical miles—*i.e.*, the departure—we shall know the d. long. it represents if we can find the length of AB in the same units. Now the circumference of a circle depends upon the length of its radius, so that equal arcs of any two circles will be respectively proportionate to their radii. To compare EF with AB we need only compare the radius DF with the radius CB. Now CB = CF because both are radii of the sphere, and so by comparing DF with CF we are comparing EF with AB. DF and CF are both in the triangle CFD, and angle DFC = angle BCF = latitude of F. If then we construct a right-angled triangle by drawing DF to scale to represent the departure, and construct an angle DFC to represent the latitude, the hypotenuse FC will represent the d. long. to scale.

Mathematically it will be seen from the triangle CFD (Fig. 20) that $\frac{\text{FC}}{\text{DF}} = \text{sec. DFC}$ or, $\frac{\text{d. long.}}{\text{dep.}} = \text{sec. lat}$—*i.e.*, d. long. = dep. × sec. lat.

Parallel Sailing. If a ship sails or an aircraft flies along a parallel of latitude, the distance sailed or flown need only be multiplied by the secant of the latitude in order to find the difference, or change, of longitude.

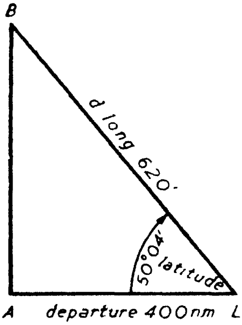


Fig. 68. Departure into D. Long.

Example. A ship (an aircraft) leaves Land's End, 50° 04′ N. 5° 43′ W. on a course (track) 270° T. What would be the longitude of the ship (aircraft) after covering 400 miles?

Fig. 68 shows how the problem of finding the d. long. may be solved graphically from a scale-drawing in which the base-line LA represents 400 miles westing while the angle L represents the latitude of Land's End 50° 04′ N. The length LB of the hypotenuse cut off by a perpendicular erected at the opposite end A of the base-line represents the difference of longitude to the same scale. Its value on measurement is found to be 620 minutes—*i.e.*, 10° 20′ W.—which when applied to 5° 43′ W. gives 16° 03′ W. as the longitude.

By calculation:

d. long. = dep. × sec.lat.
dep. = 400 n.m. log. = 2·602060
lat. = 50° 40′ log. sec. = 0·192535
∴ d. long. = 623·2′ log. = 2·794595
d. long. = 10° 23·2′ W.
Longitude of Land's End = 5° 43·0′ W.
Longitude in = 16° 06·2′ W.

To Find Distance along a Parallel. To find the distance covered along a certain parallel when the d. long. is known a solution by scale drawing may be obtained approximately as follows. Draw a base line and from one end of it draw another line to make an angle equal in value to the latitude. Along this line measure the d. long. to scale and from the end drop a perpendicular. The length of the base-line cut off by the perpendicular will represent the departure to the same scale. To find the departure by calculation the formula used earlier for finding d. long. would have to be transposed so that:

$$\frac{\text{d. long.}}{\text{dep.}} = \text{sec. lat. becomes } \frac{\text{dep.}}{\text{d. long.}} = \text{cos. lat.}$$

I.e., departure = d. long. × cos. lat.

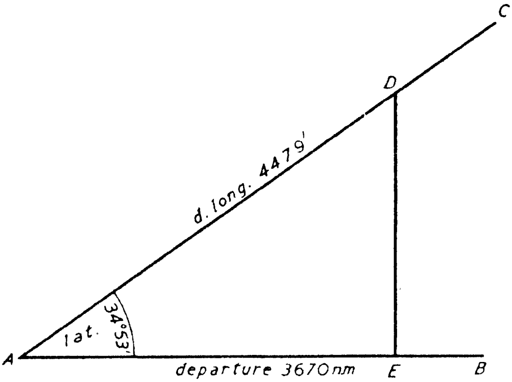


Fig. 69. D. Long. into Departure

Example. What distance would a ship (an aircraft) have to travel from Montevideo 34° 53′ S. 56° 10′ W. to the longitude of Cape Town 18° 29′ E. along a track of 090° T.

The difference, or change, of longitude is 56° 10′ + 18° 29′ = 74° 39′ E. = 4479′.

To solve graphically, draw a base-line AB Fig. 69 and from one end A draw a line AC to make an angle BAC equal to the value of the latitude. Along this line measure the d. long. AD to any convenient scale and drop a perpendicular from D. The part of the base-line AE cut off by this perpendicular will then represent the departure in the same scale as used for the d. long. On measurement the departure in this case is found to correspond to 3670 nautical miles. Owing to the small scale of the triangle this result must be regarded as only approximate. We obtain the accurate value by calculation as follows:

dep. = d. long. × cos. lat.

d.long.	= 4479′	log.	= 3·651181
lat.	= 34° 53′	log. cos.	= 9·913982
∴ dep.	= 3674 n.m.	log.	= 3·565163

Distance to travel = 3674 nautical miles.

Exercise 2

Supply the values of d. long., departure, or latitude missing from the following table:

	Latitude	*D. long.*	*Departure* (*n.m.*)
1.	25° 00′		642
2.	50° 00′		321
3.	66° 30′		560
4.	52° 20′	7° 45′	
5.	23° 30′	5° 50′	
6.	39° 40′	18° 20′	
7.		5° 00′	150
8.		11° 30′	480
9.		6° 40′	275

Mean Latitude. In making good a course or track in any direction other than east or west, the departure is not made good in one latitude only but throughout the entire difference of latitude between the two places concerned. To convert departure into d. long. under these circumstances, a parallel of latitude must be found along which the departure made good and the d. long. between the places are equivalent distances. As the ratio between dep. and d. long. cannot be the same at any two latitudes in one hemisphere, there can be but one latitude to satisfy the above conditions. Fig. 70, drawn to represent a section of a globe, shows that in going from A to B the departure must exceed the distance represented by AC but be less than the distance represented by DB. Because the dep. has taken place at a regular rate with the change of latitude, the parallel of latitude along which the departure and d. long. would correspond, must be half-way between those of the starting and finishing points. This is their arithmetic mean—found by adding the two latitudes together and dividing by two. The rule is valid only when the places are in the same hemisphere. When the equator is crossed the half-way latitude gives a somewhat different departure from that found by treating the section of course in each hemisphere separately. For example, places equidistant from the equator and different in longitude have a mean lat. of 0° and dep. equals d. long. If the sections are treated separately this is not so. Since the earth is not a perfect sphere, the usefulness of mean latitude for the calculation is limited to distances along the rhumb line up to 600 miles.

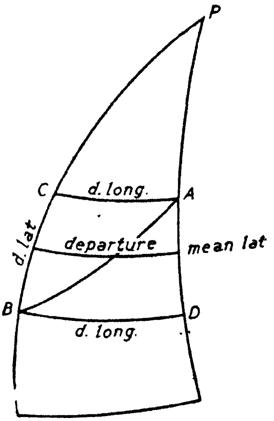


Fig. 70. MEAN LATITUDE

Middle Latitude. For distances exceeding 600 miles the use of mean latitude will not provide accurate results when converting dep. into d. long. or d. long. into dep. by calculation. This is due to the fact that, because the earth is not a perfect sphere, minutes of latitude vary slightly in length with change of latitude. Over a small change of latitude this difference is not appreciable but when the change of latitude approaches or exceeds 600′—*i.e.*, 10°—a correction must be added to the mean latitude in order to find the middle latitude now necessary in the calculation. A table of corrections is to be found in most volumes of nautical tables. Norie calls it Workman's Table.

Example. What would be the departure made good by a ship in a voyage from Madeira (33° 00′ N. 17° 00′ W.) to Queenstown (51° 51′ N. 8° 17′ W.)?

Madeira lat. =	33° 00′ N.	long. =	17° 00′ W.
Queenstown lat. =	51° 51′ N.	long. =	8° 17′ W.
	84° 51′	d. long. =	8° 43′ E.
mean lat. =	42° 25½′	d. long. =	523′
correction from Norie +	0° 45½′		
middle lat. =	43° 11′	d. lat. =	18° 51′ N.

dep. = d. long. × cos. lat.

d. long. =	523′	log. = 2·718502	log. = 2·718502
mean lat. =	42° 25½′	l. cos. = 9·868151	
mid. lat. =	43° 11′		l. cos. = 9·862827
dep. =	386·1 n.m.	log. = 2·586653	
dep. =	381·4 n.m.		log. = 2·581329
diff. =	4·7 n.m.		

The difference in departure caused by using the more accurate middle latitude in the calculation is thus seen to be 4·7 nautical miles.

Exercise 3

Convert d. long to departure in each of the following cases by using the mean latitude and give the differences of value involved when using middle latitude in the calculation:

	Latitude A	*Latitude B*	*D. long.*
1.	20° 00′ N.	28° 00′ N.	5° 00′
2.	35° 00′ S.	50° 00′ S.	8° 30′
3.	65° 00′ N.	47° 00′ N.	11° 00′
4.	24° 00′ S.	44° 00′ S.	7° 50′

Traverse Table. The nautical method of finding position from either one course or after frequent alterations of course have been made, consists of calculation rather than plotting. Each true course and distance is resolved into N.–S. and E.–W. components—*i.e.*, the amount of northing or southing and easting or westing is obtained for each course from a table known as the **traverse table,** specially designed for navigational requirements, but which may be applied to the solution of any right-angled triangle.

TRAVERSE TABLE

40 Degrees								
Distance	D. lat.	Departure	Distance	D. lat.	Departure	Distance	D. lat.	Departure
1	00·8	00·6	21	16·1	13·5	41	31·4	26·4
2	01·5	01·3	22	16·9	14·1	42	32·2	27·0
3	02·3	01·9	23	17·6	14·8	43	32·9	27·6
4	03·1	02·6	24	18·4	15·4	44	33·7	28·3
5	03·8	03·2	25	19·2	16·1	45	34·5	28·9
6	04·6	03·9	26	19·9	16·7	46	35·2	29·6
7	05·4	04·5	27	20·7	17·4	47	36·0	30·2
8	06·1	05·1	28	21·4	18·0	48	36·8	30·9
9	06·9	05·8	29	22·2	18·6	49	37·5	31·5
10	07·7	06·4	30	23·0	19·3	50	38·3	32·1
11	08·4	07·1	31	23·7	19·9	51	39·1	32·8
12	09·2	07·7	32	24·5	20·6	52	39·8	33·4
13	10·0	08·4	33	25·3	21·2	53	40·6	34·1
14	10·7	09·0	34	26·0	21·9	54	41·4	34·7
15	11·5	09·6	35	26·8	22·5	55	42·1	35·4
16	12·3	10·3	36	27·6	23·1	56	42·9	36·0
17	13·0	10·9	37	28·3	23·8	57	43·7	36·6
18	13·8	11·6	38	29·1	24·4	58	44·4	37·3
19	14·6	12·2	39	29·9	25·1	59	45·2	37·9
20	15·3	12·9	40	30·6	25·7	60	46·0	38·6
Distance	Departure	D. lat.	Distance	Departure	D. lat.	Distance	Departure	D. lat.
50 Degrees								

If you refer to the extract included here for illustration purposes, you will see that a course appears at the bottom as well as at the top, one being the complement of the other. These are the two angles of the triangle other than the right angle. Values of the three sides are given in columns side by side, so that you need know only one angle besides the right angle, and one side, in order to find the other parts. One column is headed Distance and the other two D. lat. and Departure respectively. 'Distance' is the hypotenuse of the triangle and stands for the distance made good across the meridians in the direction indicated at either top or bottom. If the course being used is the one at the top, the d. lat. will be found in the next column, and departure in the third, but if the course is the angle at the bottom, then the d. lat. will be found in the third column, and departure in the second. The reason for their interchangeability in this way will be clear on referring to Fig. 71. When the course is 40° and the distance 20·0 nautical miles, AC is the meridian, and the difference of latitude—distance made good north or south—is 15·3′, while the departure—distance made good east or west—is 12·9′. When the course is 50° and the distance still 20·0 nautical miles, BC is the meridian and the difference of latitude is 12·9′ while the departure is 15·3′.

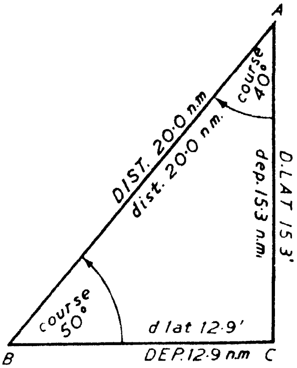


Fig. 71. Traverse Sailing

Plane, or Traverse Sailing. Small triangles on the earth's surface are scarcely distinguishable from plane triangles. To regard them as spherical triangles would be merely to complicate the calculations. Trigonometry was first applied to the sphere, and only later to the plane triangle, as a simplified special case. The particular case in Navigation based on this assumption is known as plane sailing, or traverse sailing. The three sides forming the triangle are respectively a rhumb line, part of a meridian and part of a parallel of latitude. Distance sailed is measured along the rhumb line, and the corresponding d. lat. and departure along the meridian and parallel respectively (see Fig. 74). The angle formed by the intersection of the meridian and parallel is 90°, while that formed by the intersection of rhumb line and meridian is the course, so that the remaining angle is the complement of the course.

Suppose a vessel to steer S. 40° W. (220° T.) from a position 50° 10′ N. 7° 50′ W. for 20 miles, what would be her position? As we have already seen from the section on the traverse table, the d. lat. would be 15·3′ S., making the latitude 49° 54·7′ N. The departure would be 12·9′, which must be converted into d. long. units before being applied to the longitude left.

The mean lat. is $\frac{1}{2}$ (50° 10′ + 49° 54·7′) = 50° 02·35′, which is approximately 50°, and no appreciable error will enter into the calculation for d. long. if we use this, the distance covered being so small. Fig. 72 shows that by construction the d. long. is found to be 20·0′.

By calculation:

dep. = 12·9 n.m. log. = 1·110590
lat. = 50° 00′ log. sec. = 0·191933

d. long. = 20·07′ log. = 1·302523

This result is seen to agree very closely with that found by inspection. The longitude is, therefore, 7° 50′ W. + 20′ W. = 8° 10′ W.

Do you notice any similarity between Figs. 71 and 72? They are identical, which is not surprising when we remember that in the right-angled triangle of Fig. 71 we have an angle representing a course of 50°, while in the right-angled triangle of Fig. 72 we have an angle representing a latitude of 50°. This, of course, has been arranged, but the point to be noted is that, if we treat the latitude as a course in the traverse table, the value of the d. long. may be taken from the distance column when the departure is looked up in the d. lat. column and vice versa.

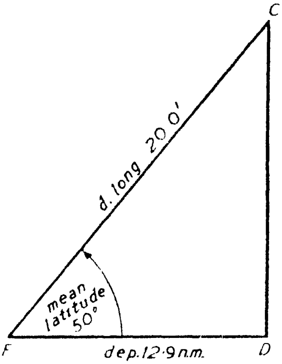


Fig. 72

Example. Find the course and distance from Flamborough Head (54° 07′ N. 0° 05′W.) to the Naze (57° 59′ N. 7° 03′E.) by plane sailing. The d. lat. between the two places is 3° 52′ N. and the d. long. 7° 08′ E., which must be converted into departure before using as a co-ordinate. A right-angled triangle similar to the one in Fig. 69, may be constructed using the mean latitude 56° 03′ and the d. long. to scale. Then, by dropping a perpendicular and measuring it to scale, the departure is found to be 239 n.m., or it may be calculated by multiplying the d. long. by the cosine of the mean latitude.

d. long. = 428′ log. = 2·631444
m. lat. = 56° 03′ log. cos. = 9·746999

dep. = 239 n.m. log. = 2·378443

A second right-angled triangle may then be constructed from the d. lat. and departure drawn to scale in their appropriate directions, when the hypotenuse will represent the distance to scale, and the angle it forms with the d. lat. line will equal the course, as in Fig. 73. These are found to be 334 n.m. and N. 46° E. respectively. Calculation by trigonometry will give more accurate results, however, when the distance is large, the method being to divide the departure by the d. lat. and so obtain the tangent of the true course, or, if using logs., the log. tangent and thence the true course.

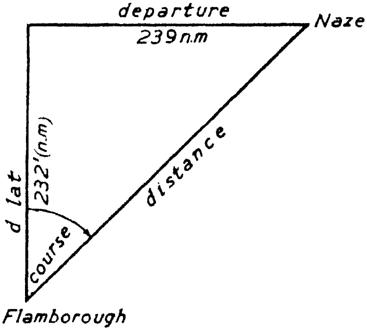


Fig. 73. PLANE SAILING

dep. = 239 n.m. log. = 2·378398
d. lat. = 232′ log. = 2·365488

Course = N. 45° 51′ E. log. tan. = 0·012910

The distance is then calculated by multiplying the d. lat. by the secant of the course.

d. lat. = 232′ log. = 2·365488
course = 45° 51′ log. sec. = 0·157054

Distance = 333·1 n.m. log. = 2·522542

The same method may be applied to longer distances provided proper cognizance be taken of true middle latitude. A voyage (Fig. 74) is being made from A to X along the rhumb line ABCDE–X on which AB, BC, CD, DE, and so on, are short, equal distances. The meridian through A will meet the parallel through B at right angles in a, the meridian through B will meet the parallel through C at right angles in b, and so on. In the triangle AaB, Aa represents the d. lat. and aB the departure between A and B. In the triangle BbC, Bb represents the d. lat. and bC the departure between B and C. Knowing AaB,

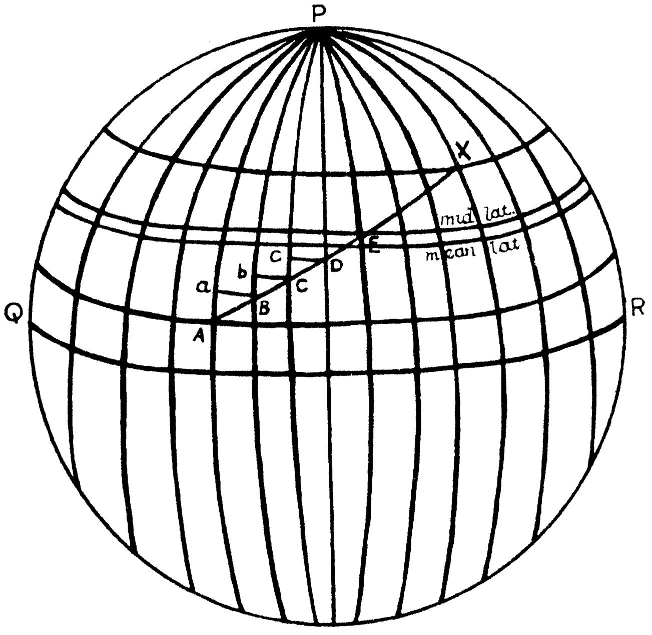


Fig. 74. Extended Plane Sailing

the course, and AB the given short distance, we can find Aa and aB from a traverse table or by calculation. Now the triangles AaB, BbC, etc., being small and considered as plane, are congruent because all their corresponding angles, and one side, the hypotenuse, are equal. The total d. lat. is then a multiple of Aa and the total dep. a multiple of aB, so that, given the course and total distance from A to X, the total d. lat. and dep. may be found in the same manner as in the small triangle AaB. Conversely, given the total d. lat. and dep., the course and total distance may be found similarly. To find the d. long. from the total dep., or the total dep. from the d. long, we should have to use the middle latitude in the calculation, and not the mean latitude.

Exercise 4

Find by plane sailing the course and distance from A to B in each of the following cases.

	A		*B*	
	Lat.	*Long.*	*Lat.*	*Long.*
1.	54° 20′ N.	17° 42′ W.	51° 12′ N.	20° 36′ W.
2.	18° 55′ S.	65° 15′ E.	20° 16′ S.	68° 42′ E.
3.	61° 15′ N.	2° 24′ E.	63° 50′ N.	1° 32′ W.
4.	1° 42′ S.	105° 18′ W.	2° 06′ N.	101° 54′ W.
5.	30° 35′ S.	46° 42′ W.	34° 20′ S.	49° 25′ W.
6.	12° 10′ N.	59° 27′ E.	10° 14′ N.	63° 56′ E.
7.	31° 17′ S.	178° 50′ E.	26° 46′ S.	176° 28′ E.
8.	27° 48′ N.	177° 11′ W.	29° 35′ N.	179° 16′ E.

A Continued Traverse or Day's Work. Details of all the courses steered by a ship, and any other information relative to its navigation, are kept in a log-book—just as you may keep account of your activities in a diary. The navigator of an aircraft writes similar information on a log-sheet as, unlike ships, aircraft do not usually remain navigable for longer periods than twenty-four hours without refuelling. A ship, however, may be at sea for weeks without seeing land, and one of the items required to be entered in the log-book is the position of the ship at noon each day, as near as it can be calculated. This means that if no position has been obtained by observation of celestial objects it must be deduced from the record of courses and distances or speeds kept in the log-book—*i.e.*, found by dead reckoning. After the compass courses have been corrected for deviation, variation, and leeway, the traverse table is entered with each course and distance, and the corresponding d. lat. and dep. taken out. These are arranged in columns under their appropriate headings—*i.e.*, N., S., E., or W. and added up. The total d. lat. and dep. when looked up side by side in the traverse table will give the course and distance made good during the period under review—*i.e.*, since the last position was obtained.

Before the complete position can be found, however, the total departure must be converted into difference of longitude, when both total d. lat. and d. long. can be applied to the latitude and longitude of the previous position, to find the required position. To demonstrate the method, the example given on p. 148 for the air plot has been adapted to suit the capabilities of a ship and reads as follows:

Example. At 05.44 hours the position of a ship steering S. 70 W. true (250 T.) at a speed of 15 knots was found by bearings of shore objects to be latitude 49° 50′ N. longitude 5° 20′ W., and the following alterations of course were subsequently made:

07.24 hrs.	S. 30° W.	true	(210° T.)
08.24 hrs.	South	true	(180° T.)
09.44 hrs.	S. 50° E.	true	(130° T.)
10.56 hrs.	N. 40° E.	true	(040° T.)

What was the vessel's position at noon assuming a current to have set N. 60° W. true (300° T.) at 2 knots since 05.44 hours.

Fig. 75 illustrates the development of the problem, the plotting of which may be done on either squared or plain paper.

On plotting the courses from A and using a scale of 10 nautical miles to 1 inch or the side of a small square to represent a nautical mile, the successive positions B, C, D, and E are the dead reckoning (D.R.) positions en route, and F the final dead reckoning position at noon. If the current were allowed for, represented in the diagram by the component FG, G will be the estimated position. SAF will then represent the bearing, and AF the distance of the vessel's D.R. position from A, and SAG the bearing, and AG the distance of the vessel's estimated position from A. These are found to be S.10° W. True (190° T.) 41¾ n.m. and S. 27° W. True (207° T.) 39 n.m. respectively. To give these positions in terms of latitude and longitude, the components AK and KF, and AH and HG must be applied to the position of A. AK represents 41 nautical miles, or minutes of latitude, south, and when applied to the latitude of A, 49° 50′ N., will give 49° 09′ N. as the latitude of K

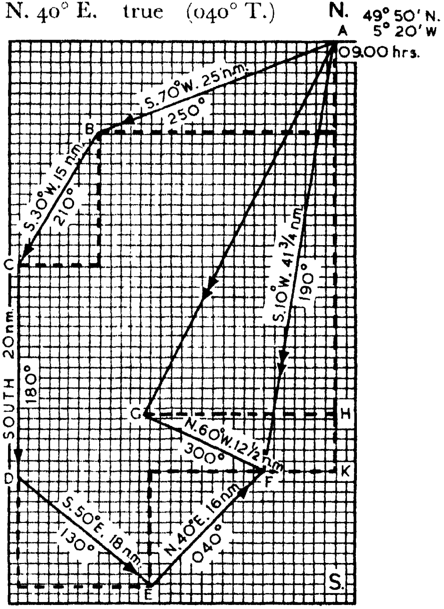


Fig. 75. A Continued Traverse

and, hence, of F. KF, being in nautical miles, must be converted into minutes of longitude before it can be applied to the longitude of A, to find the longitude of F. The position of G is found similarly.

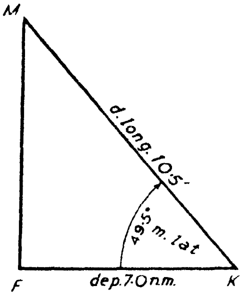


Fig. 76

In Fig. 76, FKM represents the mean latitude 49° 30′, KF the departure, 7·0 nautical miles, and KM the difference of longitude, which is found from the Figure to be 10·5′ W.; the longitude of F thus becomes 5° 30·5′ W. Using the same method to find the position of G, the d. lat. is found to be 34·6′ S., the departure 17·7 n.m. W., and the d. long. 27·2′ W., making the final latitude and longitude 49° 15·4′ N. and 5° 47·2′ W. respectively. The same results can be obtained by tabulating and adding the values of the co-ordinates, but a traverse table or other table solving right-angled triangles is needed. The totals are seen to agree almost exactly with those found on the graph (Fig. 75), and the same positions would naturally result.

		D. lat.		*Departure*	
Course	*Distance (n.m.)*	*N.*	*S.*	*E.*	*W.*
S. 70° W.	25	—	8·6	—	23·5
S. 30° W.	15	—	13·0	—	7·5
South	20	—	20·0	—	—
S. 50° E.	18	—	11·6	13·8	—
N. 40° E.	16	12·3	—	10·3	—
		12·3	53·2	24·1	31·0
			12·3		24·1
			40·9		6·9
Current					
N. 60° W.	12·5	6·3			10·8
			34·6		17·7

To find the position of F, the D.R. position:

Lat. A = 49° 50·0′ N.
d. lat. AK = 40·9′ S.

lat. F = 49° 09·1′ N.

mean lat. = 49° 29·5′ log. sec. = 10·187382
dep. KF = 6·9 n.m. log. = 0·838849

d. long. = 10·62′ log. = 1·026231

Long. of A = 5° 20′ W.
d. long. = 10·6′ W.

Long. of F. = 5° 30·6′ W. Lat. of F = 49° 09·1′ N.

To find the position of G, the estimated position:

Lat. A =	49° 50·0′ N.		
d. lat. AH =	34·6′ S.		
lat. G =	49° 15·4′ N.		
mean lat. =	49° 32·7′	log. sec. =	10·187856
dep. HG =	17·7 n.m.	log. =	1·247973
d. long. =	27·28′	log. =	1·435829
Long. of A =	5° 20·0′ W.		
d. long. =	27·3′ W.		
Long. of G =	5° 47·3′ W.	Lat. of G =	49° 15·4′ N.

To find angle HAG, the estimated course:

$$\tan. \text{HAG} = \frac{\text{HG}}{\text{AH}}\ i.e.,\ \tan. \text{course} = \frac{\text{dep.}}{\text{d. lat.}}$$

dep. HG =	17·7 n.m. W.	log. =	1·247973
d. lat. AH =	34·6′ S.	log. =	1·539076
Course =	S. 27° 05·5′ W.	log. tan. =	9·708897

To find AG, the estimated distance:

AG = AH × sec. HAG—*i.e.*, distance = d. lat. × sec. course.

d. lat. AH =	34·6′	log. =	1·539076
course HAG =	27° 05·5′	log. sec. =	0·050474
Distance AG =	38·86 n.m.	log. =	1·589550

Exercise 5

1. A ship in latitude 49° 40′ N. 5° 55′ W. steamed the following courses and distances:

Course (*T.*)	*Distance* (*n.m.*)
255°	82
196°	75
132°	72
035°	60
334°	56

Find by means of the traverse table the vessel's final D.R. position. If in the meantime a current set 045° T. for 40 n.m., find also the vessel's estimated position as well as the course and distance made good throughout.

2. From a position 13 n.m. south (T.) of Fastnet Rock (51° 23′ N. 9° 35′ W.) a ship steams the following courses and distances:

Course (*T.*)	*Distance* (*n.m.*)
260°	150
320°	165
200°	195
120°	160
070°	120

Find the vessel's final D.R. position.

If a current set 090° T. for 50 n.m., what would be her estimated position, and the course and distance made good?

3. From a position 10 n.m. east (T.) of Flamborough Head (54° 07′ N. 0° 05′ W.) the following courses and distances were steamed:

Course (*T.*)	*Distance* (*n.m.*)
105°	100
020°	150
320°	200
250°	90

Find the vessel's D.R. position.

If during this time the tide set 160° T. for 25 n.m. and then 300° T. for 30 n.m., what would be the vessel's estimated position and course, and distance made good?

4. Using Deviation Card No. 2 (p. 396) and variation 15° W. correct the following courses, and find the ship's D.R. position after covering the given distances from a position with Lizard lighthouse (49° 58′ N. 5° 12′ W.) bearing 311° M. distant 7 n.m.

Course (*C.*)	*Distance* (*n.m.*)
262°	150
177°	82
090°	58
350°	79

If a current set 070° T. for 28 n.m. in the meantime, what would be the vessel's estimated position, course, and distance made good, and also the final bearing and distance of Lizard lighthouse?

Mercator Sailing. In solving a triangle to find the course and distance from one place to another, it is necessary to measure the sides in terms of the same units—that is, if one side is measured in nautical miles so must they all be. That is why we need to change d. long. into departure in plane sailing, but we can find the course just as simply if we change the d. lat. into d. long. units instead of d. long. into departure. This is the main difference between plane sailing and Mercator sailing. You remember that in discussing the construction of a Mercator chart we measured the distance of any parallel of latitude from the equator in terms of longitude units which we called meridional parts. If, then, we convert d. lat. into D.M.P. (difference of meridional parts), we can use it with the d. long. to find the course. If the two places are in the same hemisphere, the D.M.P. is found by taking the difference between the meridional parts for each latitude, but if they are in opposite hemispheres, the sum of the meridional parts is the D.M.P. The rule is the same as for finding d. lat.

Example. Find the course and distance from The Lizard to Bermuda.

The Lizard:	lat. 49° 57′ N.	long. 5° 12′ W.	merid. pts. 3469·81
Bermuda:	lat. 32° 15′ N.	long. 64° 52′ W.	merid. pts. 2046·10
	d. lat. = 17° 42′ S.	d. long. = 59° 40′ W.	d. merid. pts. = 1423·71 S.
	d. lat. = 1062′ S.	d. long. = 3580′ W.	

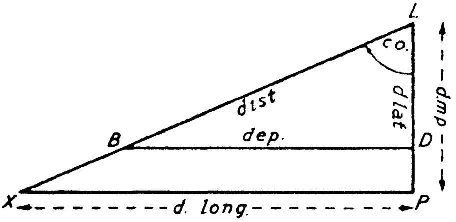


Fig. 77. MERCATOR SAILING

Fig. 77 is drawn to scale to illustrate the values, but no more than approximate results can be expected when the sides represent large distances.

To find the course, using longitude units:

In the triangle LPX, $\frac{\text{d. long.}}{\text{D.M.P.}}$ = tan. course.

d. long. =	3580′	log. =	3·553883
D.M.P. =	1423·7′	log. =	3·153419
Course =	S. 68° 18¾′ W.	log. tan. =	10·400464

To find the distance, using latitude units:

the similar triangle LDB, $\frac{\text{dist.}}{\text{d. lat.}}$ = sec. course.

I.e., distance = d. lat. × sec. course.

d. lat. =	1062′	log. =	3·026124
course =	68° 18¾′	log. sec. =	10·432344
Distance =	2874 n.m.	log. =	3·458468

Example. Find by Mercator sailing the course and distance from Land's End to Gt. Bahama Is.

Land's End:	lat. 50° 04′ N.	merid. pts. 3480·70	long. 5° 43′ W.
Gt. Bahama Is.:	lat. 26° 45′ N.	merid. pts. 1666·70	long. 78° 30′ W.
	d. lat. = 23° 19′ S.	D.M.P. = 1814′	d. long. = 72° 47′ W.
	d. lat. = 1399′		d. long. = 4367′

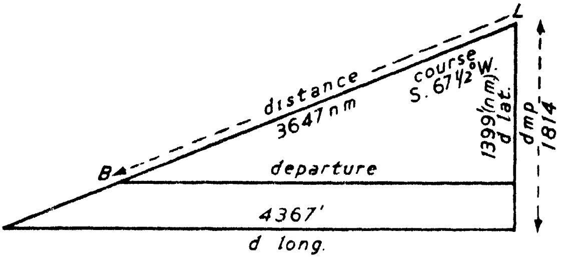


Fig. 78

See Fig. 78 and compare with Figs. 18 and 23.

$$\text{Tan. course} = \frac{\text{d. long.}}{\text{D.M.P.}}$$

d. long. =	4367′	log. =	3·640183
D.M.P. =	1814′	log. =	3·258637
Course =	S. 67° 26½′ W.	log. tan. =	10·381546

Distance = d. lat. × sec. course.

d. lat. =	1399′	log. =	3·145818
course =	67° 26½′	log. sec. =	10·416094
Distance =	3647 n.m.	log. =	3·561912

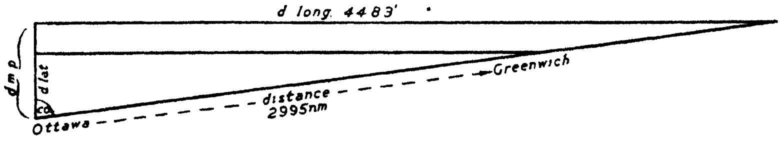


Fig. 79

Example. Find by Mercator sailing the course and distance from Ottawa to Greenwich.

Ottawa: lat. 45° 20′ N.	merid. pts. 3058·31	long. 74° 43′ W.
Greenwich: lat. 51° 28′ N.	merid. pts. 3613·52	long. 00° 00′
d. lat. = 6° 08′ N.	D.M.P. = 555·21	d. long. = 74° 43′ E.
d. lat. 368′ N.		d. long. = 4483′ E.

See Fig. 79 and compare with Fig. 15.

$$\text{Tan. course} = \frac{\text{d. long.}}{\text{D.M.P.}}$$

d. long. =	4483′	log. =	3·651569
D.M.P. =	555·2′	log. =	2·744449
Course =	N. 82° 56½′ E.	log. tan. =	10·907120

Distance = d. lat. × sec. course.

d. lat. =	368′	log. =	2·565848
course =	82° 56½′	log. sec. =	10·910520
Distance =	2995 n.m.	log. =	3·476368

Exercise 6

Find by Mercator sailing the course and distance from A to B in each of the following cases:

	A		*B*	
	Lat.	*Long.*	*Lat.*	*Long.*
1.	50° 00′ N.	7° 30′ W.	40° 00′ N.	68° 15′ W.
2.	7° 45′ N.	156° 12′ W.	37° 45′ N.	122° 10′ W.
3.	33° 20′ S.	15° 10′ E.	7° 40′ S.	31° 30′ W.
4.	11° 18′ N.	52° 55′ E.	34° 16′ S.	113° 40′ E.
5.	56° 45′ S.	67° 15′ W.	34° 40′ S.	18° 25′ E.
6.	21° 10′ N.	158° 27′ W.	36° 50′ S.	175° 15′ E.
7.	14° 40′ N.	17° 30′ W.	35° 10′ N.	75° 30′ W.

Great-circle Navigation. The distances found by Mercator sailing would be those covered when making the voyage or flight along a rhumb line, but, as explained in Chapter I, the great-circle distance would be less. As only one great circle can pass through two places not diametrically opposite, the shortest distance between them consists of the length of great-circle arc joining them. A minute of any great-circle arc is equivalent to a nautical mile, so that this arc when converted into minutes gives the distance in nautical miles. It is calculated by means of the haversine formula adapted from the fundamental cosine formula. (See Appendix.)

Fig. 80 shows how the foregoing example (Land's End to Gt. Bahama

Is.) may be illustrated by projection; but a graphical solution could be only very approximate. The circle represents the earth, P and P_1 the poles, and QQ_1 the equator. L (Land's End) and B (Gt. Bahama Is.) are inserted as follows. B has the lesser latitude and is marked for convenience of construction on the circumference, the arc 26° 45′ is accordingly measured upward from Q towards P. The difference, or

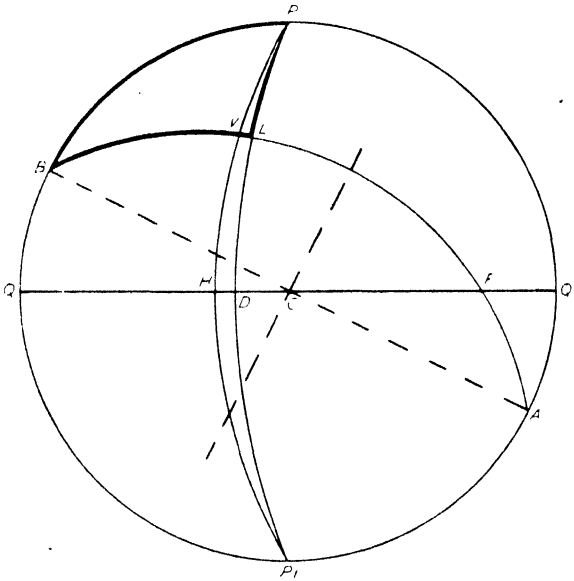


Fig. 80. Great-circle Navigation

change, of longitude between B and L is 72° 47′, so from Q a distance representing the d. long. to scale is measured eastward along the equator. The position D thus reached must be the point where the meridian of L crosses the equator. The centre of this meridian will lie along DQ_1 or DQ_1 produced. B is now joined to C, the centre of the circle, and produced to meet the circumference in A. Through C a perpendicular is drawn to BA, and along it will be found the centre of a circle passing through BL and A. This represents the great circle passing through B and L. It crosses the equator at F, and as the meridian of vertex must be 90° removed from the meridian through F, a distance representing 90° is measured along the equator from F to H on the meridian of the northern vertex, there being, of course, always two vertices. The point V at which the meridian through H crosses the great circle will represent the required vertex, which is seen to fall just inside the spherical triangle. This will not always be the case. In the triangle LPB we have PL, PB, and LPB—*i.e.*, two sides and the included angle. We require to find the other side.

Now here is the formula and calculation:

Haversine BL = hav. BPL × sine PB × sine PL + hav. (PB ∼ PL)
(in which ∼ represents difference)

or in words:

Hav. dist. = hav. d. long. × sine co-lat. of B × sine co-lat. of L + hav. (co-lat. B ∼ co-lat. L)

D. long. (BPL) = 72° 47′	log. hav. = 9·54655	Add
Co-lat. B (PB) = 63° 15′	log. sine = 9·95084	
Co-lat, L (PL) = 39° 56′	log. sine = 9·80747	
	sum = 9·30486	
	antilog. of sum = 0·20177	Add
PB ∼ BL = 23° 19′	nat. hav. = 0·04083	
Distance (BL) = 59° 01′	nat. hav. = 0·24260	
Distance (BL) = 3541 n.m.		

On comparing this distance with that found by Mercator sailing we notice a difference of 106 n.m.

The course to destination may also be calculated, although in practice tables solving spherical triangles are often resorted to. The formula used is a transposed form of the first one. We now have three sides and require an angle. Suppose we want the initial course.

$$\text{Haversine PLB} = \frac{\text{hav. PB} - \text{hav. (PL} \sim \text{BL)}}{\text{sin. PL} \times \text{sin. BL}}$$

Co-lat. B (PB) = 63° 15′	nat. hav. = 0·27495	
PL ∼ BL = 19° 05′	nat. hav. = 0·02748	Subtract
	difference = 0·24747	
	log. diff. = 9·39352	
Co-lat. L (PL) = 39° 56′	log. cosec. = 10·19254	Add
Distance (BL) = 59° 01′	log. cosec. = 10·06686	
Initial course (PLB) = N. 84° 13½′ W.	log. hav. = 9·65292	
Initial course (PLB) = 275° 46½′		

Parallel sailing has sometimes to be combined with great-circle sailing, when owing to obstructions in high latitudes—*e.g.*, ice—it is not possible to follow a great-circle route all the way. Two great circles are then drawn, one from point of departure and the other from destination, tangential to the parallel of latitude along which it is desired to make the easting or westing. This is known as composite great-circle sailing.

Exercise 7

Find the distance along the great-circle track, and the initial course from A to B in each of the following cases:

	A		*B*	
	Lat.	*Long.*	*Lat.*	*Long.*
1.	34° 40′ S.	18° 25′ E.	36° 50′ S.	114° 30′ E.
2.	56° 45′ S.	67° 15′ W.	18° 00′ S.	178° 30′ E.
3.	59° 50′ N.	44° 00′ W.	55° 30′ N.	7° 25′ W.
4.	49° 50′ N.	5° 10′ W.	24° 30′ N.	81° 45′ W.
5.	35° 30′ N.	140° 00′ E.	49° 30′ N.	123° 00′ W.
6.	33° 00′ S.	71° 40′ W.	41° 15′ S.	174° 40′ E.
7.	34° 50′ S.	56° 10′ W.	34° 30′ S.	18° 20′ E.

Comparison of Nautical and Aeronautical Terms. A certain amount of difference in meaning exists between navigational terms used both nautically and aeronautically. The following table has been drawn up to help students distinguish between them. It should be remembered that, whereas an aircraft is subject to the movement of the wind, a ship may be subject to the movement of both wind and water.

	SEA	*AIR*
Air position		Position calculated from a knowledge of: (i) true course or courses (ii) T.A.S. × time
D.R. position	Position calculated from a knowledge of: (i) true course or courses (ii) distance registered along the courses	Position calculated from a knowledge of: (i) true course or courses (ii) T.A.S. × time (iii) Wind effect
Estimated position	D.R. position plus current and wind effect	
Course made good	Direction made good over the ground	
Track made good		Direction made good over the ground
Set	Direction in which a current moves	
Drift	Distance drifted by vessel due to current effect	Angular effect of wind on an aircraft's course—*i.e.*, angle between course and track
Leeway	Angular effect of wind on a vessel's course	

Wind is always named according to the direction *from* which it comes. It is measured in either M.P.H. or knots.

Current is named according to the direction *towards* which it goes. It is measured only in knots.

General Exercises

1. What do you understand by the term 'departure'?

Draw figures to scale, and by means of the appropriate formula complete the following table.

	Mean lat.	*D. long.* (*ch. long.*)	*Departure* (*n.m.*)
(*a*)	55° 00′	10° 10′	
(*b*)		3° 40′	150
(*c*)	47° 00′		600
(*d*)	35° 00′	7° 15′	
(*e*)		1° 52′	100
(*f*)	68° 00′		310
(*g*)	60° 00′		50

2. A ship steams or an aircraft flies the following courses and distances from (F. lat. 54° 10′ N. long. 00° 00′).

040° T.	80 n.m.
100° T.	65 n.m.
220° T.	75 n.m.
300° T.	40 n.m.

W/V 160°/20 knots for one hour, or set of current 340° T., drift 20 n.m.

Find by means of plotting or by calculation:

the ship's (*a*) D.R. position;	the aircraft's (*a*) air position;
(*b*) estimated position;	(*b*) D.R. position;
(*c*) course and distance made good;	(*c*) bearing and distance from F.

3. A ship steams or an aircraft flies the following courses and distances successively from a position 5 n.m. 000° T. of Malin Head (55° 23′ N. 7° 23′ W.).

270° T.	120 n.m.
020° T.	90 n.m.
110° T.	80 n.m.
240° T.	50 n.m.

Give (*a*) ship's D.R. position; (*b*) aircraft's air position.

If the actual position were 56° 10·5′ N. 9° 55′ W., give (*a*) average set and drift; (*b*) W/V. (T.A.S. 170 knots.)

4. In what latitude will the d. long. (ch. long.) be 1½ times the departure?

5. At what speed in knots is Edinburgh (55° 57′ N.) being carried round by the earth?

6. A craft left Butt of Lewis (58° 31′ N. 6° 16′ W.) on a track of 270° T. What would be her longitude after covering 400 n.m.?

7. After covering a distance of 520 n.m. in a south-westerly direction from Fastnet Rock (51° 23′ N. 9° 36′ W.) a craft was found to be in latitude 44° 50′ N. What was the track, or course made good, and what the longitude?

8. Find the course and rhumb-line distance between the following places:

(a) Spurn Head 53° 34′ N. 0° 07′ E.
to Esbjerg 55° 28′ N. 8° 29′ E.
(b) Land's End 50° 04′ N. 5° 42′ W.
to Cape Finisterre 42° 53′ N. 9° 15′ W.
(c) Duncansby Head 58° 39′ N. 3° 01′ W.
to Bergen 60° 21′ N. 5° 20′ E.
(d) Barcelona 41° 22′ N. 2° 11′ E.
to Bizerta 37° 30′ N. 10° 00′ E.
(e) Hong Kong 22° 16′ N. 114° 09′ E.
to Manila 14° 58′ N. 121° 00′ E.
(f) Saigon 10° 59′ N. 106° 43′ E.
to Singapore 1° 15′ N. 103° 50′ E.
(g) Fernando Po 3° 38′ N. 8° 30′ E.
to Cape Lopez 0° 45′ S. 8° 50′ E.
(h) Phœnix Is. 4° 00′ S. 172° 30′ W.
to Gilbert Is. 1° 20′ N. 173° 00′ E.

9. From St John's (Newfoundland) 47° 20′ N. 52° 50′ W., you travel 000° T. for 500 n.m., then 090° T. for 500 n.m., then 180° T. for 500 n.m., and finally 270° T. for 500 n.m. How far would you still have to go to reach St John's?

10. St John's (Newfoundland) and Nantes are approximately on the same parallel of latitude, 47° 20′ N. Their longitudes are 52° 50′ W. and 1° 33′ W. respectively. What is their distance apart along the parallel and along a great circle? How does Nantes bear from St John's?

11. Find the initial course and distance along the great circle also the rhumb-line course and distance, using meridional parts, between the following places. Calculate the percentage increase in rhumb-line distance over great-circle distance and draw conclusions.

(a) Cape Wrath 58° 38′ N. 5° 00′ W.
to Belle Isle 51° 53′ N. 55° 22′ W.
(b) San Francisco 37° 45′ N. 122° 41′ W.
to Honolulu 22° 18′ N. 157° 52′ W.
(c) Honolulu 22° 18′ N. 157° 52′ W.
to Sydney 33° 50′ S. 151° 18′ E.
(d) Lagos 6° 26′ N. 3° 38′ E.
to Rio de Janeiro 22° 50′ S. 43° 44′ W.
(e) Mauritius 20° 08′ S. 57° 29′ E.
to Fremantle 32° 03′ S. 115° 44′ E.
(f) Leningrad 60° 00′ N. 30° 30′ E.
to Bermuda 32° 20′ N. 65° 00′ W.
(g) Harbour Grace 47° 43′ N. 53° 08′ W.
to Victoria (Vancouver) 48° 25′ N. 123° 23′ W.
(h) Yokohama 35° 30′ N. 139° 30′ E.
to San Francisco 37° 45′ N. 122° 41′ W.
(i) Wellington (N.Z.) 41° 17′ S. 174° 47′ E.
to Valparaiso 33° 00′ S. 71° 40′ W.

12. Find the shortest possible distance between Glasgow (55° 52′ N. 4° 17′ W.) and Halifax, N.S. (44° 40′ N. 63° 40′ W.). Calculate the total alteration of course necessary in following this route.

13. At 23.20 hrs. local apparent time on February 27, a steamer left a position 55° 30′ N. 7° 14′ W. steering 270° T. at a speed of 15 knots. On March 2 at 06.40 hours local apparent time, an aircraft, G.S. 190 knots., passed over the same position in pursuit of the steamer. In what position would the aircraft overtake the steamer, and at what local apparent time?

CHAPTER VIII

TERRESTRIAL POSITION LINES

Use of charts—Pinpointing—Bearings—Position lines from bearings—Fix by simultaneous bearings—Fix by relative bearings—'Cocked hat'—Distance by vertical angle—Distance of sea horizon—Fix by bearing and distance—Fix by bearing and horizontal angle—Fix by horizontal angles—Running fix—Doubling the angle on the bow—Four-point bearing—Track by three bearings—Practical problem.

Use of Charts. When proceeding from one place to another by air the navigator will connect the two on a chart by a straight line; if the distance is short, by a rhumb line on a Mercator chart, but if it is long, by a great circle on a gnomonic chart. Such a procedure cannot always be followed at sea, as obstacles such as land or ice may intervene and have to be circumvented. Much navigating has, therefore, to be done in narrow waters or within sight of land when changes of course are frequent and allow of little error in computation. In clear weather no difficulties need arise if the technique has been mastered and properly applied. The correct use of charts is an art which should commend itself to all with a flair for adventure and sufficient imagination to place themselves in command of a ship's bridge, the plan being to seek the shortest route within the limits of safety.

Pinpointing. One of the chief occupations of a navigator is that of finding the position of the ship or aircraft. If he makes sure of this at the time of setting a course, he may reasonably expect to estimate the craft's progress with tolerable accuracy. A common method of finding one's position when flying is by noting the time at which some clearly defined and recognizable object is vertically below the aircraft. This easy and accurate method of establishing position relative to the ground is known as **pinpointing,** and the position so found a pinpoint (P.P.) position.

Bearings. For a ship, the departure position cannot generally be given as immediately above a particular spot, because definite points on the sea-bed are neither easily located nor readily recognizable. Positions must, therefore, always be related to some fixed object in a horizontal direction. This is made possible by taking bearings—*i.e.*, by measuring **the horizontal angle between some standard direction and the direction in which the object is situated from the observer.** The principal standard directions are those referring to north (true, magnetic, or compass), the longitudinal axis of a ship or aircraft, objects in line—*i.e.*, in transit, or the predetermined direction of any other object. Fig. 81 illustrates all of these. OT and OM represent the direction of the true and magnetic meridians respectively, OC the direction of the compass needle, HOA the longitudinal axis of the ship (or aircraft), K and L two objects in transit with O (usually expressed LϕK, the farther object being named first), X an object whose direction OX has been established, and Z the object whose direction is desired relative to one of the others. If the direction of Z is observed by compass, then COZ is the compass bearing and MOZ the magnetic bearing obtained by applying to COZ the easterly deviation MOC, as explained in Chapter V. TOZ, the true bearing, is obtained either by applying the variation TOM to MOZ, or observing the direction of the object by gyro compass. The angle HOZ is known as the relative bearing, and when added to TOH, the true course, will give the true bearing of Z. KOZ is the angle subtended at O between the fixed direction KL and the required direction OZ, and when added to TOK gives the true bearing TOZ. It may be measured either by pelorus or sextant held horizontally. XOZ is the angle subtended at O between the known direction of X, say TOX, and the direction of Z. This also may be found by pelorus or sextant angle, and when added to TOX gives the true bearing TOZ.

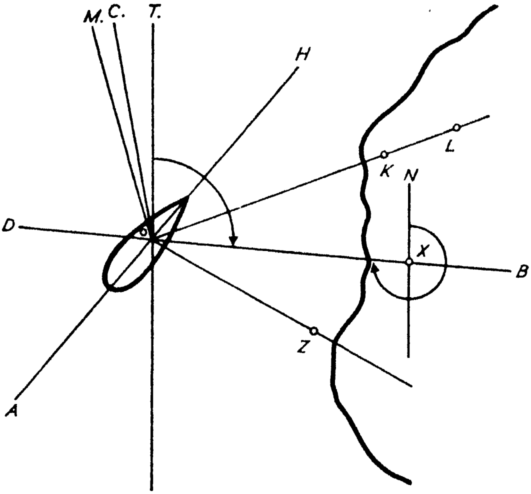


Fig. 81. TERRESTRIAL BEARINGS

Position Lines from Bearings. A bearing gives us the direction of an object from an observer, but as the position of the ship or aircraft is still unknown, we are unable to plot the bearing from the observer. The position of the object is known, however, so if we reverse the bearing—*i.e.*, find its reciprocal by adding or subtracting 180°—we can draw a line in this direction from the object and know it will pass through the observer's position. He must be somewhere along it, and so we call it a **position line.** In Fig. 81 TOX is the true bearing of X from O. Then, because TO and NX represent meridians on a Mercator chart they are parallel, so that angle NXB = angle TOX. As OX, being short, is indistinguishable from a rhumb line, the direction of O from X must be the reflex angle NXO as indicated in the diagram—*i.e.*, NXB + 180°—which equals TOX + 180°. By adding 180° to TOX, therefore, we obtain the direction of O from X. The observer at O must then be situated along the line XOD.

Fix by Simultaneous Bearings. If two bearings are taken simultaneously, each of different objects, two separate position lines

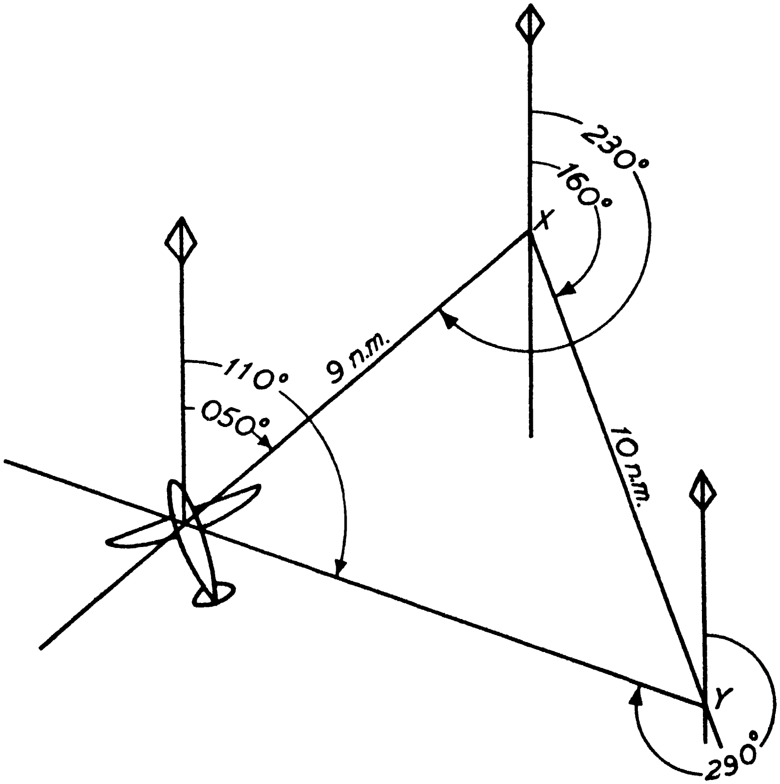


Fig. 82. Fix by Cross-bearings

will be obtained, and, provided the objects are not in line with the observer, these position lines will intersect. As the observer must be situated somewhere along each of them, he can be only at their point of intersection. The observer's position is thus fixed by two bearings.

Example. An observer on a ship or aircraft finds the true bearing of an object X to be 050°. At the same time an object Y, 10 n.m. 160°

from X, bears 110° T. What is the observer's position as a bearing and distance from X?

Fig. 82 shows how the problem may be solved on plain paper.

Mark position X and through it draw a line to represent the true meridian. From the direction of true north measure an angle of 160° to obtain the direction of Y. Draw a line in this direction and along it measure to scale a distance representing 10 nautical miles. This will give the position of Y. As the bearing of X from the observer is 050°, a position line in the reciprocal direction 230° is drawn from X. Similarly, because the bearing of Y from the observer is 110°, a position line in the reciprocal direction 290° is drawn from Y. The point at which these two position lines intersect will be the observer's position. His bearing from X is 230°, and the distance on measurement is found to be 9 miles. On a chart the positions of X and Y would be marked already, so that the true bearings need only be reversed and drawn from their respective object to find the position.

Example. Newarp light-vessel bore 290° T. and Cross Sand light-vessel bore 230° T. Find the position in terms of latitude and longitude.

Lay off the reciprocal of the bearings through Newarp light-vessel and Cross Sand light-vessel respectively. The point of their intersection will give the position, latitude 52° 44′ N. longitude 2° 08′ E. (position B on the example chart (in pocket at end of book)).

Exercise 1

Mark the positions of A and B on plain paper, lay off from them their true bearings reversed, and fix the observer's position relative to A and B in each of the following cases.

	Bearing of A from observer	*Bearing and distance of B from A*		*Bearing of B from observer*
	(*T.*)	(*T.*)	(*n.m.*)	(*T.*)
1.	045°	160°	5·0	110°
2.	300°	180°	7·0	225°
3.	130°	270°	10·0	200°
4.	250°	020°	6·0	330°
5.	105°	350°	12·0	030°
6.	225°	100°	8·5	140°
7.	340°	215°	4·8	260°

Fix by Relative Bearings. Directions measured clockwise from the forward direction of a craft's longitudinal axis, or fore-and-aft line, are termed **relative bearings,** being relative to the direction of the craft's head. As already explained in this chapter, it is necessary only to add the relative bearing of an object to the craft's true course in order to obtain the object's true bearing.

Example. From a ship or aircraft steering 140° T. the relative bearing of an object A was found to be 080°, while at the same time the relative bearing of an object B, 8 n.m. north of A, was found to be 150°. What was the craft's bearing and distance from B?

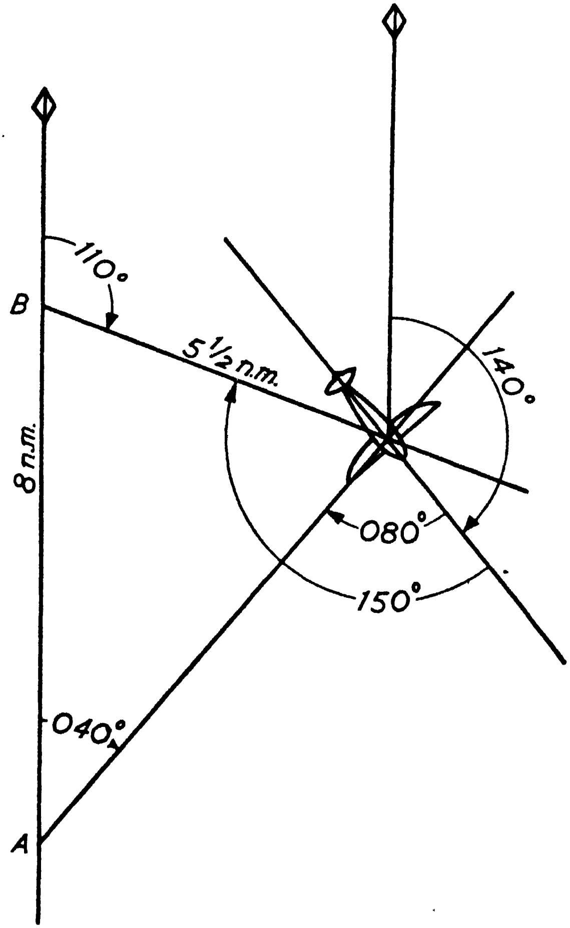


Fig. 83. Fix by Relative Bearings

Fig. 83 shows how the problem may be solved on plain paper.

Mark position A and through it draw a line to represent the true meridian. Measure 8 nautical miles to scale along it to the north and mark this point B. As the true course is 140°, the true bearing of A must be 140° + 80° = 220° and the true bearing of B 140° + 150° = 290°. To obtain the observer's position the reciprocals of these bearings must be drawn in from their respective objects. The position lines are shown in the diagram to be 040° from A and 110° from B. They intersect at a point 5½ nautical miles from B. The position relative to B is, therefore, 110°B 5½ n.m.

When using a chart the positions of suitable objects would be marked already, and after adding each relative bearing to the course separately, the respective reciprocal directions would be drawn in to give the position at their point of intersection, as explained above. The following example is plotted for illustration on the example chart.

Example. While steering 145° T. the relative bearings of Outer Dowsing light-vessel and Outer Dowsing buoy were respectively 235° and 305°. Find the vessel's position by graticule reference.

Find the respective true bearings by adding the relative bearings to the course—*i.e.*,

145° + 235° = 020° T., bearing of Outer Dowsing light-vessel
and 145° + 305° = 090° T., bearing of Outer Dowsing buoy.

The reciprocals being then laid off give the position, latitude 53° 26′ N. longitude 00° 53′ E. (position C on the chart).

Exercise 2

Mark on plain paper the positions of A and B, find their true bearings in each of the following cases, and by laying off the corresponding position lines, fix the observer's position relative to A and B.

	True course	*Relative bearing of A*	*True bearing and distance of B from A*		*Relative bearing of B*
				(n.m.)	
1.	060°	030°	200°	10	085°
2.	140°	065°	315°	7	130°
3.	325°	100°	166°	12	165°
4.	250°	040°	042°	4·5	100°
5.	175°	180°	250°	6·2	122°
6.	024°	350°	174°	9·7	094°
7.	218°	220°	328°	8·8	154°

'Cocked Hat.' A third bearing is sometimes taken to confirm the position obtained from the first two. Confirmation, however, is not always forthcoming, as, instead of passing through the point of intersection formed by the other two, the third position line may pass to one side or the other, so completing a triangle which is then known as a 'cocked hat.' This is usually due either to a small personal error produced by the faulty judgment of the observer or to a small instrumental error caused by maladjustment of the instrument used for taking the bearings. The navigator will then be in a trilemma, as there are now three points of intersection, in fact, the case is even less satisfactory, as he may not be at one of the corners of the triangle at all, nor even within the triangle, and further observations may only serve to complicate matters. In this case the nautical practice is to assume a position which puts the observer on the triangle at its nearest point to danger. The air navigator assumes the centre of the triangle to be his position.

Distance by Vertical Angle. If, instead of taking the bearing of an object, we were to measure our distance from it by rangefinder or otherwise, we should know that our position lay somewhere along the circle whose radius was equivalent to that distance. Such a circle is known as a circle of position. When the approximate direction of the object is known only a portion of the circle need be plotted for position-finding purposes. This is then called a position arc. Distances found simultaneously from two objects provide two position arcs which fix the position at their point of intersection. As comparatively few ships are supplied with fixed rangefinders, the commonest method of obtaining distance at sea is by measuring with a sextant the angle subtended vertically by an object whose height is known. Fig. 84 shows how an observer at sea finds his distance from a lighthouse by this method. Angle A represents the vertical sextant angle, and h the height of the lighthouse whose axis makes an angle of 90° with the sea or ground. It is thus very simple to construct such a triangle to scale and find the distance d when the angle A is appreciable. If, however, as is usual, the angle A is small—*i.e.*, less than 5°—the triangle is unwieldy and calculation must be resorted to unless suitable tables are available.

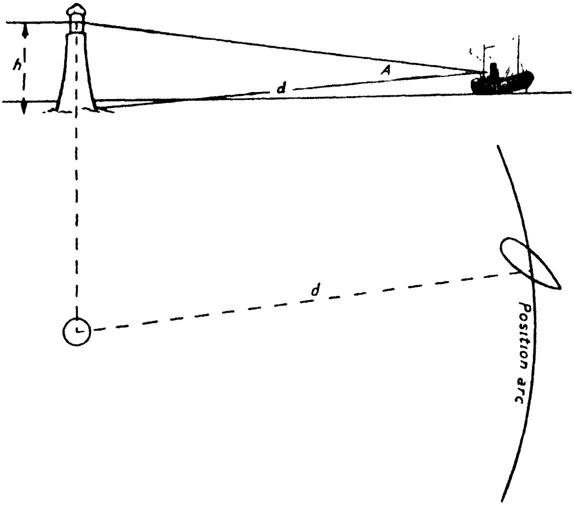


Fig. 84. Distance by Vertical Angle (i)

Example. The height of Eddystone lighthouse—*i.e.*, the height of its focal plane above mean high-water, spring tides—is 133 feet. What will be its distance off when the subtended vertical angle measured by sextant is 2° 17′?

From Fig. 84 it can be seen that $\frac{\text{d}}{\text{h}} = \text{cot. A.}$

I.e., $\frac{\text{distance from object}}{\text{height of object}} = \text{cotangent of subtended vertical angle}$

or, distance = height × cot. vertical angle.

Height of object = 133 ft. log. = 2·123852
Subtended angle = 2° 17′ log. cot. = 1·399323

Distance from object = 3336 ft. log. = 3·523175

Distance from object = $\frac{3336}{6080}$ nautical miles = 5·5 cables.

Volumes of nautical tables generally include a table from which the distance may be obtained in nautical miles direct. When it is desired to pass a safe distance round an object with the least delay the angle A, Fig. 84, is found in the table or calculated and set on the sextant, so that as long as the angle subtended by the object does not exceed the sextant reading, the vessel is outside the assumed danger zone. The course may be altered gradually to enable the subtended angle to remain constant, in which case the vessel will be moving along an arc whose centre is at the object and whose radius will be the distance from the object. Such an arc belongs to a circle of position, as without further information the subtended angle will place the navigator anywhere along the arc or even the circle. When the angle has been set and the distance off is correct, the direct and reflected views of a lighthouse would appear through the sextant as shown in Fig. 85, the reflected view being below the direct view when the reading is positive—*i.e.*, on the arc—and above the direct view when the reading is negative—*i.e.*, off the arc.

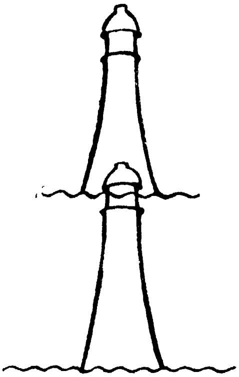


Fig. 85. DISTANCE BY VERTICAL ANGLE (ii)

Distance of Sea Horizon. When the object is on the horizon a simple formula giving the distance of the horizon in nautical miles may be used.

Viz., distance (n.m.) = $1{\cdot}15\sqrt{\text{height (ft.)}}$

When the object is below the horizon and only the top visible, as when a powerful light comes into view in clear weather (a condition known as *dipping*), the formula must be applied separately to both the height of the object and the height of the observer, as illustrated in Fig. 86.

The formula then becomes:

$$\text{distance (n.m.)} = 1{\cdot}15\,[\sqrt{\text{ht. of light (ft.)}} + \sqrt{\text{ht. of observer (ft.)}}]$$

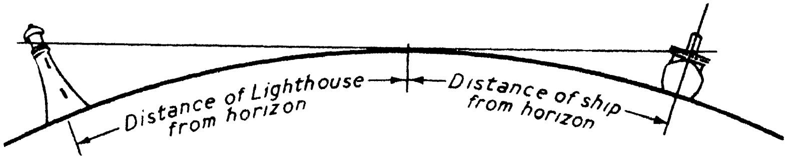

Distance of Lighthouse from horizon
Distance of ship from horizon

Fig. 86. MAXIMUM RANGE OF VISIBILITY

Example. Find the distance of an object on the horizon from an observer whose height above sea-level is 49 feet.

$$\text{By the formula, distance (n.m.)} = 1{\cdot}15\,\sqrt{\text{height (ft.)}}$$

$$\therefore \quad \text{distance} = 1{\cdot}15\,\sqrt{49} = 1{\cdot}15 \times 7$$

$$\textit{i.e.},\ 8{\cdot}05 \text{ n.m.}$$

Example. At what distance would a light whose height is 100 feet be visible to an observer 49 feet above the sea at mean high-water, spring tides.

$$\text{By the formula, distance} = 1{\cdot}15\,[\sqrt{\text{ht. of object}} + \sqrt{\text{ht. of observer}}]$$

$$\therefore \quad \text{distance} = 1{\cdot}15\,(\sqrt{100} + \sqrt{49}) = 1{\cdot}15 \times 17$$

$$= 19{\cdot}55 \text{ n.m.}$$

The visibility of lights as shown on charts is given for an assumed observer situated 15 feet above sea-level.

Fix by Bearing and Distance. It will now be seen that by combining a position line obtained from a bearing with a position curve, a **fix** is obtained, and positions of objects are often reported by this method, the bearing *from* being given first, then the name of the reference point, and lastly the distance—*e.g.*, 200° Beachy Head 10 n.m.

The following examples are worked on the chart at end of book.

Example. The bearing of Flamborough Head lighthouse was 240° T. and its distance by vertical angle 8 nautical miles. What was the vessel's position by graticule reference?

Plot the reciprocal of 240°—*i.e.*, 060° T.—from Flamborough Head lighthouse and measure 8 nautical miles—*i.e.*, minutes of latitude—along it from the lighthouse. The latitude of the vessel will then be found to be 54° 11½′ N. and the longitude 0° 07′ E. (position A on the chart).

Example. Cross Sand light-vessel bore 037° C. (dev. 2° E.) and the vertical angle subtended by Lowestoft lighthouse (ht. 123 ft.) was 0° 14′.

Give the position relative to Yarmouth.

Bring the compass bearing to true by applying the given deviation and the variation as shown on the chart.

Viz., 037° C., 039° M., 030° T. (Var. 9° W.)

Lay off 210° the reciprocal of the true bearing from Cross Sand light-vessel and find the distance from Lowestoft lighthouse as follows:

Height of Lowestoft lighthouse =	123 ft.	log. =	2·089905
Subtended angle =	0° 14′	log. cot. =	2·390143
Distance from Lowestoft lighthouse =	30203 ft.	log. =	4·480048
		log. 6080 =	3·783904
∴ Distance from Lowestoft lighthouse =	4·968 n.m.	log. =	0·696144

Using this distance as radius and Lowestoft lighthouse as centre, the point of intersection with the bearing of Cross Sand light-vessel will give a position 120° Yarmouth 8 n.m. (position E on the chart).

Exercise 3

1. Supply the missing values in the following cases.

	Height of object	*Vertical angle*	*Distance*
(*a*)	150 ft.	10° 00′	
(*b*)	120 ft.	2° 16′	
(*c*)	185 ft.	0° 45′	
(*d*)	200 ft.	0° 30′	
(*e*)	160 ft.		2·0 n.m.
(*f*)	220 ft.		3·0 n.m.
(*g*)	250 ft.		2·5 n.m.

2. Find the distance of the sea horizon from an observer whose height is:

(*a*) 36 ft.; (*b*) 81 ft.; (*c*) 56 ft.; (*d*) 24 ft.; (*e*) 110 ft.

3. What would be the maximum range of visibility under the following conditions?

	Height of observer	*Height of object*
(*a*)	60 ft.	145 ft.
(*b*)	40 ft.	160 ft.
(*c*)	30 ft.	90 ft.
(*d*)	20 ft.	115 ft.

4. At what height above sea-level must the crow's nest of a ship be placed, so that the look-out man may have a visibility range of 8 miles?

5. A look-out man at a height of 60 ft. above sea-level reports a light, whose indentity he recognizes, as just dipping. Ten minutes later the officer of the watch from a height of 35 ft., sights the same light. What is the ship's speed?

6. A lighthouse (height 105 ft.) is visible when dipping at a distance of 20 miles to an observer on a loaded ship. At what distance will it

be visible to him when the ship is in ballast if the difference in draught at the two conditions is 15 feet?

Fix by Transit Bearing and Horizontal Angle. When two prominent fixed objects are seen in line—*i.e.*, in transit—a position line may be obtained immediately by joining their positions, as given on the chart, and producing the line seaward. A second position line may then probably be obtained from the compass bearing of some object or other or from a distance obtained by vertical angle. A quite different and often preferable choice, however, is sometimes open to the observer. He may find it more convenient to measure by sextant the horizontal angle between the objects in transit and some other prominent landmark. This horizontal angle will provide a position arc which when combined with the position line already obtained will fix the position. The method is explained by means of the following problem.

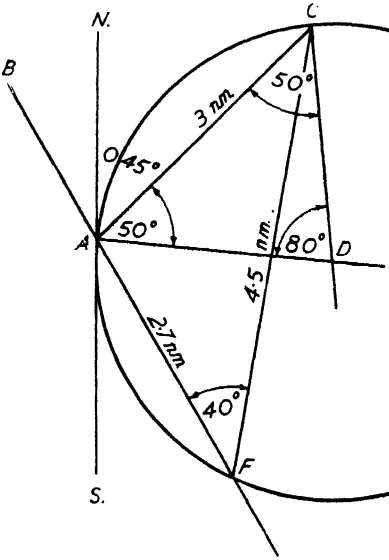


Fig. 87. Fix by Bearing and Horizontal Angle

Example. Two objects A and B were observed to be in transit, bearing 330° T. when the horizontal angle between object A and a landmark C, 045° from A, 3 n.m. distant, was found to be 40°. What was the distance of the observer from A and from C?

In Fig. 87 NAS represents the meridian of A. A and B are in transit on a bearing of 330° T. and B is therefore placed on a line from A in this direction. BA produced will then give a position line along which the observer is situated. The horizontal angle between

A and a position C must now be used to locate him exactly. Position C lies in a direction 045° from A, distant 3 n.m. This is drawn in to scale and C marked. Angles of 50°, the complement of the horizontal angle, are constructed at A and C. The lines forming them intersect at a point D, which when used as a centre with radius DA or DC provides a circle of position. This intersects the transit bearing at F, which must be the observer's position. PROOF: because angle DAC + angle DCA = 100°, angle ADC = 80°, and, as the angle at the centre of a circle is always twice the angle at the circumference when standing on the same chord, angle AFC = 40°. Since, therefore, A and B are in the transit and the horizontal angle between A and C is 40°, F must be the observer's position. His distances from A and C are then found to be 2·7 n.m. and 4·5 n.m. respectively.

The following problem is worked on the chart.

Example. The transit bearing of Haisbro light-vessel and Haisbro lighthouse was observed to be 192° C., and, at the same time, Cromer lighthouse bore 232° C. Find the position in terms of latitude and longitude.

The true bearing of Haisbro light-vessel and Haisbro lighthouse in transit may be obtained by joining them on the chart and is found to be 185°. Because the compass bearings differ by 40°, that must be the horizontal angle between them which, when applied to the true transit bearing obtained from the chart, will give the true bearing of Cromer lighthouse as 225° T. By drawing the reciprocals through their points, the bearings will be found to cross in latitude 53° 05′ N. longitude 1° 36′ E. (position D). It will be noticed that there was no need to apply the compass error in order to find the true bearing of Cromer lighthouse, although it could easily be found by taking the difference between the true and compass transit bearings, as the true transit bearing was already known. A similar treatment can be applied when any other true bearing is known.

Exercise 4

Using the following data, find the observer's bearing and distance from both A and B in each problem.

	Bearing of A from observer	*Bearing and distance of B from A*		*Horizontal angle subtended by A and B*
	(T.)	*(T.)*	*(n.m.)*	
1.	270°	020°	5·0	50°
2.	340°	100°	3·0	60°
3.	045°	284°	9·0	72°
4.	156°	040°	7·5	65°
5.	083°	327°	6·2	48°
6.	284°	134°	8·8	56°
7.	307°	082°	4·7	78°
8.	164°	291°	11·3	90°
9.	022°	156°	10·2	110°
10.	210°	030°	12·0	180°

Fix by Horizontal Angles. If no true bearing is deducible, the difference between the compass bearings of any two fixed points taken on the same compass will give a horizontal angle which can be used for finding a position circle on which the observer must be situated. Three different bearings of fixed points will provide two horizontal angles and two position circles at the intersection of which the observer must be situated. This is illustrated in Fig. 88 where A, B, and C represent three fixed points. A is 000° from B, distant 2·4 n.m., and C is 208° from B, distant 2·0 n.m. Suppose the horizontal angle between A and B, measured either by compass or sextant, to be 50°, and the horizontal angle between B and C to be 35°. To construct the position circles and find the ship's position it will be necessary to proceed as follows. Join A to B, and at both A and B construct an angle equal to the complement of 50°—*i.e.*, 40°. The point where the two lines forming the angles intersect will be the centre D of the position circle whose radius will be DA and DB. Now join B to C, and at both B and C make an angle equal to the complement of 35°—*i.e.*, 55°. The centre of the position circle is found as before, and when the circles are completed their point of intersection will give the craft's position indicated by F in the Figure. This is 094° from B, distant 1·75 n.m. It can be seen that the angle ADB = 180° – 80° = 100°, therefore the angle AFB at the circumference = 50°. Similarly angle BEC = 70°, therefore angle BFC = 35°, and F being on both position circles must be the position given by the subtended angles.

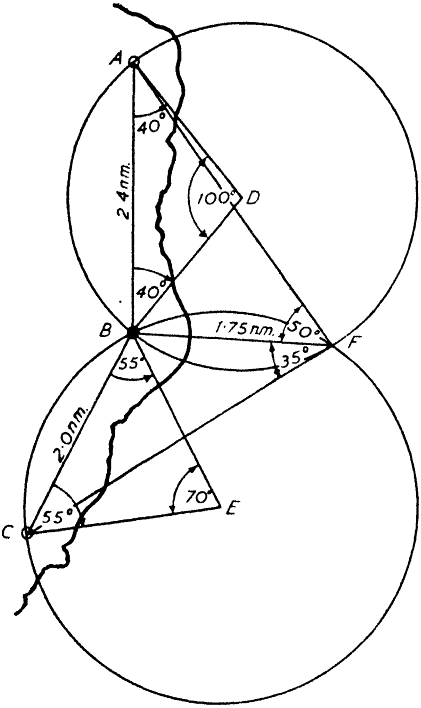


Fig. 88. Position by Horizontal Angles

The reliability of such fixes depends upon certain conditions. The objects should be in the same horizontal plane. The distance from the middle object should be the shortest and as short as possible. The circles should intersect at an angle as near 90° as possible and never at an angle less than 30°. Errors may occur due to inaccurate observation but these should never be large if the distances are short. The sextant measures the angles far more accurately than the compass, but angles cannot be plotted on the chart to a greater accuracy than about half a degree.

Exercise 5

Find the observer's position as a bearing and distance from B in each of the following problems:

	True bearing of A from B	*(n.m.)*	*Angle subtended by AB*	*Approx. bearing of B*	*Angle subtended by BC*	*True bearing of C from B*	*(n.m.)*
1.	240°	4	50°	northward	35°	045°	3
2.	100°	5	40°	northward	60°	300°	4
3.	330°	3	70°	south-westward	30°	180°	7
4.	075°	6	20°	eastward	100°	220°	5
5.	270°	2	0°	westward	45°	010°	3½
6.	000°	4½	90°	south-westward	37°	140°	2
7.	180°	4	180°	northward	40°	060°	3

Running Fix. It is not always possible to see three suitable objects when a position is required, nor even two, and use has often to be made of only one. If the height of the object cannot be ascertained a position cannot be obtained immediately, so a first bearing is taken

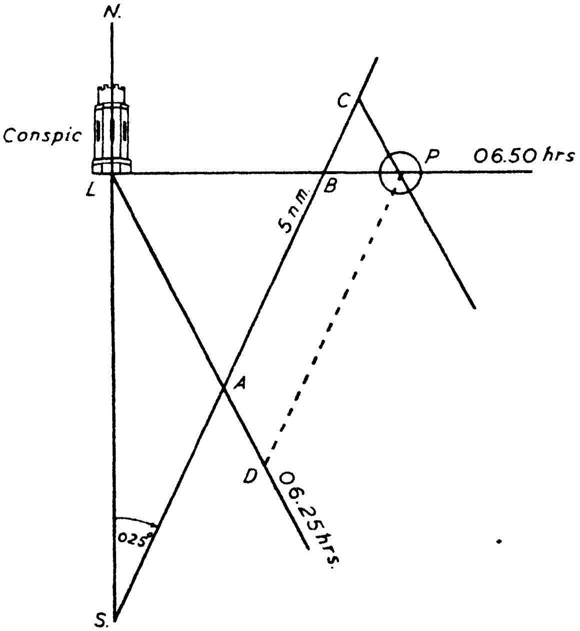


Fig. 89. The Running Fix (i)

followed by another after an interval of time, during which the craft will have covered sufficient distance to produce a change in bearing large enough to give a satisfactory intersection of position lines. This method of finding position is known as a **running fix** and is illustrated

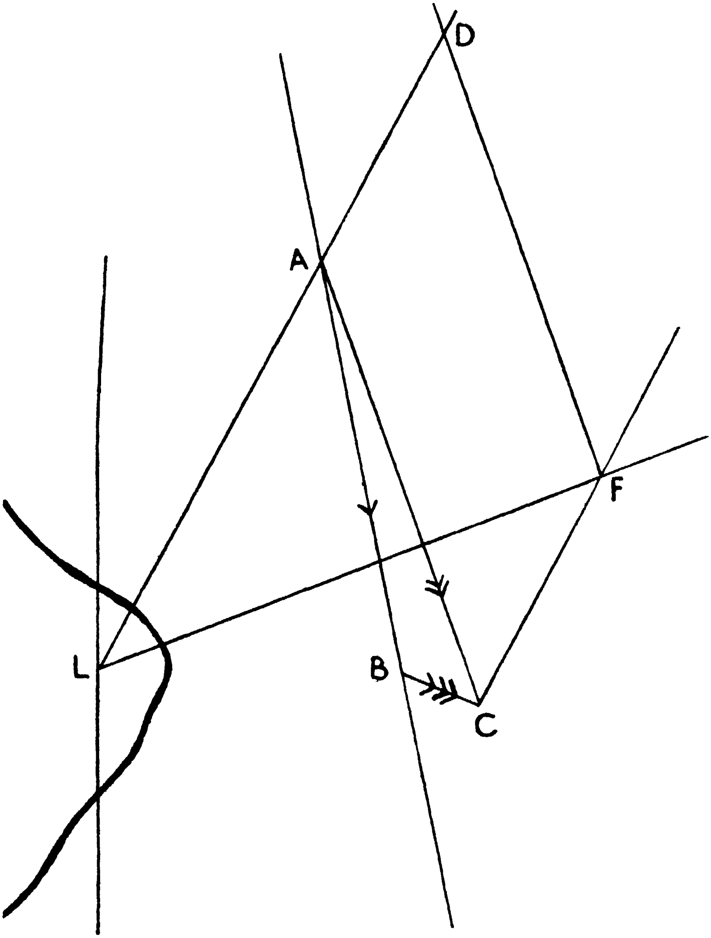


Fig. 90. The Running Fix (ii)

in Fig. 89, where L represents the position of a conspicuous object, AL, the first bearing 330° T. at 06.25 hours, and BL, the second bearing 270° T. at 06.50 hours. The track, or course made good, over the ground is 025° T., shown by the angle LSA, and the speed is estimated at 12 knots. To construct the Figure lay off through L the reciprocals of the two bearings. Across these lay the track of course made good, and from the point A, where it crosses the first bearing, measure the distance travelled in the interval—*i.e.*, 5 n.m.—between the two bearings to scale along the track, or course line. If the end C does not fall on the second bearing a line must be drawn through it parallel to the first bearing LA, and the point P where this intersects the second bearing will give the position when the second bearing was taken. This is found to be 090° from L distant 4·7 n.m. CP is really the bearing LA transferred in the direction of the track, or course made good, for the amount of distance travelled by the vessel in the interval. Lay the reversed course back through P, and where it intersects the first bearing, produced if necessary, will give the position D where the vessel must have been when the first bearing was taken. PD = AC because ACPD is a parallelogram.

When the course over the ground—*i.e.*, the track, is not known accurately, the course steered and estimated current or wind are used. In this case the course and current or wind vectors are linked together as shown in Fig. 90, in which AL represents the first bearing 209° T., AB the distance steamed or flown, 5·5 n.m., and BC the current or wind vector for the time taken to pass between the bearings. LA is transferred through C, the end of the wind vector, and intersects the second bearing at F, giving the position on the second bearing as 069° T. from L, distant 7 n.m. AC is the resultant of AB and BC, and a line parallel laid back through F will give the point D, at which the vessel or aircraft must have been on the first bearing. If leeway has to be included with current effect, it should be applied to the course at the point where it crosses the first bearing—to starboard when the wind is on the port side, and to port when the wind is on the starboard side—the distance steamed being then marked along the adjusted course.

Exercise 6

Using the following information, find the observer's bearing and distance from the object at the time of taking both the second and first bearings.

	First bearing (*T.*)	*Course and distance between bearings* (*T.*)	(*n.m.*)	*Second bearing* (*T.*)	*Current*	*or*	*Wind*
1.	240°	200°	4·0	330°	nil		nil
2.	320°	270°	5·0	045°	nil		nil
3.	100°	150°	7·2	040°	090°/1·0 n.m.		270°/1·0 n.m.
4.	260°	300°	10·4	180°	210°/2·0 n.m.		030°/2·0 n.m.
5.	060°	025°	6·9	126°	105°/1·5 n.m.		285°/1·5 n.m.
6.	174°	127°	8·1	248°	350°/1·8 n.m.		170°/1·8 n.m.
7.	310°	342°	3·7	264°	020°/0·7 n.m.		200°/0·7 n.m.
8.	047°	018°	9·8	125°	198°/1·3 n.m.		018°/1·3 n.m.

Doubling the Angle on the Bow. A special case arises when the angle between the course and bearing—*i.e.*, angle on the bow—is doubled. An isosceles triangle is formed in which the distance covered by the vessel or aircraft is equal to the distance from the lighthouse at the time of taking the second bearing, as shown in Fig. 91 where LAB = 30° and LBC = 60°. It can be seen that ALB also = 30°, so that LAB is an isosceles triangle in which AB the distance steamed or flown = LB the distance off.

When the first angle is 45° the second will be 90°, and the distance covered in the interval will equal the distance off when the object is abeam—*i.e.*, at right angles to the longitudinal axis. This latter case is generally referred to as the **four-point bearing problem** (4 × 11¼° = 45°), but whenever the distance covered between the bearings is deemed to be the distance off, it is assumed that the course steered is the course made good over the ground, and the speed has been estimated correctly. Otherwise the effect of wind, current, or tide must be taken into account as for a running fix and the estimated distance measured along the track.

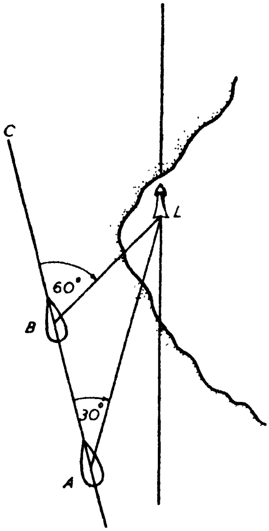


Fig. 91. Doubling the Angle on the Bow

Track by Three Bearings. Three bearings of one object are sometimes taken in order to ascertain the track, or course made good. Fig. 92 illustrates the method. L represents a lighthouse or prominent landmark, AL the first bearing 033° T., taken at 09.30 hrs., BL the second bearing 012° T., taken at 09.40 hrs., and CL the third bearing 338° T., taken at 10.00 hrs. A line DE is drawn through L at right angles to the centre bearing LB and distances are measured along it from L to correspond to the ratio of the intervals of time taken to pass from one bearing to another. The intervals in this case were ten minutes and twenty minutes, a ratio of 1 to 2, so the distances LK and LH are in the same ratio. Lines drawn through K and H parallel to the centre bearing LB intersect the first and third bearings in J and M respectively. The line joining J to M then represents the track in direction but not in position. Suppose, however, that at the time of taking the third bearing of L, the bearing of another landmark P were noted to be 040° T., a fixed position X would be obtained, and if a line parallel to JM were drawn through X, YX would represent the track both in direction and position—Y being the position at 09.30 hrs. and X the position at 10.00 hrs. The course would probably differ from the track, and on being plotted would extend, say, to Z in half an hour. Z would, therefore, represent the D.R. position of a ship or the air position of an aircraft, and on being joined to X would give, when measured to scale, either the set and drift of a current or the wind velocity in half an hour.

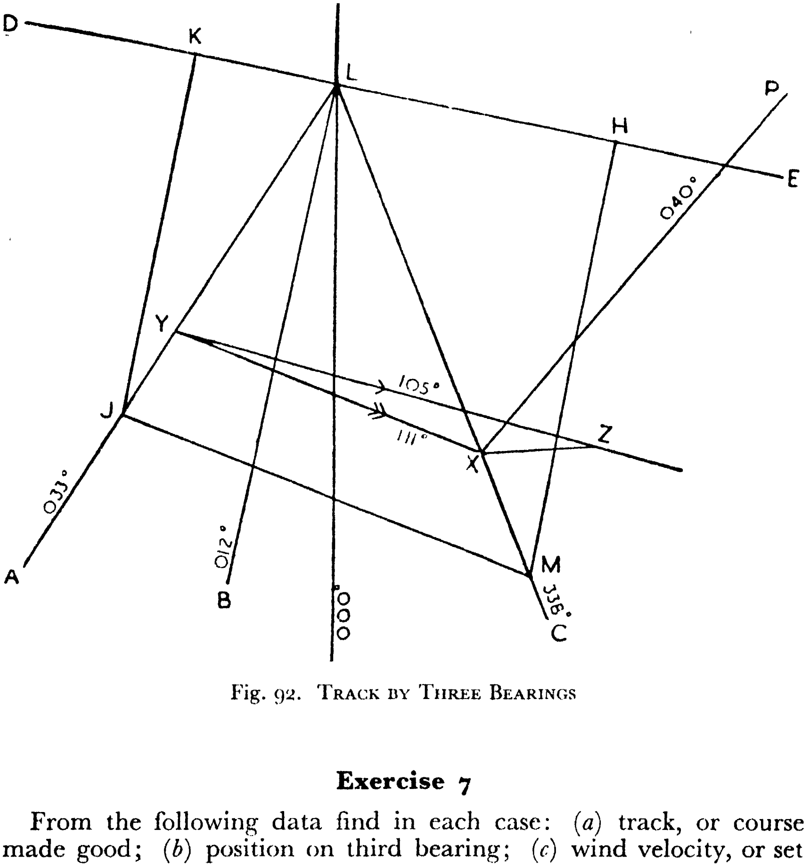


Fig. 92. Track by Three Bearings

Exercise 7

From the following data find in each case: (*a*) track, or course made good; (*b*) position on third bearing; (*c*) wind velocity, or set and drift of current. True air speed of aircraft 150 knots.

	Course (T.)	*True bearings of L*			*Distances covered*		*Bearing and distance of P from L*		*True bearing of P*
		1	2	3	(*n.m.*)		(*T.*)	(*n.m.*)	
1.	010°	055°	094°	152°	4·0	6·5	020°	8·0	070°
2.	118°	062°	010°	344°	5·2	3·6	090°	6·0	025°
3.	355°	310°	272°	228°	6·1	7·4	000°	10·0	295°
4.	280°	338°	024°	052°	5·8	4·5	270°	7·5	350°
5.	077°	118°	162°	215°	3·9	6·2	085°	5·6	163°
6.	212°	264°	308°	355°	7·2	9·4	200°	12·3	257°

Use of Examples in a Practical Problem. Some of the examples illustrated in this chapter have been incorporated in a problem on the chart at end of book as follows:

06.00 hrs. Position A. Find the compass course to pass 5 n.m. to the westward of Outer Dowsing light-vessel. Speed of vessel 10 knots. Deviation from Deviation Curve No. 1. Variation 10° W.
11.00 hrs. Position C. Find the set and drift of current and compass course to Cross Sand light-vessel to counteract the current. At what distance would you expect to pass the Haisbro and Newarp light-vessels, and when would you expect to reach the Cross Sand light-vessel?
16.35 hrs. Cross Sand light-vessel. Altered course to 186° by compass.
17.20 hrs. Lowestoft lighthouse bears 260° by compass.
18.08 hrs. Lowestoft lighthouse bears 300° by compass.
Give position at 18.08 hrs., assuming the tide to have set 200° T. for 2 n.m. in the interval.

Here is an explanation of the working.

With radius 5 miles draw an arc to the westward of Outer Dowsing light-vessel and from A draw a tangent to the arc. The direction of this line is the required true course (145° T.), which when corrected for variation 10° W. gives 155° (M.), and the deviation 6° E. from the card gives 149° (C.).

The position at 11.00 hrs. is C, whereas the D.R. position is at P, 50 n.m. along the course line from A and just abeam of the light-vessel. The tide must have carried the vessel in the interval in the direction of and over a distance covered by PC, making the set 170° T. and the drift 6 n.m. or 1·2 knots. To find the course to the Cross Sand light-vessel to counteract the effect of this tide, we join C to the Cross Sand light-vessel and from C draw a line in the direction of the tide, that is, extend PC and mark along it a distance to correspond to the amount of set for any convenient interval. For our purpose CR has been made equal to PC, and so from R a distance corresponding to 50 n.m. has been taken as a radius and is found to intersect the direct line at T. RT is then the direction of the course and is found to be 138° T., 148° M., and 142° C. Distances from Haisbro and Newarp light-vessels are 2½ and 4 n.m. respectively. The time taken to reach the Cross Sand light-vessel is found by either drawing a line parallel to RT through the Cross Sand light-vessel and dividing its distance from CR produced by the steaming speed, or finding the speed over the ground along CT and dividing it into the distance from C to the light-vessel. The former method is the more convenient but when the angle of intersection at T is small the latter method will prove more accurate. In either case the time here is found to be five and a half hours, which when added to the time of leaving C gives 16.30 hrs. as the time of arrival at the Cross Sand light-vessel. The actual time proved to be 18.35 hrs., and the course was then altered to 186° by compass which was south true. Forty-five minutes later Lowestoft lighthouse bears 260° C., which when corrected for the deviation as found for the course and the variation gives 254° T. The second bearing taken after steaming for forty-eight minutes at 10 knots was 300° by compass, giving 294° T. when corrected for the same deviation and variation as the first bearing. The reciprocals of these are drawn through Lowestoft lighthouse, and 8 n.m. measured along the course from its point of intersection S with the first bearing. This gives point Q from which the current is drawn for a distance of 2 n.m. Through the terminal-point W a line parallel to the first bearing is drawn, and where it cuts the second bearing at Z is the vessel's position at 18.08 hrs. This is latitude 52° 23′ N. longitude 2° 05′ E., or 115° Lowestoft lighthouse 14 n.m.

General Exercises

1. What do you understand by the term 'fix'?

2. Enumerate the methods employed in obtaining position lines or curves.

3. What is the difference between a course and a bearing?

4. Describe the formation of a 'cocked hat.' What reliance should be placed on the information obtained from it?

5. Round Island lighthouse (49° 59′ N. 6° 19′ W.) bore 240° T., and its height, 180 ft., subtended an angle of 00° 30′. By plotting on plain paper give the vessel's position in latitude and longitude.

6. When is a light said to be 'dipping'? Would you see it farther at high water or at low water?

7. St Catherine's Point light has a charted visibility of 18 n.m. How far would it be visible to an observer 50 ft. above sea-level?

8. The Lizard light (49° 58′ N. 5° 42′ W.) 230 ft. high, bore 050° T. when dipping to an observer 50 ft. above sea-level. What was the latitude and longitude of the vessel?

9. Longships lighthouse (50° 04′ N. 5° 45′ W.) bore 045° T. and Wolf Rock lighthouse (49° 57′ N. 5° 48′ W.) bore 140° T. Find the position in latitude and longitude and distance from Longships lighthouse.

10. Cape la Hague lighthouse (49° 43′ N. 1° 57′ W.) bore 150° T., Casquets lighthouse (49° 43′ N. 2° 23′ W.) bore 240° T. Find the latitude and longitude and the distance from Casquets lighthouse.

11. Shipwash light-vessel (52° 02′ N. 1° 42′ E.) *ϕ* Orfordness lighthouse (52° 05′ N. 1° 34′ E.) bearing 303° T. Buoy (51° 58′ N. 1° 38′ E.) subtends an angle 50° to the left. Find the position relative to Shipwash light-vessel.

12. While proceeding along the East Coast the horizontal angle between Southwold lighthouse (52° 19½′ N. 1° 41′ E.) and Aldeburgh Napes buoy (52° 09½′ N. 1° 42′ E.) was found to be 56°. At the same time the horizontal angle between Aldeburgh Napes buoy and Orfordness lighthouse (52° 05′ N. 1° 35′ E.) was 48°. Find the position of the vessel relative to Orfordness lighthouse.

13. While steering 315° T. the relative bearing of Haisbro light-vessel (52° 59′ N. 1° 35′ E.) was 030° and the relative bearing of Cromer lighthouse (52° 55′ N. 1° 20′ E.) was 315°. Give the position of the craft in latitude and longitude.

14. Steering 190° C. Lowestoft lighthouse (52° 29′ N. 1° 45′ E.) bore 30° on the starboard bow. After covering 4 n.m. it bore 60° on the bow. Find the position in latitude and longitude. Deviation 4° W., variation 10° W.

15. Ailsa Craig (55° 15′ N. 5° 06′ W.) bore 110° T. Corsewall Point (55° 00′ N. 5° 09′ W.) bore 170° T. Find the position in latitude and longitude and relative to Corsewall Point.

16. Steering 100° T. Oversay Is. lighthouse bore 050° T. and after covering 15 n.m. it bore 350° T. Find the position relative to the lighthouse.

17. Minehead lighthouse (52° 00′ N. 7° 35′ W.) bore 020° T. Ballycottin lighthouse (51° 49′ N. 7° 59′ W.) bore 270° T. Find the position in latitude and longitude.

18. While steering 080° T. the relative bearing of Barrels light-vessel was 340° and after covering 4 n.m. the relative bearing was 290°. Find the distance off at the time of taking the second bearing and when abeam.

19. The following horizontal angles were measured simultaneously from a craft off the east coast of Scotland:

Clyth Ness (58° 18½′ N. 3° 12′ W.)
} 50°
Noss Head (58° 28½′ N. 3° 03′ W.)
} 60°
Duncansby Head (58° 38′ N. 3° 01′ W.)

Give the position relative to Duncansby Head.

20. Being to the southward of Eddystone lighthouse the following horizontal angles were measured simultaneously. Find the position of the craft as a bearing and distance from Eddystone lighthouse.

Eddystone lighthouse (50° 11′ N. 4° 16′ W.)
} 80°
Bolt Head (50° 12½′ N. 3° 47′ W.)
} 50°
Start Point (50° 13′ N. 3° 38′ W.)

21. Swilkie Point lighthouse (58° 42′ N. 3° 07′ W.) bore 170° T.
Dunnett Head lighthouse (58° 40′ N. 3° 22′ W.) bore 250° T.
Find the position in latitude and longitude and relative to Dunnett Head.

22. Steering 045° T. Clyth Ness (58° 18½′ N. 3° 12′ W.) bore 030° T. After covering 8 n.m. its relative bearing was 250°. If the effect of the wind or current were in the direction 220° T. for 2 n.m., what would be the position of the craft relative to Clyth Ness at the time of taking the second bearing and the first bearing?

23. Steering 200° C. Berry Head (50° 24′ N. 3° 29′ W.) bore 255° C., and after covering 9 n.m. it bore 320° C. Find the position of the craft in latitude and longitude at the second bearing and relative to Berry Head. Deviation 2° E., variation 12° W.

24. Steering 210° C. Strumble lighthouse (52° 02′ N. 5° 04′ W.) bore 102° C. and S. Bishop lighthouse (51° 51′ N. 5° 25′ W.) bore 162° C. Find the position of the craft in latitude and longitude and relative to S. Bishop lighthouse. Deviation 3° E., variation 15° W.

25. Kingswear lighthouse (50° 21′ N. 3° 34′ W.) bore 300° C. when Start Point lighthouse (50° 13′ N. 3° 38½′ W.) and Prawle Point (50° 12′ N. 3° 43′ W.) were in transit bearing 255° C. Find the position of the craft in latitude and longitude.

26. Being to the northward of Bull Point the horizontal angles between The Foreland lighthouse (51° 15′ N. 3° 47′ W.), Bull Point lighthouse (51° 12′ N. 4° 12′ W.), and Lundy Is. N. lighthouse (51° 12′ N. 4° 41′ W.) were 80° and 75° respectively. Find the position of the craft in latitude and longitude and relative to Bull Point lighthouse.

27. Steering 020° C. Ushinish lighthouse (57° 18′ N. 7° 12′ W.) bore 350° C. and after covering 5 n.m. it bore 257° C. If the effect of the wind or current were 1 n.m. in the direction 045° T., what would be the positions at the time of taking the first and second bearings? Deviation 2° E., variation 14° W.

28. Being to the south-eastward of C. Henlopen the horizontal sextant angle between Cape Henlopen lighthouse (38° 47′ N. 75° 05′ W.) and Overfalls light-vessel (38° 48′ N. 75° 01′ W.) was 15° when Cape May light (38° 56′ N. 74° 58′ W.), height 164 ft., was dipping to an observer 20 ft. above sea-level. Find the position of the craft in latitude and longitude.

29. Steering 238° T. Tuskar Rock lighthouse (52° 12′ N. 6° 13′ W.) bore 280° T. After covering 6 n.m. it bore 340° T., and after covering a further 4 n.m. it bore 020° T. when Barrels light-vessel (52° 08½′ N. 6° 23′ W.) bore 290° T. Find the course made good (track), the positions at the time of taking the first and third bearings, and the set and drift of current or wind velocity. Ship's speed 12 knots; aircraft's speed 240 knots.

30. From a vessel anchored in a river the vertical angle subtended at high water by a factory chimney (height 120 ft. above high water) was 2° 15′. What would be the subtended angle at low water supposing the range of tide to be 20 ft?

CHAPTER IX

TACTICAL NAVIGATION

Relative velocity—Course to steer to maintain a constant relative bearing—Interception (simple case)—Course to steer by an aircraft to maintain a constant relative bearing from a ship—Interception of a ship by an aircraft—Relative wind—Radius of action (simple case)—Radius of action (returning to the same base)—Radius of action (returning to a different base)—Radius of action (returning to a moving base)—Fixed square-search.

Relative Velocity. Up to the present we have discussed the velocity of ships and aircraft in relationship to an apparently stationary object, the earth. We are now going to consider the movement of one moving object as seen from another. This is generally referred to as relative velocity. When you look through the window of a moving train, stationary objects by the lineside appear to be moving backward in a direction opposite to your own but at the same speed as yourself—*i.e.*, the velocity of a signal-box relative to you is the reverse of your own velocity. Suppose, however, a friend of yours in another train were to pass in the opposite direction, his relative velocity would be in a direction the reverse of your own but at the sum of your individual speeds. This is the same as if he were stationary and you were travelling at the sum of your individual speeds. So if we imagine one of the moving objects to be brought to a standstill, we can maintain the relationship between them by giving to the second object the first object's velocity reversed. If you, in a train travelling at 40 M.P.H., were to overtake another train travelling on an adjacent line at 30 M.P.H., the relative velocity of it to you would be 10 M.P.H. backward, which result could also be obtained by bringing yourself to a standstill and applying your velocity reversed to that of the other train—*i.e.*, $30 - 40 \doteq -10$ M.P.H. When objects are moving obliquely to each other the application of this rule is not so obviously true. A consideration of Fig. 93 in connexion with the following problem should, however, clear up any misapprehensions.

Example. Two aircraft (or two ships) A and B set course simultaneously from an aerodrome (anchorage) D, A steering 030° T. at 120 knots (12 knots) and B 090° T. at 150 knots (15 knots). Find their relative velocities.

In Fig. 93 D represents the position of the aerodrome (anchorage), DK the course of craft A, and DL the course of craft B. DA_1 represents 2 n.m. and DA_2 4 n.m., so that A_1 is the position of A after one minute's flying (ten minutes' steaming) and A_2 its position after two minutes' flying (twenty minutes' steaming). Similarly DB_1 represents $2\frac{1}{2}$ n.m. and DB_2 5 n.m., so that B_1 and B_2 are the positions of B after one (ten) and two (twenty) minutes respectively. Now the relative bearing of B from A is found to be 110° in each case and their distance apart 2·3 n.m. after one minute's flying (ten minutes' steaming), and 4·6 n.m. after two minutes' flying (twenty minutes' steaming), making a speed of 138 knots (13·8 knots). The velocity of B relative to A is therefore 110° T. 138 knots (13·8 knots), and that of A relative to B 290° T. 138 knots (13·8 knots). Suppose A brought to a standstill at D and A's reversed velocity applied to B, then B has the two velocities DM and MP, which when combined provide a resultant DP. This by measurement and calculation can be proved to be the same in direction and amount as A_1B_1, showing that the problem is solved graphically by either applying each velocity to its respective body or bringing one to a standstill and adding its reversed velocity to that of the other.

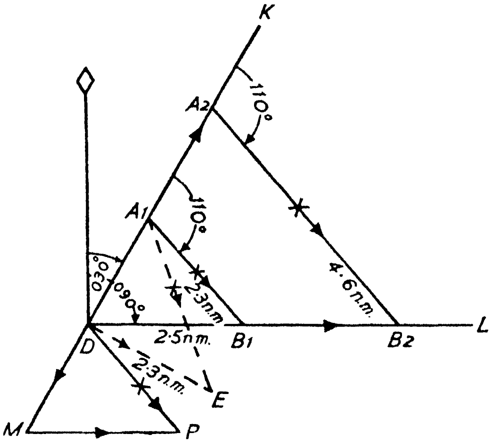


Fig. 93. Relative Velocity

Exercise 1

Find the relative velocity of craft B to craft A in each of the following cases.

	A *True course*	*A* *Speed (knots)*	*B* *True course*	*B* *Speed (knots)*
1.	000°	15·0	090°	12·0
2.	240°	12·0	315°	20·0
3.	200°	18·0	080°	15·0
4.	075°	16·5	350°	13·8
5.	300°	9·0	060°	14·7
6.	108°	24·5	172°	13·2

For aircraft multiply the speeds by 10.

Course to Steer to Maintain a Constant Relative Bearing. In the last problem the velocity of one craft relative to the other was conditional on their respective velocities being maintained. The three velocities were in equilibrium and could therefore be represented by three vectors forming a triangle (DAB, Fig. 93). If any other relative velocity were to be prescribed, however, the side of the triangle representing it would have to be altered in direction or length or both, resulting in a consequent adjustment of one of the other sides to maintain equilibrium. Suppose that in the last problem the relative velocity of B with regard to A were to be altered to 130° T. at 180 knots (18 knots), the direction and length of A_1B_1 (Fig. 93) would have to be altered as shown by the pecked lines, and the course and speed of B would then be readjusted accordingly. The new course and speed of B is thus found to be 120° T. at 138 knots.

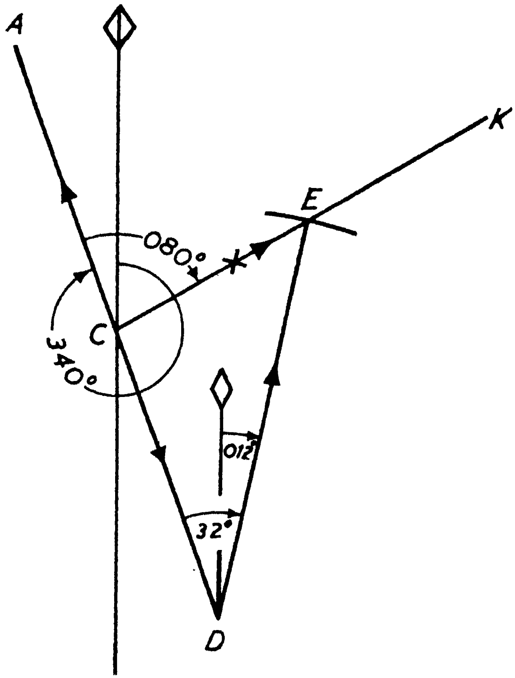


Fig. 94. Course to Steer to Maintain a Relative Bearing

If only the relative bearing is given and the speed of opening not specified, the course of B will be determined by whatever speed is assigned to B.

Example. An aircraft (ship) B flying (steaming) in close formation with an aircraft (ship) A on a course of 340° T. at 180 knots (18 knots) has instructions to proceed on and maintain a relative bearing of 080° from A at a true air (steaming) speed of 240 knots (24 knots). Find B's course.

Fig. 94 provides a solution of the problem. Suppose both craft to be at position C, steering 340° T., when craft B is directed to proceed on a relative bearing of 080°; CK is to be the direction of B from A. If we apply A's reversed velocity to both craft we shall make A stationary at C while B is pushed back to D but has a velocity yet to be determined. As the direction of opening is fixed and B's speed is also fixed, the course for B to steer can be found by using B's speed as a radius and with D as centre to intersect with an arc the direction of opening CK, so completing the triangle of velocities. The point of intersection determines the direction of DE—*i.e.*, the course B must steer, found to be 012° T.—and the distance CE, the speed of opening, 130 knots.

Exercise 2

Find the true course that B must steer to maintain the following constant relative bearing from A, and give the speed of opening.

	A's course (T.)	*A's speed (knots)*	*B's speed (knots)*	*Relative bearing*
1.	180°	14·0	17·0	060°
2.	235°	12·0	20·0	340°
3.	300°	24·0	30·0	100°
4.	045°	16·2	24·8	330°
5.	146°	15·4	13·5	125°
6.	345°	18·8	26·3	050°

For aircraft multiply the speeds by 10.

Interception (Simple Case). Instead of opening out from another craft it is often desirable to close with one—*i.e.*, intercept it. In this case the procedure for finding the course to steer is very similar to that just described for the action of maintaining a constant bearing.

Example. A ship A, speed 25 knots (aircraft 250 knots), has instructions to intercept a ship (aircraft) B bearing 260° T. from her, distant 50 n.m., and steering 020° T. at 15 knots (150 knots). Find the course A must steer to intercept her, and the time taken.

In Fig. 95 A and B represent the relative positions of both craft. Because A has to intercept B in the shortest possible time, AB must represent the line of approach, the direction of constant bearing. To find the course to steer in order to maintain this bearing, we must make B appear stationary and apply to A the reversed velocity of B. A thus appears to start from C, which, when used as the centre for

A's speed to scale as radius, provides an arc cutting AB in D, so completing the triangle of velocities. The reflex angle TCD is then the course, 290° T., and the distance AD represents the speed of approach to scale. This is found to be 29 knots (290 knots for the aircraft). As the total distance to approach is 50 n.m. the time to intercept taken by the ship will be $\frac{50}{29}$ hours = 1 hour 43 minutes. For the aircraft the time would be one tenth of this—*i.e.*, 10·3 minutes. On drawing A's course parallel to CD through A it intersects B's course at a point E. Only in cases where the water or air is still, however, can we say that the point of intersection of the two courses will also represent the geographical position of interception. The water through which the ship travels or the air through which an aircraft flies is itself usually in motion, and the effect of this on the craft concerned must be allowed for in determining the geographical position. When the craft involved are both ships or both aircraft they will each be moving through the same medium, and, generally speaking, the effect on one ship or aircraft will be the same as that on the other. Allowance for this may be left until the end of the problem when it can be applied to the position at which the courses intersect. If in our example a current had set 180° T. or a wind blown from 000° T. for 5 n.m. in the interim, it would be represented by EF (Fig. 95), so that the position of interception would be F not E.

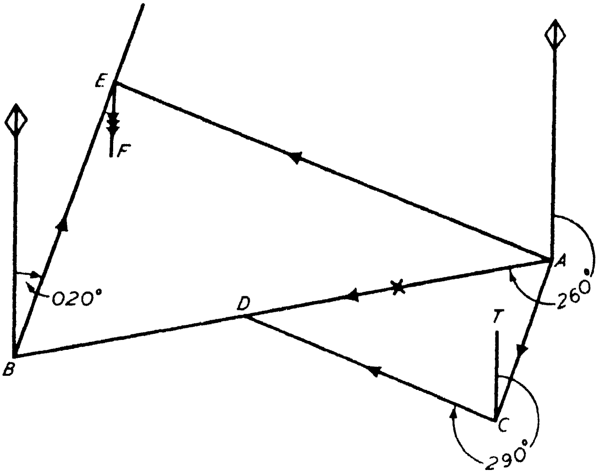


Fig. 95. INTERCEPTION (SIMPLE CASE)

Exercise 3

Find the course A must steer to intercept B, and the duration of the operation.

	B's bearing and distance from A		*B's course and speed*		*A's speed*
	(T.)	*(n.m.)*	*(T.)*	*(knots)*	*(knots)*
1.	020°	100	250°	12·0	18·0
2.	200°	75	310°	10·2	15·5
3.	330°	120	090°	16·0	14·5
4.	145°	90	060°	13·4	20·3
5.	070°	48	340°	11·9	16·4
6.	240°	60	165°	18·6	24·1

For aircraft multiply the speeds by 10.

Course to Steer by an Aircraft to Maintain a Constant Relative Bearing from a Ship. When an aircraft leaves a carrier

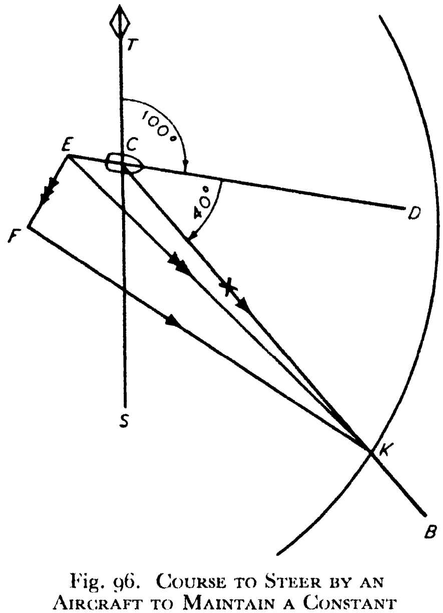


Fig. 96. Course to Steer by an Aircraft to Maintain a Constant Relative Bearing from a Ship

and proceeds in a given direction towards some shore object, the direction and speed of the aircraft as seen from the carrier is constant and will be its relative velocity. This may be represented by a straight line joining the carrier's position to that of the aircraft at any subsequent time, as long as no alteration of course or speed is made by either craft. The aircraft's track, however, must be represented by the straight line joining the starting-point to the aircraft's subsequent positions. Fig. 93 may be taken to illustrate this, if it is remembered that the carrier's speed is only a fraction of that of an aircraft. If A be regarded as the carrier and B the aircraft, AB will be the aircraft's relative velocity and DB its track.

If the bearing to be maintained is specified, the track must be arranged to suit. In this case we treat the carrier's reversed velocity as an additional wind to be combined with the actual wind.

Example. The pilot of an aircraft is instructed on leaving the carrier to patrol at T.A.S. 150 knots on a bearing of 40° to starboard. Course of carrier 100° T., speed 20 knots, W/V 030°/30 knots. What would be his true course, track and G.S., and speed of opening?

Fig. 96 illustrates how the problem may be solved by plotting. Point C represents the position of the carrier, angle TCD the carrier's true course, and angle DCB the constant relative bearing. CE represents the carrier's reversed velocity to scale, and EF the wind vector. Then, with F as centre and the air speed to scale as radius, an arc is drawn to intersect CB. The point of intersection is K. FK is then the course vector, crossing the meridian TS at an angle of 124°. The track vector is obtained by joining E to K, giving a track of 135° at a G.S. of 155 knots. As the aircraft will be at K after one hour's flying, CK must represent the speed of opening, which on measurement is found to be 139 knots. The problem may also be solved by means of the air-speed circle circumscribed about a compass rose on a chart. Longitude units are then used as the constant scale for vectors. In this case F would be the centre of the circle and FK the radius, FE would be plotted to scale up-wind and EC would represent the carrier's course and speed. A line making an angle of 140° with the meridian at C would intersect the circle at K, whence a line joining F to K would give the course and EK would represent the track and G.S. Similarly CK would represent the speed of opening.

Exercise 4

Find the course an aircraft A must steer to maintain from a ship B the following constant relative bearings.

	B's course (T.)	*B's speed (knots)*	*A's speed (knots)*	*Relative bearing*
1.	090°	12·0	150	060°
2.	140°	15·0	120	330°
3.	300°	18·0	180	045°
4.	220°	16·4	145	310°
4.	045°	21·7	166	030°
6.	250°	20·5	138	300°

Interception of a Ship by an Aircraft. When an aircraft has to intercept a moving ship the problem is the same as that of an aircraft wishing to return to a carrier whose bearing is known. The method of solution in each case is similar to that described in the preceding section.

Example. An aircraft flying at a true air speed of 180 knots leaves a point A to intercept a vessel then at B, bearing 347° T. from A, distant 220 n.m., and steering 240° T. at 20 knots, the wind velocity being from 210° T. at 30 knots.

What will be the true course to steer in order to intercept the vessel in the shortest possible time? Where will the interception take place, and after how long?

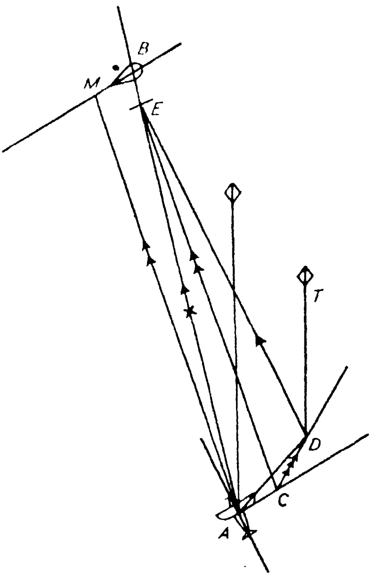


Fig. 97. INTERCEPTION OF SHIPS BY AIRCRAFT

Fig. 97 illustrates the problem. A and B represent the respective positions of aircraft and ship, and when joined represent the line of approach, or constant bearing, along which the aircraft will appear to travel towards the ship. In order to counteract the vessel's movement, a velocity equal in amount to that possessed by the ship but opposite in direction is applied to both craft. The ship is brought apparently to a standstill while the aircraft has to allow for this new contingency by applying it as an additional vector. AC then represents the ship's velocity reversed, and CD is the vector representing the wind. From D the air speed is taken as a radius, and the point at which its arc intersects AB will mark the position of the aircraft relative to the ship after whatever interval has been used for the vectors. If it were an hour, then AC would represent 20 knots, CD 30 knots, and DE 180 knots. AE would represent the speed of approach, and is found on measurement to represent 203 knots. The true course given by the reflex angle TDE is found to be 335°. Now, because CD represents the wind velocity and DE the course and air speed, CE must represent the track and ground speed. If the track is drawn through A, the starting-position of the aircraft, the position of interception will be indicated by the point M, where this track crosses the vessel's track. The time taken to reach the point of interception can be found either by dividing the ground speed CE into the distance AM along the track, or by dividing the speed of approach AE into the distance AB between aircraft and ship at the start. In either case the time taken works out at 1 hour 06 minutes.

Exercise 5

Find the course an aircraft A must steer to intercept a ship B. Give also speed of approach and time taken.

	B's bearing and distance from A		*B's course and speed*		*A's speed*	*W/V*
	(*T.*)	(*n.m.*)	(*T.*)	(*knots*)	(*knots*)	
1.	000°	50	240°	12·0	160	150°/30 knots
2.	200°	120	300°	15·0	145	045°/24 knots
3.	090°	80	010°	11·5	180	240°/35 knots
4.	140°	105	220°	18·0	150	180°/20 knots
5.	045°	96	120°	14·8	200	300°/40 knots
6.	320°	150	000°	16·2	175	090°/32 knots

Relative Wind. If in the foregoing problem you were to display a flag from the intercepting aircraft, in which direction would it fly? Down-wind? Well, yes, if by down-wind you mean the wind as felt by you in the aircraft, but not if you mean the wind relative to the ground. To the pilot of an aircraft in straight and level flight the wind *appears* to be always ahead; it is the wind produced by the movement of the aircraft through the air; a flag, if displayed, would always fly dead astern no matter what the direction and strength of the wind relative to the ground might be.

The case for the ship is different, however, as although like the aircraft the ship produces a wind due to its own movement, unlike the aircraft the ship is not carried along by the wind. The wind as felt aboard ship is similar to that which would be felt by you on a bicycle, except that a ship does sometimes make a little leeway. It is a combination of the true wind as experienced by a stationary person and that produced by the ship. If, then, we draw from the same point vectors to represent both the true wind and the artificial, the relative wind will be represented in direction and amount by their resultant,

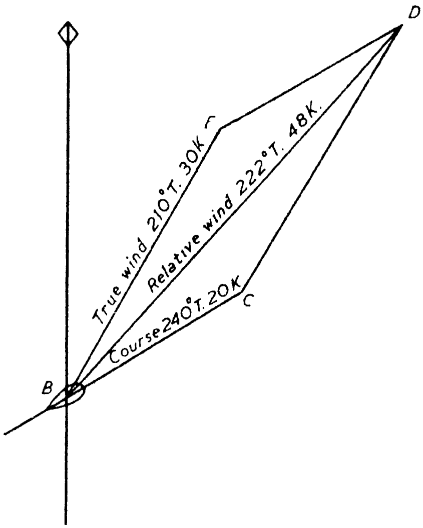


Fig. 98. Relative Wind

the diagonal of the parallelogram of which the first two are adjacent sides. Fig. 98 illustrates this in the case of the ship referred to in the foregoing example.

The true wind 210° T. at 30 knots is represented down-wind by the vector BE, the course 240° T., and speed 20 knots of the vessel, are represented reversed by BC. On completing the parallelogram the resultant vector BD is found by measurement to represent a relative wind of 222° T. 48 knots.

CD, being parallel and equal to BE, may be taken to represent the true wind and we then have a triangle of velocities which on comparison will be found to be the same as the triangle ACD in Fig. 97. The aircraft, in intercepting a moving ship, must then set a course so as to counteract the ship's relative wind. When this is known a simple triangle of velocity ADE (Fig. 97) is all that is required to find the course and speed of approach. The position of interception in this case would have to be found by marking the position reached by the vessel along her course in the time taken by the aircraft to reach her.

To find the true wind when the course, speed, and relative wind are known, two vectors representing the known values are drawn from the same point and the triangle completed. The third side will represent the true wind to scale, the direction being as indicated in Fig. 98 by CD—*i.e.*, from reversed course to relative wind.

Exercise 6

Complete the following table by finding the true wind velocity or relative wind velocity as required.

	Ship's course and speed		*True W/V*	*Relative W/V*
	(T.)	*(knots)*		
1.	270°	15·0	160°/30 knots	
2.	080°	12·5	200°/24 knots	
3.	140°	18·0	090°/32 knots	
4.	300°	22·4		330°/45 knots
5.	045°	10·2		200°/20 knots
6.	210°	16·8		150°/35 knots

Radius of Action (Simple Case). A ship at sea or an aircraft in flight may be called upon to leave a squadron and proceed on a given course at increased speed, with the object of making an investigation and rejoining the squadron at a stated time. The navigator has to calculate the time at which the course must again be altered, and what that course must be in order to return at the required time.

Example. At 08.00 hrs. a ship (an aircraft) receives instructions to leave a squadron, which is steering 090° T. at a speed of 15 (150) knots, and proceed at 20 (200) knots on a course (track) of 120° T., so as to rejoin the squadron at 10.00 hrs. (08.12 hrs.)

Fig. 99 shows how the problem may be solved graphically. A represents the position of the ships (aircraft) at 08.00 hrs., and AB the distance covered by the squadron in 2 hours (12 minutes). AC represents the distance which would be covered by the single ship (aircraft) on the new course at increased speed during the same period. The ship (aircraft) should by that time be at position B with the squadron. To make this possible a point D must be found along AC so that DC = DB and BCD becomes an isosceles triangle. This may be accomplished by joining BC and constructing an angle CBD equal in size to the angle BCA. Then DC will equal DB and AD + DB will equal AC, so that the ship (aircraft) will arrive at B at the same time as the squadron. The point at which course must be altered is D, and the distance AD divided by the speed will give the interval of time since leaving A. This on measurement is found to be $\frac{25}{20}$ = 1 hour 15 minutes for the ship, making the time of alteration 09.15 hrs. For the aircraft the interval would be $\frac{25}{200}$ hour—*i.e.*, 7½ minutes—making the time of alteration 08.07½ hrs. The new course for both craft is the same, and found from the Figure to be 035° T.

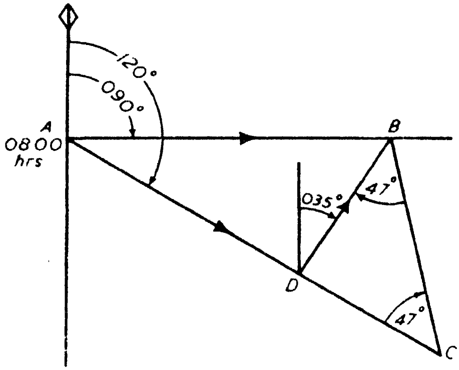


Fig. 99. Radius of Action (Simple Case)

Exercise 7

Find the course to return, and the time to alter course in each of the following cases.

	Squadron's course (T.)	*Squadron's speed (knots)*	*Scout's course (T.)*	*Scout's speed (knots)*	*Time interval*
1.	045°	22 (220)	010°	30 (300)	3 hrs. (18 mins.)
2.	200°	18 (180)	250°	24 (240)	4 hrs. (24 mins.)
3.	170°	20 (200)	150°	33 (330)	2 hrs. (12 mins.)
4.	340°	15 (150)	020°	25 (250)	3½ hrs. (21 mins.)
5.	180°	16 (160)	240°	20 (200)	5 hrs. (30 mins.)
6.	015°	21 (210)	330°	29 (290)	4½ hrs. (27 mins.)

Radius of Action (Returning to the same Base). Aircraft and ships are sometimes required to patrol along a given track at a constant speed and return to the starting-point at a fixed time. The track back is the reciprocal of the track out. If there is no wind or current, the course back will be the reciprocal of the course out and the time occupied on each part of the journey will be the same. It is thus an easy matter to calculate when and where an aircraft or a ship should turn. When a wind or a current has to be counteracted, however, the course back can be the reciprocal of the course out only if the wind or current happens to be up or down the track. The ground speed out would then be different from the ground speed back. It could be the same only if the wind or current were at right angles to the track. The following example is typical:

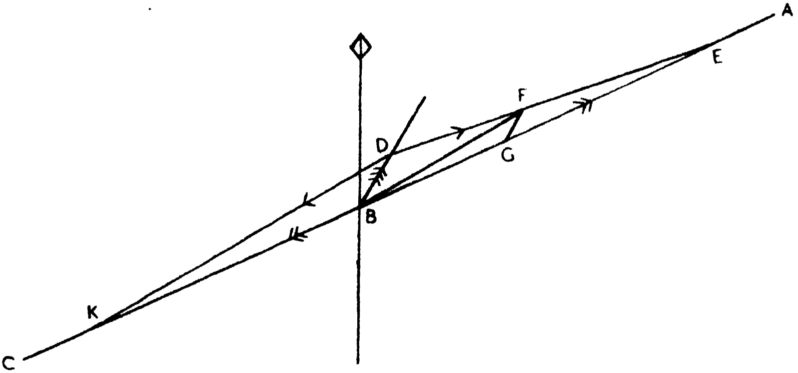


Fig. 100. RADIUS OF ACTION (RETURNING TO THE SAME BASE)

Example. An aircraft (a ship) has to patrol on a track of 065° T. and turn so as to be back at the starting-point in 3 (30) hours. T.A.S. 150 knots (ship's speed 15 knots), W/V 210°/24 knots (current 030° T. 2·4 knots). What would be the courses (T.) and ground speeds out and back, the radius of action, and the time to turn?

Fig. 100 illustrates how the problem may be solved graphically. BA represents the track out, and, as the craft must return to the same base, BC will represent the track back. By laying off the wind (current) vector for an hour and completing the triangles by making DE and DK represent the T.A.S. (or steaming speed), BE and BK will represent the G.S. out and back respectively.

To find how far the aircraft or ship could proceed along the track and be back in an hour, it will be necessary so to combine the two courses that their resultant will be equivalent to the wind vector for the same period. This can best be achieved geometrically as follows: BF is drawn parallel to KD, and through F a line is drawn parallel to DB. Where this cuts the track at G will be the point at which the aircraft should turn in order to be back at base in an hour. As BG represents the radius of action for an hour we need only multiply this distance by the number of hours allowed, in this case three, in order to find the radius of action for the full time. The time to turn can then be found by dividing the ground speed out, represented by BE, into the radius of action. The full results are as follows:

Course out 070° T. Course back 240° T.
G.S. out 168 knots (16·8 knots) G.S. back 130 knots (13·0 knots)
Radius of action in one hour 72 n.m. (7·2 n.m.)
Radius of action in three hours 216 n.m. (21·6 n.m.)
Time to turn $= \frac{216}{168} =$ 1 hour 17 minutes after leaving base.

Exercise 8

Returning to a stationary base, find from the following data: (*a*) courses and G.S. out and back; (*b*) radius of action (R. of A.); (*c*) time to turn (T.T.T.).

	Track out (T.)	*T.A.S. (knots)*	*Steaming speed (knots)*	*W/V*	*(Current)*	*Total time*
1.	150°	120	12	260°/20 knots	080°/2 knots	4 (40) hrs.
2.	300°	200	20	000°/30 knots	180°/3 knots	3 (30) hrs.
3.	040°	180	18	170°/25 knots	350°/2·5 knots	2½ (25) hrs.

Radius of Action (Returning to a Different Base). An aircraft may have to return at a stated time to an aerodrome different from that from which it set out, or to a moving carrier. In either case the method of finding courses and G.S., radius of action, and time to turn is the same.

Example. From an aerodrome B you are instructed to patrol on a track of 050° T. and return to an aerodrome A, 325° T. from B, distant 135 n.m., in 2½ hours. T.A.S. 150 knots, W/V 120°/25 knots. Find the courses and G.S. out and back, the track back, radius of action, and time to turn.

Mark the position of aerodrome B (Fig. 101) and from it draw a line in the direction of A, and measure along it a distance DE corresponding to the desired rate of progress towards A per hour, in this case $\frac{2}{5} \times 135 = 54$ n.m. Lay off the given track BG, 050° T., and put in the wind vector BC for an hour. Measure the true air speed to the same scale from C and join up to the point D at which it intersects the track. Join D to E and produce sufficiently far to allow an hour's air speed measured from C to intersect it at a point F. BF then represents the track and G.S. back.

The results obtained from the Figure are as follows:

	Out	*Back*
Course .	059° T.	265° T.
Track .	050° T.	269° T.
G.S. .	140 knots	170 knots

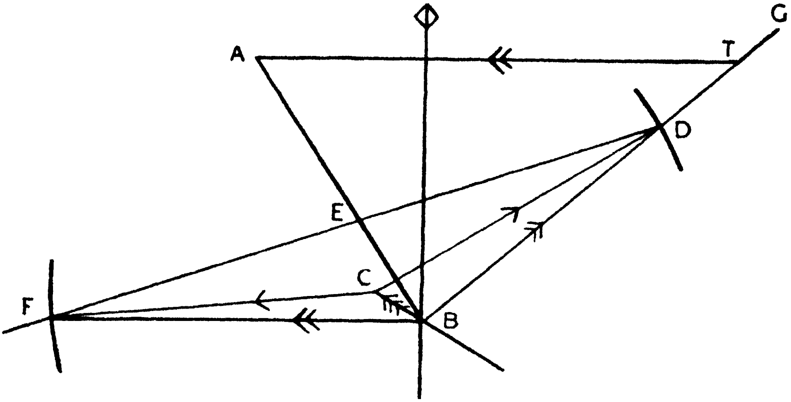


Fig. 101. Radius of Action (Returning to a Different Base)

The radius of action may be found from the Figure by drawing the reciprocal of BF through the second aerodrome A to intersect the track out at a point T. BT then represents the radius of action which is found on measurement to be 190 n.m. This divided by the G.S. out gives the time to turn,

$$Viz., \frac{190}{140} = 1\frac{5}{14} \text{ hours} = 1 \text{ hour } 21{\cdot}5 \text{ minutes.}$$

The radius of action may be found by calculation from the formula:

$$\text{R. of A.} = \text{Interval} \times \frac{\text{G.S. out} \times \text{G.S. back}}{\text{G.S. out} + \text{G.S. back}}$$

or briefly,

$$R = \frac{I \times O \times B}{O + B}$$

which becomes in our problem

$$\frac{5 \times 140 \times 170}{2(140 + 170)} = 192 \text{ n.m.}$$

The time to turn is found as before by dividing the G.S. out into the radius of action thus:

$$\frac{192}{140} = 1\frac{13}{35} \text{ hours or } 1 \text{ hour } 22 \text{ minutes.}$$

The geographical position of the point at which to turn is found by the intersection on a chart of the track out with the reciprocal of the track back drawn from the starting- and finishing-points respectively.

Radius of Action (Returning to a Moving Base). When an aircraft operates from a carrier the position of the carrier to which the aircraft must return is that found by allowing for the carrier's movement in the meantime. The course to be made good by the carrier is plotted, and the distance to be covered up to the time of the aircraft's return is measured along it. This gives a second position similar to that used in the preceding example, and the problem is solved in a similar way. The following example will show the development.

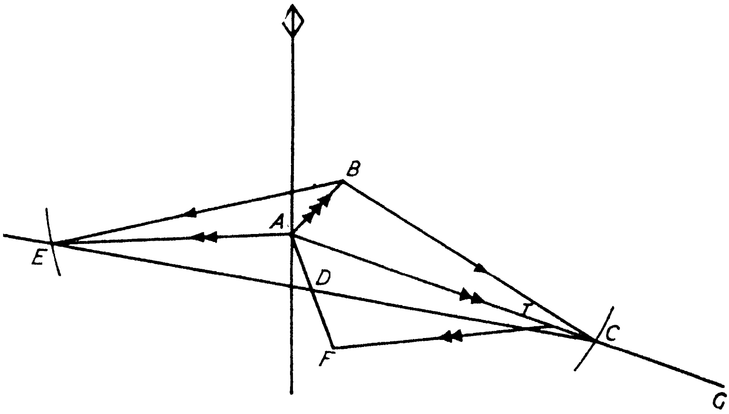


Fig. 102. RADIUS OF ACTION (MOVING BASE)

Example. An aircraft operating from a carrier has to reconnoitre along a track of 110° T. and return in 2 hours. The carrier is steaming 160° T. at 25 knots, T.A.S. of aircraft is 120 knots, and W/V 230°/30 knots.

What will be: (i) course and G.S. out; (ii) course, track and G.S. back; (iii) radius of action; (iv) time to turn.

In Fig. 102 position A represents the starting-point, and AG is drawn in the direction 110° T. to represent the track out. The wind vector for W/V 230°/30 knots is then drawn from A down-wind to B, and the triangle of velocities completed by using the T.A.S. as a radius on the same scale to intersect the track in C. BC is then the course vector, and the course out is found on measurement to be 123° T. The G.S. represented by the distance AC is found to be 132 knots.

The carrier is proceeding on a course of 160° T. so AD is drawn to represent its progress in one hour. C is then joined to D and the line produced sufficiently far to allow the T.A.S., taken to scale as a radius from B, to intersect it at a point E. AE then represents the track and G.S. back. These on measurement are found to be 266° T. and 95 knots respectively. The course is 258° T.

The reciprocal of the track back when drawn through F, representing the carrier's second position, is found to intersect the track out at a point T. AT then represents the radius of action, and is found to be 110 n.m. This divided by the G.S. out, 132 knots, gives the time to turn—*i.e.*, 50 minutes after leaving the carrier.

By calculation:

$$\text{R. of A.} = \frac{\text{I} \times \text{O} \times \text{B}}{\text{O} + \text{B}} = \frac{2 \times 132 \times 95}{132 + 95} = 110{\cdot}5$$

$$\text{T.T.T.} = \frac{110{\cdot}5}{132} = 50 \text{ minutes (to nearest minute).}$$

Tabulating the result:

(i) OUT: course 123° T. track 110° T. G.S. 132 knots.
(ii) BACK: course 258° T. track 266° T. G.S. 95 knots.
(iii) Radius of action 110 n.m.
(iv) Time to turn 50 minutes after leaving carrier.

Exercise 9

Using the following information find (*a*) the aircraft's courses out and back, (*b*) track back, (*c*) R. of A., (*d*) T.T.T.

	Carrier's course and speed		*Aircraft's T.A.S.*	*Track out*	*Duration of flight hours*	*W/V*
	(T.)	*(knots)*	*(knots)*	*(T.)*		
1.	090°	20·0	150	120°	3	045°/30 knots
2.	210°	16·0	120	150°	4	180°/20 knots
3.	340°	22·5	170	020°	2½	270°/24 knots
4.	045°	18·8	200	350°	3½	170°/36 knots
5.	140°	17·6	180	200°	1¾	300°/32 knots

The Fixed Square-search. If an aircraft comes down into the sea or a ship requires assistance, an endeavour is made on the part of the crew to indicate their plight and position by any means whatever. Some time may have elapsed since a fix was obtained, which means that the position given is probably only approximate. When news is received by wireless that an aircraft is in distress in a certain position, either a plane or a high-speed launch will be dispatched to the spot to effect a rescue. Should survivors not be located immediately, a systematic search of the neighbourhood must be made in the event of the position being in error, or a change of position having taken place due to the effect of wind or current.

Having assessed the visibility in nautical miles, the method generally adopted for the search is to proceed first either upwind or downwind for a distance equal to twice the visibility, then turn on to a track 90° to the right or left for the same distance, and continue turning in the same direction on to a track 90° different after covering distances on each successive pair of tracks longer by twice the visibility than the preceding pair.

An illustration of this is given in the following example.

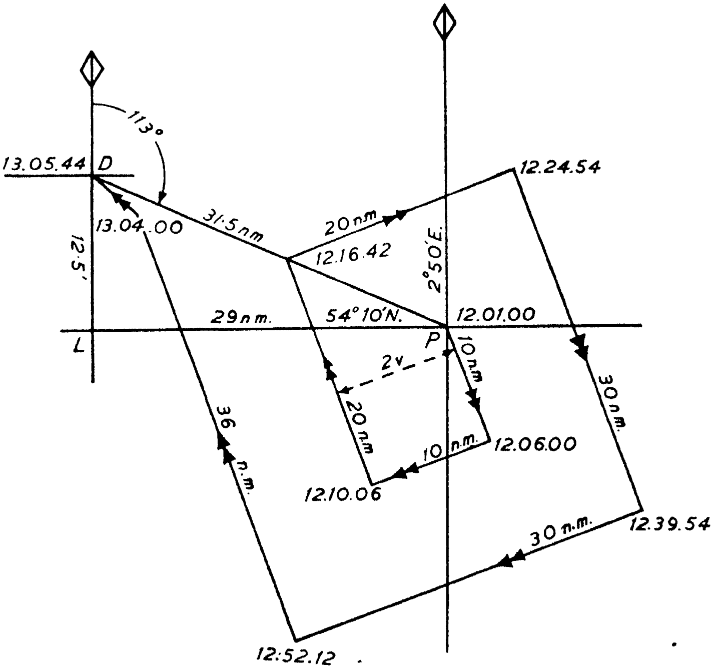


Fig. 103. The Fixed Square-search

Example. In response to an SOS sent out by an aircraft in distress, a rescue plane leaves its base and proceeds to the position given—*i.e.*, lat. 54° 10′ N. long. 2° 50′ E. where it arrives at 12.01 hrs. T.A.S. 150 knots, W/V 160°/30 knots.

No sign of aircraft or survivors is to be seen, so a search is decided upon, heading first up-wind and then right-handedly, visibility being 5 n.m.

At 13.04 hrs. a rubber dinghy with survivors is sighted on a relative bearing of 330° at maximum visibility. What would be its position relative to the reported position?

On turning to Fig. 103 you will see that on reaching the given position P, the aircraft turns up-wind for 10 n.m., the visibility being

5 n.m., then right for another 10 n.m., then right for 20 n.m., and then right for another 20 n.m. Because the perpendicular distance between any two tracks is never more than 10 n.m., it will be understood than an object of size situated between the tracks should be seen from one of them if the visibility remains the same. It may be more convenient to make the first track down-wind, this being advisable if it is thought that the boat looked for might have made leeway, although around our coasts there is tide to consider, too. For the first and every alternate course no allowance need be made for drift, as the course is either up-wind or down-wind, but on the other tracks an allowance must be made unless the air is calm, but it will be the same in amount though opposite in name. The G.S. on odd-numbered courses will be either the difference or sum of the T.A.S. and wind velocity, but on the even-numbered courses the G.S. will be the same because these vector triangles are congruent. To avoid overcrowding the figure the course and wind vectors have been omitted, but full answers are given below and the student is advised to verify these by plotting the triangles independently.

	1	2	3	4
Track .	160° (T.)	250° (T.)	340° (T.)	070° (T.)
Course .	160° (T.)	238° (T.)	340° (T.)	082° (T.)
G.S. .	120 knots	147 knots	180 knots	147 knots

It will be found convenient to calculate the time required on each track to cover a distance of twice the visibility, and take multiples of these for subsequent parallel tracks. This will simplify the procedure for finding when to turn, and so maintain a record of positions.

	First track	*Second track*	*Third track*	*Fourth track*
10 n.m. covered in .	5 minutes	4·1 minutes	3·3 minutes	4·1 minutes

The time-table of course alterations would be as follows:

		hrs. mins. secs.
Arrive reported position . . .	A/C 160° T.	12. 01. 00
10 n.m. at G.S. 120 knots 5·0 mins. .	A/C 238° T.	12. 06. 00
10 n.m. at G.S. 147 knots 4·1 mins. .	A/C 340° T.	12. 10. 06
20 n.m. at G.S. 180 knots 6·6 mins. .	A/C 082° T.	12. 16. 42
20 n.m. at G.S. 147 knots 8·2 mins. .	A/C 160° T.	12. 24. 54
30 n.m. at G.S. 120 knots 15·0 mins. .	A/C 238° T.	12. 39. 54
30 n.m. at G.S. 147 knots 12·3 mins. .	A/C 340° T.	12. 52. 12
	Sighted dinghy at	13. 04. 00
	5 n.m. at G.S. 173 knots	1. 44
	Arrive dinghy	13. 05. 44

The positions in Fig. 92 are all D.R. positions, so by joining the actual position D to the reported position P the bearing and distance may be obtained. The bearing from D is found to be 113° T. and the distance 31·5 n.m., so that the position of the dinghy relative to position P is 293° T. 31·5 n.m.

The latitude of D may be found by applying the ch. lat. LD, 12·5′, to the lat. of P, giving 54° 22·5′ N.

To find the longitude, the departure PL 29 n.m. would have to be converted into ch. long. as explained in Chapter VII. It is found to be 49′, making the longitude of D 2° 01′ E.

The same final position would be obtained by making an air plot, and the student should verify this as an exercise.

Exercise 10

An aircraft sets course from base to locate an object reported in a position bearing 080° T., distant 100 n.m. On arrival at the indicated position a square search was begun down-wind and continued right for 20 mins. when the object was located ahead, distant 4 n.m. What was its bearing and distance from the aircraft's base? Visibility 4 n.m. W/V 240°/30 knots, T.A.S. 150 knots.

General Exercises

1. From Cromer (52° 55′ N. 1° 14′ E.) you are to proceed on a track of 340° T. and return in 1 hour. T.A.S. 120 knots, W/V 280°/36 knots.

Find: course and G.S. out; course and G.S. back; R. of A.; T.T.T.; air position and D.R. position on turning.

2. From Margate (51° 20′ N. 1° 25′ E.) you are to proceed on a track of 035° T. and return in 3 hours. T.A.S. 120 knots, W/V 250°/30 knots.

Find: course and G.S. out; course and G.S. back; R. of A.; T.T.T.; air position and D.R. position on turning.

3. From 53° 15′ N. 0° 15′ E. proceed on a track of 080° T. and return in 3 hours to 54° 10′ N. 00° 30′ W. T.A.S. 120 knots, W/V 300°/36 knots.

Find: course and G.S. out; course and G.S. back; R. of A.; T.T.T.; air position and D.R. position on turning.

4. From 52° 40′ N. 1° 30′ E. proceed on a track of 060° T. and return in 2½ hours to an aircraft-carrier in a position 080° Flamborough Head 50 n.m. at time of leaving and steaming 320° T. at 20 knots. T.A.S. 120 knots, W/V 120°/40 knots.

Find: course and G.S. out; course and G.S. back; R. of A.; T.T.T.; air position and D.R. position on turning.

5. From an aircraft-carrier in 53° 30′ N. 1° 20′ E. steaming 070° T. at 20 knots, proceed on a track of 080° T. and return in 2 hours.

Find: course and G.S. out; course and G.S. back; R. of A.; and T.T.T. T.A.S. 120 knots, W/V 020°/25 knots.

6. At 10.00 hrs. you leave 52° 40′ N. 0° 20′ E. to intercept a vessel in a position 53° 55′ N. 6° 12′ E. steering 256° T. at 15 knots. T.A.S. 120 knots, W/V 334°/32 knots.

Find: course out; estimated time of interception (E.T.I.); and course back to base, supposing you accompany the vessel for 20 minutes.

7. From an aerodrome A set course at 10.10 hrs. to intercept a ship reported 10 minutes earlier to be in a position bearing 071° T., distant 138 n.m., and steaming 207° T. at 10 knots. T.A.S. 100 knots, W/V 135°/30 knots.

Give: true course out; G.S.; E.T.I.; and D.R. position of interception relative to A.

Supposing you accompany the vessel for 15 minutes, what would be the course back to A and your E.T.A.?

8. From an aerodrome A set course at 13.10 hrs. to intercept a ship bearing 074° T. distant 133 n.m., steering 210° T. at 12 knots. T.A.S. 120 knots, W/V 040°/10 knots.

Give: true course out; G.S.; E.T.I.

Owing to bad visibility you miss the ship, and at 14.20 hrs. pinpoint in a position 083° T. from A, distant 143 n.m.

Find the W/V experienced since leaving A, and readjust course accordingly to intercept the vessel, giving the speed of approach, new E.T.I., and position of interception.

After accompanying the vessel for 15 minutes, what would be the true course back to A and E.T.A.?

9. At 07.00 hrs. set course from A 51° 50′ N. 0° 30′ E. for Maas light-vessel (52° 02′ N. 3° 54′ E.) T.A.S. 150 knots, W/V 160°/36 knots.

07.25 hrs. pinpoint 52° 00′ N. 2° 04′ E. Find the W/V experienced, and readjust course accordingly.

07.40 hrs. alter course to intercept a ship reported in a position 53° 22′ N. 4° 35′ E. and steering 210° T. at 12 knots. Give the E.T.I.

After accompanying the vessel for 10 minutes set course for A and give the E.T.A. there.

10. Leaving Orfordness (52° 04′ N. 1° 34′ E.) at 20.00 hrs. a ship has to join another vessel which left Corton light-vessel (52° 31′ N. 1° 51′ E.) at 18.30 hrs. bound for Ijmuiden (52° 27′ N. 4° 35′ E.) at a speed of 8 knots. What course (C.) will the first ship have to steer at a speed of 12 knots to intercept the second, and when will the interception take place? No allowance need be made for effect of tide. Variation 10° W., deviation card No. 2.

11. A ship leaves Yarmouth (52° 34′ N. 1° 44′ E.) at 18.00 hrs. to intercept another vessel which left Sunk light-vessel (51° 51′ N. 1° 35′ E.) at 16.00 hrs. steering 050° T. at 10 knots. The tide sets south (M.) at 2 knots. Give the course (C.) the first vessel must steer in order to intercept the second, if the former's speed were 15 knots. Variation 10° W., deviation card No. 2. When would the interception occur?

12. Based at Mablethorpe (53° 20′ N. 0° 18′ E.) you receive at 10.00 hrs. information that a vessel in a position at 09.20 hrs. of 070° Flamborough Head 120 n.m., steaming 200° T. at 12 knots, requires

the services of a doctor. You set course 15 minutes later to intercept the vessel, allowing for a tide setting 150° T. at 3 knots, the speed of the launch being 30 knots. Give the compass course out, E.T.I., and position of interception as a bearing and distance from Spurn Head (53° 35′ N. 0° 07′ E.). Variation 10° W., deviation card No. 2.

13. Based at Yarmouth you receive a report at 08.10 hrs. of a small ship in distress in a position 080° Spurn Head 120 n.m. At 08.20 hrs. set course in a rescue-launch (speed 35 knots) to render assistance, counteracting a tide setting 150° T. at 3 knots. Give the E.T.A. On arrival at the given position no craft is sighted, so a fixed square-search is begun down-tide and thence right-handedly, the visibility being 4 n.m. At 13.50 hrs. the ship is sighted on a relative bearing of 040°, distant 4 n.m. Give its position in latitude and longitude. Twenty minutes later the rescue operation is complete. What is the true course back to base across a tide setting 360° T. at 3 knots? Give the E.T.A. at base.

14. At 10.00 hrs. you receive information that a ship had been sighted in distress an hour earlier in 53° 50′ N. 3° 30′ E. You set course in a launch immediately from 53° 34′ N. 0° 13′ E. to effect a rescue, counteracting a tide setting 150° T. at 2 knots. At what time would you expect to reach the given position, supposing your speed to be 36 knots? When the ship is not found as expected, a square-search is begun on a track of 150° T. and thence right-handedly, the visibility being 5 n.m. At 14.30 hrs. the craft is sighted at maximum visibility on a relative bearing of 320°. What is its position, and your course back from there, assuming the tide to be now 340° T. at 2 knots? Give E.T.A.

15. At 08.15 hrs. you set course from an aerodrome A to drop relief supplies to survivors on a life-raft reported on a bearing of 108° T., distant 70 n.m. T.A.S. 150 knots, W/V 250°/40 knots, visibility 5 n.m.

Give: true course out; G.S.; E.T.A.

On arrival at the given position no object is sighted, and a square search is begun down-wind and thence right-handedly.

At 09.12 hrs. you sight the raft right ahead at maximum visibility. What would you give as its position relative to your base, and what would be your course back from that position and E.T.A.?

16. In response to an SOS message from a craft giving as her position 53° 40′ N. 3° 10′ E., you set course at 07.44 hrs. from an aerodrome in 52° 50′ N. 1° 10′ E. to locate her exactly. T.A.S. 120 knots, W/V 200°/24 knots, visibility 4 n.m. At the same time a rescue-launch (speed 30 knots) leaves Yarmouth to co-operate. When no craft is sighted at the given position course is altered down-wind and then right-handedly in continuation of the search. At 09.45 hrs. survivors are sighted on a relative bearing of 020°, distant 4 n.m. What would you give as their bearing from the launch?

PART II

ASTRONOMICAL NAVIGATION

material away from the head. The earth has passed through the tail of a comet several times with no ill effect. Some comets are known to return to the sun at regular intervals—*e.g.*, Halley's comet, last seen in 1910, due again in 1986—but the majority appear once only. Whether they dissipate their substance in space can only be a matter for conjecture. Their paths are regarded as parabolic curves.

Meteors, commonly known as shooting stars, are pieces of solid material, usually iron or stone, which, during the course of their wanderings through space, come under the influence of the earth and are attracted to it. In their passage through the earth's atmosphere at terrific speed they become hot due to friction and burn, often emitting a bright light, and are thereby wholly or partially consumed. Those that reach the earth are known as meteorites. The majority, however, are very small and probably resolve into fumes and dust. The way in which fixed stars fit into the scheme is discussed in the next chapter.

COMPARISON OF CELESTIAL WITH CORRESPONDING TERRESTRIAL TERMS

Earth	*Celestial Sphere*
(concentric with and rotating eastward within celestial sphere)	*(fixed but apparently rotating westward about earth's extended axis)*
Poles (ends of axis).	Poles (ends of earth's axis produced indefinitely).
Equator.	Equinoctial.
Meridian.	Hour circle.
Prime meridian (through Greenwich).	Hour circle through First Point of Aries (♈).
Latitude.	Declination.
Longitude E. from Greenwich (0°—180°).	Right ascension E. from ♈ (0—24 hours).
Longitude W. from Greenwich (0°—180°).	Sidereal H.A. W. from ♈ (0°—360°).
Greenwich Hour Angle of Aries (G.H.A. ♈) *(variable link between rotating earth and fixed sky)*	
Observer.	Zenith.
Antipodal point.	Nadir.
Great-circle bearing (000°—360°).	Azimuth N. E. 180° S. W.

General Exercises

1. Which are the superior and inferior planets?
2. What are minor planets?
3. Name the navigational planets, and describe the movements of one of them.

4. Draw a figure to show when Venus is a morning star and when an evening star.
5. Explain with a figure what is meant by the Zodiac.
6. What do you understand by the terms, celestial latitude and celestial longitude?
7. What circumstances produce a total eclipse of the sun?
8. When does an annular eclipse of the sun occur?
9. Describe the causes of a partial eclipse of the moon.
10. Define the term gibbous.
11. Give an account of the phases of the moon.
12. State Kepler's three laws of planetary motion.
13. Explain why the earth does not move along its orbit at a constant speed.
14. What are the characteristics of an ellipse?
15. What is meant by Precession of the Equinoxes?
16. Calculate the orbital speed of Mars.
17. Explain what is meant by universal gravitation.
18. Give definitions for Declination and Right Ascension.
19. What are the Ecliptic and the First Point of Aries?
20. Explain the terms aphelion, perihelion, apogee, and perigee.
21. What is the difference between a comet and a meteor?
22. Explain why α Draconis was once used as the Pole Star.
23. Describe the relationship between sun, moon, and earth.

CHAPTER XI

THE CELESTIAL SPHERE

Our universal dome—How stars are named—Star recognition—Northern stars—Equatorial stars—Winter stars—Spring stars—Summer stars—Autumn stars—Southern stars—The Milky Way—Stellar magnitudes—The daily round—Circumpolar stars—Twilight.

Our Universal Dome. Imagine yourself sitting by the camp-fire one evening in August waiting and watching, waiting for your comrades who have gone in search of fuel, and watching the sun, that much bigger fire, slip slowly and silently like a noiseless golden ball behind the western horizon. Nothing is now left but the clear sky, the dome which was blue but whose colour is now beginning to deepen. Perhaps you wonder how far above you it really is; you know that it returns no echo to your call, that a Spitfire with a ceiling of 35,000 feet never even approaches it. You have heard that the atmosphere extends to a height of 50 miles, and in the last chapter you were told that the sun you have just seen setting has an average distance of 93 million miles from us, without being fixed to the dome.

What then is the limit? Will the stars which are just beginning to peep through supply the answer? Let us examine them as they appear. The first one to show up is nearly overhead, the next is about 15° to the south of west and about 35° vertically above the horizon. Another becomes visible a little to the east of north, quite low down, while a fourth soon follows, about 30° to the east of south and about 40° above the horizon. In fact as the darkness deepens it is difficult to keep count of each new arrival without feeling overwhelmed by the number of pinpoint lights which continue to crowd in.

And so, as we pursue our quest and as each fresh star appears, the story of the heavens unfolds and new limits in distance are reached, but even when we have probed the sky with the most powerful telescope and have examined the Milky Way and the nebulæ, we fail to pierce the veil and wrest Nature's secret from her bosom. There still remains a beyond whose unfathomed depths are as yet a mystery but can never be completely solved, for the bounds of heaven are infinity. As we thus gaze into space, so we look back into time. The light reaching us from the nearest star left that orb more than four years ago, and were we privileged to enlarge our view of more distant stars we might see ancient history in the making, not, of course, on the star itself, but on one of its possibly many attendant satellites warming themselves around their magnificent camp-fire that never seems to need replenishing.

How the Stars are Named. Owing to the vast distances of the stars from the earth their positions relative to each other do not appear to change, and so shapes can be made by joining them together or by drawing lines round the stars forming particular groups.

From very early times names were given to groups of stars because they were assumed to resemble beings or things associated with everyday life—*e.g.*, goat, lion, bear, water-bearer, or events in history or mythology. In all probability this is the way in which stories were handed down at a time when the art of writing was out of the reach of the great majority. Nomadic tribesmen would sit outside their tents as darkness fell and recount their stories written indelibly in the volume of the sky. For them the hours unrolled the scroll of history, and it is thus that the Flood may have become identified with Eridanus and Noah's ark with Argo, while Columba, the dove, flew nightly across the sky, as Cetus, the whale, sounded the depths of heaven.

Little, if any, resemblance to actual things is seen in the groups to-day, but the names still persist. We saw in the last chapter how twelve of them were fitted into the belt known as the Zodiac, and there are many others outside its limits.

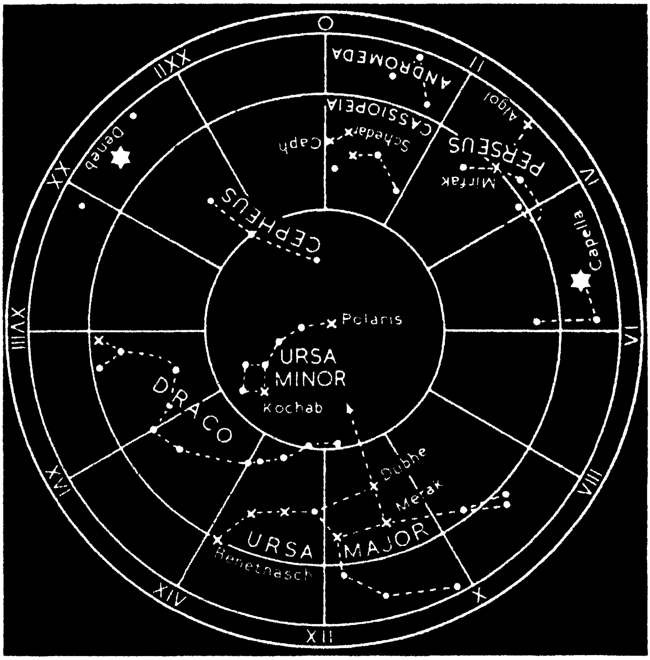
The names of the groups or constellations are invariably Latin—*e.g.*, Canis Major (great dog), Leo (lion), and Cetus (whale), but individual stars within the constellations are known by a letter from the Greek alphabet, and named from α (Alpha) onwards according to their importance in the constellation. The name of the constellation must then be put in the genitive case. We thus refer to the North (or Pole) Star as α Ursæ Minoris—*i.e.*, the principal star in the constellation of Ursa Minor (the lesser bear). The brightest stars usually have a personal or proper name in addition to the family, or group, name. Many such still retain their Arabic form—*e.g.*, Achernar, Aldebaran, and Dubhe, while others have acquired Latin names—*e.g.*, Capella, Regulus, and Sirius.

In recent times the fifty most useful navigational stars have been given numbers to facilitate rapid reference. Achernar is numbered 1 and Capella 10, as may be seen in the extract from the *Air Almanac* (p. 388).

Star Recognition. Until something more than a passing glance is cast in the direction of the night sky, the general arrangement of the stars will appear to be not a little chaotic, as if the hand of some giant sower had scattered indiscriminately a whole mountain of sparkling crystals, so far away that any observer on the earth appears to be at their centre. For purposes of astronomical navigation, except when using the moon, the earth may, in fact, be treated as a point in space, so that the positions of observer and earth's centre do coincide, the radius of the earth being comparatively negligible.

The best way of recognizing the stars is by watching them in the sky and noting their positions relative to other stars or groups. Reference must be made to star charts from time to time, in order to know which fresh stars should be looked for and what names should be given to those already known by sight. A word of guidance is necessary here. Star charts are generally constructed so that they give an approximately correct appearance of the sky when held reversed overhead. When looking down on the chart the westerly stars will appear on the right and the easterly stars on the left.

The Northern Stars. If, when in middle northern latitudes, you face north, the sun having set on your left, you can scarcely miss



Star Chart 1. Stars Visible all Night in Latitude 50° N.

noticing a prominent group of seven stars, shaped like a butcher's cleaver but known in classical times as Ursa Major (the great bear)—since when, however, it has been frequently described as the Plough or the Big Dipper. These stars are not among the twenty brightest in the sky, being only of second magnitude, but at least two of them, Dubhe and Benetnasch, are regarded as useful for navigational purposes, and will be found readily on the Star Chart No. 1.

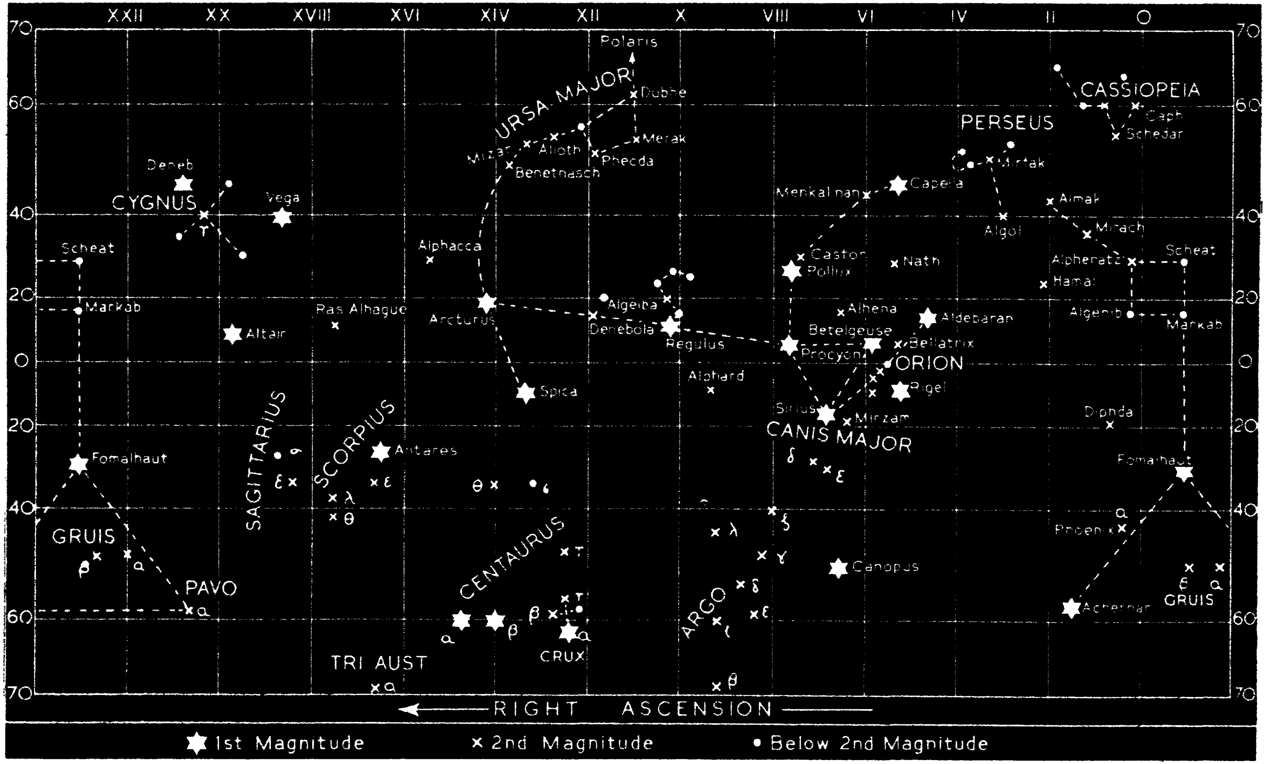
Dubhe, and its partner Merak, are known as the pointers because, as indicated in the chart, a line drawn from Merak through Dubhe points out the position of **Polaris,** the Pole Star, situated at an angular distance of only 61′ from the North Pole of the sky, and hence often referred to as the North Star. The usefulness of such a star for directional purposes needs no emphasis when it is known that in these latitudes its direction from true North never differs by more than about 1½°. Besides having this and other navigational values, Polaris, being of second magnitude, is the principal star in the constellation of Ursa Minor (the lesser bear), occupying a position in it similar to that occupied by Benetnasch in Ursa Major.

On the opposite side of Polaris from Ursa Major, and at about the same distance, we find a constellation in the shape of sprawling M or, viewed the other way, a big W. This is Cassiopeia, containing stars Schedar and Caph of secondary navigational value. Beyond Cassiopeia we come to the constellation of Andromeda, which, however, passes south of an observer in latitude 50° N. and is discussed later, while to the east and slightly south of Cassiopeia we see a constellation in the shape of a bow concave to the Pole. This is Perseus containing the two useful stars Mirfak and Algol, the latter being imagined at the tip of an arrow. Between Cassiopeia and Polaris is situated the less prominent constellation of Cepheus. In Greek mythology Cepheus and Cassiopeia were the King and Queen of Ethiopia and parents of Andromeda. Cassiopeia offended the sea god Neptune, who demanded as retribution that Andromeda be delivered to a great sea monster. While awaiting her fate, chained to a rock, Andromeda was rescued by the gallant Perseus who was returning from his victory over the fabled Gorgon.

Draco (the dragon), mid-way between Ursa Major and Ursa Minor, is the only other constellation we need mention here. It consists of a long, loose line of stars, the brightest of which is a second magnitude star situated at the head of the monster, the tail being not far from Dubhe the more northerly of the pointers. A noteworthy member of this constellation is the star Thuban, which, some 4700 years ago, was the Pole Star when Polaris was much farther away from the celestial pole than it is now.

Equatorial Stars. Stars which pass across the sky to the southward of middle north latitudes will have low declinations—*i.e.*, will not be far from the equator—and are therefore referred to as equatorial stars. They are given on Star Chart 2. As different groups of this collection become prominent at different times of the year, they will be divided into four sections according to the season of their longest night appearance.

Winter Stars. If, in the winter-time, you stand with Polaris behind you—*i.e.*, facing south—you would be confronted with a constellation of great distinction. This is named Orion, after the mighty warrior. Three second magnitude stars close together in a straight line mark the position of his belt, while the variable reddish coloured first magnitude star **Betelgeuse** indicates the position of his right shoulder.



Star Chart 2. Navigational Stars on Mercator's Projection

The second magnitude star, Bellatrix, marks the position of his left shoulder. A straight line drawn from Betelgeuse through the centre of the belt leads to the blue-white first magnitude star **Rigel,** while from Bellatrix a straight line through the centre of the belt leads to the second magnitude star κ Orionis.

Not only are Betelgeuse and Rigel of great navigational importance but the whole constellation stands out as a signpost. Continuing Orion's belt to the south-eastward we come to **Sirius** the sparkler, and brightest star in the sky.

Sirius is in the constellation of Canis Major (the great dog), and is therefore sometimes referred to as the Dog Star. Continuing the belt in the opposite direction, a straight line passes close to the reddish coloured first magnitude star **Aldebaran** in the constellation of Taurus (the bull). A line from the centre of the belt passing mid-way between Betelgeuse and Bellatrix will go north through Nath to the yellow first magnitude star **Capella,** which may also be identified on a dark clear night by the proximity of a tall isosceles triangle. A straight line from Rigel through the centre of the belt to Betelgeuse will pass close to the heavenly twins, Castor of second magnitude and **Pollux** of first magnitude. An arc with centre near Aldebaran will pass from Sirius through Procyon, Pollux, Castor, and Menkalinan to Capella. **Procyon,** of first magnitude, is the brightest star in the constellation of Canis Minor (the lesser dog). The three bright stars Sirius, Betelgeuse, and Procyon when joined by straight lines form an almost perfect equilateral triangle. Alhena lies about half-way between Betelgeuse and Pollux.

Lastly a straight line from Betelgeuse to Procyon will, when produced, pass close to the bright star **Regulus** in the constellation of Leo (the lion), sometimes called the Sickle. This constellation looks like a question-mark facing the wrong way, with Regulus as the full stop. Regulus is almost exactly on the ecliptic.

Spring Stars. Prominent stars in the spring night sky are far less numerous than those of winter. The most notable is **Arcturus,** the principal star in the constellation of Boötes (the herdsman). It may be located by extending the line joining Procyon to Regulus, and also by continuing the curve of the Great Bear's 'tail.' It is a first magnitude star with a diameter twenty-seven times that of our sun, and its distance from us is 38 light years. Light travels at the rate of 186,000 miles per second. If the curve of the Great Bear's tail is continued beyond Arcturus it leads to a bright white star named **Spica,** a first magnitude star, distant 181 light years. If it were placed at the same distance as our sun, it would give us 870 times more light. Spica may also be identified by the proximity of four stars belonging to the constellation of Corvus (the crow)—known to sailors as 'the cutter's mainsail,' whose shape they represent.

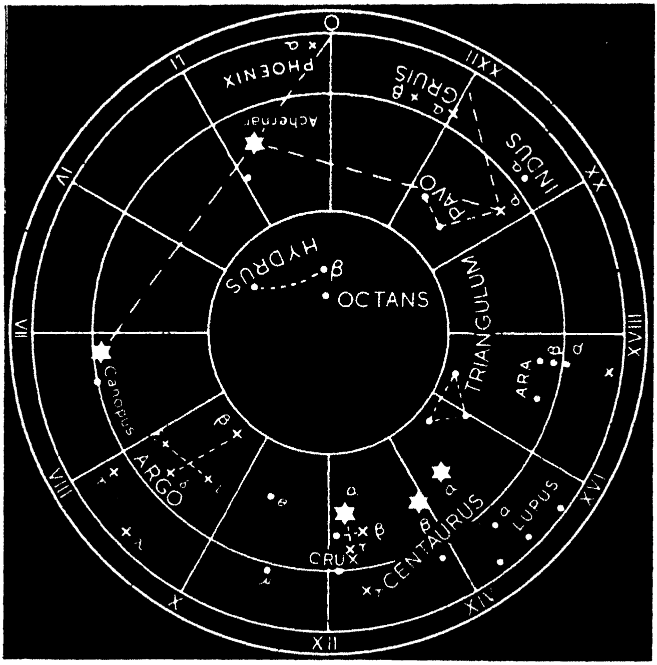
Summer Stars. A constellation, prominent at this time of the year is Cygnus (the swan), often called the Northern Cross, because five of its stars may be linked together to form a tolerably recognizable cross. It may even be imagined to be a boy's kite. The star at the head of the cross is **Deneb,** of first magnitude, and one of the remotest of the bright stars. Its distance is estimated to be probably more than 1600 light years. A brighter star, however, is that of **Vega,** to the westward in the constellation of Lyra (the lyre). It is the brightest star north of the equinoctial, and distant 23 light years from us. To the southward of the Northern Cross lies **Altair,** in the constellation of Aquila (the eagle). It is one of the nearest first magnitude stars, but comparatively small in bulk, being no larger than our sun.

Two stars of lesser magnitude are situated one on either side of Altair and equidistant from it. The three being thus disposed in a straight line become easily recognizable. Well to the southwest, and almost in line with the axis of the Northern Cross, you will find the first magnitude star **Antares,** in the constellation of Scorpius (the scorpion). It also is disposed between two smaller stars, but the line joining them is not quite straight. Antares is the largest known star, having a radius greater than that of the orbit of Mars. It has a distinctly ruddy tint, indicative of its low temperature when compared with that of most other visible stars, including our sun.

Autumn Stars. The stars in the autumn night sky are not of outstanding brilliance, but with Altair sinking in the west and Aldebaran rising in the east they form a connexion between the brighter stars of summer and winter. High up to the southward may be seen four stars disposed in a large square, known as the Square of Pegasus, the winged horse on which Perseus rode out to destroy the sea monster that threatened Andromeda. Of the four stars, Alpheratz at the north-east corner is of second magnitude, while the others are somewhat less bright. Alpheratz is often assigned also to the constellation of Andromeda, whose other second magnitude stars Mirach and Almach lead north-eastward in an almost straight line from Alpheratz to Mirfak, in Perseus. Alpheratz, owing to its small Right Ascension of 00 hrs. 05 mins. 20 secs., lies almost on the hour circle of the First Point of Aries. A straight line joining Algenib, the other eastern star of the Square, to Alpheratz runs north, and when produced leads to Polaris, thus providing an alternative means of locating the Pole Star. A straight line joining the two western stars Scheat and Markab, when produced southward, leads to the first magnitude star **Fomalhaut** in the constellation of Piscis Australis (the Southern Fish), thereby forming a useful link between northern and southern constellations.

Southern Stars. First magnitude stars with a southerly declination greater than those of Antares or Formalhaut are not visible to an observer in middle northern latitudes. But, although you may not learn their dispositions at first-hand until you travel south, you will find the knowledge concerning them obtained from Star Charts 2 and 3, together with the following guidance, very useful whenever you are called upon to navigate in more southerly latitudes.

Fomalhaut, as seen from Star Chart 2, forms with two other stars a large equilateral triangle. Of the other two stars, **Achernar** of first magnitude belongs to the constellation of Eridanus (the river Eridanus), while Peacock (α Pavonis) is of second magnitude, but very useful owing to the dearth of brighter stars in this part of the sky. The Fomalhaut-Achernar side of the triangle when produced leads to **Canopus,** a first magnitude star in the constellation of Argo (the ship Argo).



Star Chart 3. Stars never Visible in Latitude 50° N.

On the opposite side of the pole from Achernar, and at about the same distance away from it, is situated a group of four stars in the form of a cross. This is Crux, the Southern Cross. Only α, or **Acrux** as it is sometimes called, the southernmost of the four, is of first magnitude, but owing to the compactness of the group it is quite distinctive, and has been chosen as the emblem of Australia. A little to the eastward of the cross stands Centaurus (the centaur), whose two brightest stars α and β outshine Acrux.

α Centauri, sometimes known as **Rigel Kent,** is the nearest known star to the earth above magnitude 11.0, being only four light years away. A little to the east and south of Centaurus lies Triangulum Australis (the southern triangle).

About mid-way between the Southern Cross and Canopus lies a group of stars often referred to as the False Cross, but actually forming part of the constellation of Argo.

The Milky Way. On any dark clear night you will observe in the sky an irregular band of faint illumination stretching across the sky. In the early spring at evening it passes almost overhead in our latitude, extending from the south-western to the north-eastern horizon. As with the known stars, its position naturally varies according to the season and the time of night. It passes within 30° of the north celestial pole, and through the constellations of Perseus, Cassiopeia, Cepheus, Cygnus, Aquila, over the equinoctial, which it crosses at an angle of about 60°, into the southern hemisphere, and so through the constellations of Sagittarius, Scorpius, Centaurus, Crux, Argo, Canis Major, and, back in the northern hemisphere, through Gemini and Auriga. It is brightest when near the south celestial pole.

When examined with a powerful telescope the Milky Way is seen to consist of myriads of stars of many sizes, whose distances must be truly enormous.

Stellar Magnitudes. The importance of a star for navigational purposes depends largely upon its brightness. As we all know, stars differ greatly in their degree of brightness—*i.e.*, they differ in the amount of light we receive from them—and in order that we may compare their usefulness they are arranged in categories according to the amount of illumination they give us, although two stars are hardly ever alike in this respect. These categories are known as magnitudes, but mass does not enter directly into their calculation, as the effect of a star's size is limited by those of its constitution and its distance from us.

The Greek astronomer Hipparchus, second century B.C., who could have known little, if anything, about the diameter, constitution, or distance of stars, instituted the six magnitudes into which visible stars were divided. The brightest stars were regarded as being of the **first** magnitude, while those which could just be seen by the naked eye under the best night conditions were of the **sixth** magnitude. The remainder were arranged into **second, third, fourth,** and **fifth** magnitudes.

With the improvement in methods of observation and measurement a certain amount of rearrangement among the stars became necessary, but the same number of magnitudes has been retained. A type of photometer is now used to compare the brightness of the stars. This instrument shows that the average bright star gives about 100 times as much light as a star just visible to the naked eye on a dark clear night. As these two types are selected to represent the first and sixth magnitudes respectively, the others can be arranged in their intermediate categories by placing in the fifth magnitude those stars giving 2½ times as much light as those of the sixth magnitude, and in the fourth magnitude those giving 2½ times as much light as those of the fifth magnitude, and so on. Stars marked 1·0, 2·0, 3·0, etc., have the average brightness of their respective magnitude, so that stars marked up to 1·5 are regarded as being of the first magnitude, those from 1·5 to 2·5 of the second magnitude, and so on.

The Daily Round. We are all familiar with the apparent movement of the sun, which after rising in the east (Latin, *orient*, rising), climbs up and across the sky until having reached its highest point at midday, when on the observer's meridian, it declines towards the west and finally reaches the horizon and sets (Latin, *occident*, falling).

The direction of the sun at rising or setting to a stationary observer depends upon the time of the year because the sun's declination changes continually. As a star's declination may be regarded as practically constant throughout the year, the direction of a star when rising or setting is always the same. No two navigational stars have exactly the same declination however, so that none of them rises or sets in the same direction as another. They all appear to revolve round the earth, but, owing to their different declinations and consequent different angular distances from the celestial pole, they all appear to be swinging round it at the end of ropes of differing length, as if the pole were the head of a maypole at whose base were placed the earth. The comparison can apply, of course, to only one hemisphere at a time. If the observer were at the earth's pole—*i.e.*, directly under the head of the maypole—he would see the stars going round him in circles at the same level. As the observer receded from the earth's pole, the celestial pole would cease to be directly over his head but the stars would still trace out concentric circles for him about the celestial pole. These circles would now become lower on the Polar side of the observer until eventually the lowest of them, that farthest from the pole, would dip down behind the curved surface of the earth. Thus, as the observer retreated from the pole, more and more arcs would become hidden, but others belonging to the other hemisphere would come into view.

A picture of this cavalcade, the silent march past of the principal stars as we review them in latitude 50° N. on an August evening, may be obtained from a study of Fig. 113, which illustrates the appearance of the eastern half of the sky as darkness is falling. The numbers marking the stars are those used in the *Air Almanac* where possible.

We see that number 10, Capella, is low down, bearing a little to the east of north. It has just crossed the observer's meridian from west to east below the elevated celestial pole P without dipping down below the horizon, and is, therefore, said to be circumpolar. The path it will follow is indicated by its circle of declination. Its altitude above the horizon will increase slowly at first, but more rapidly, as, following number 11, Deneb, it approaches the observer's meridian, which it crosses this time above the pole, its upper transit, and south of the observer's zenith Z. Dubhe has not yet crossed the observer's meridian below the pole and so does not appear in the eastern sky, but as the night proceeds it will transit at D and follow the indicated path round the pole until it crosses the observer's meridian above the pole at T twelve hours later. Star 22, Vega, is just about to cross the meridian south of the observer when the sun has set about August 20. When it crosses the meridian north of the observer—*i.e.*, at its lower transit—it will be just on the horizon at N if the observer's latitude is 52° N.

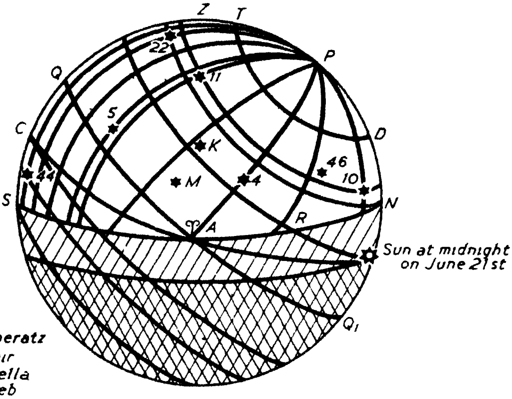


Fig. 113. THE STARS IN THEIR COURSES

The remainder of the stars shown in Fig. 113 cannot be circumpolar to an observer in latitude 52° N. Number 4 (Alpheratz), for example, will rise at a point R and, following a path parallel to those of the stars already mentioned, cross the observer's meridian some nine hours later, staying above the horizon altogether for a matter of eighteen hours. Star 5 (Altair) will rise at a point A and cross the meridian—*i.e.*, reach its culminating point—about seven hours later, remaining above the horizon for approximately fourteen hours. ♈, the First Point of Aries, is the spot on the equinoctial QQ_1 at which the sun, moving along the ecliptic from the direction of C, crosses about March 21.

Being a point on the equinoctial, ♈ rises due east, transits six hours later, and sets due west after being above the horizon for twelve hours. The star δ Orionis, in the belt of Orion, declination 0° 21′ S., would likewise rise due east and set due west, dividing the day into two equal parts. Stars with a declination farther south, however, would be above the horizon for less than twelve hours, the length of time decreasing as the declination increased, so that number 44 would be above the horizon for no more than about four hours. Stars with a southerly declination greater than QS would always be below the horizon to an observer at Z—*i.e.*, when latitude of observer plus declination of body in the opposite hemisphere exceeds 90° the star is never visible.

The general appearance of the sky at 20.00 hrs. on August 20 is substantially the same as at midnight on June 21 when the sun reaches its maximum declination of 23° 27′ N. as shown in Fig. 113. The reason for this apparent acceleration is fully explained in the chapter on Time.

Circumpolar Stars. If the observer's latitude is less than 52° N. Vega will transit below N, but if greater than 52° N. above N. What then causes a celestial body to be circumpolar—*i.e.*, above the observer's horizon throughout the whole twenty-four hours? When a celestial body is just on the horizon at N its angular distance from the observer's zenith is 90°. To be circumpolar, then, to an observer, it must be not more than 90° from his zenith when it crosses his meridian below the pole—*i.e.*, the angular distance of the observer from the pole plus the angular distance of the celestial body from the pole must be less than 90°, or the angular distance of the observer from the terrestrial equator plus the angular distance of the body from the celestial equator must be more than 90°. This, expressed succinctly, means, observer's latitude + declination of body must exceed 90°.

Twilight. The position of the sun at its lower transit in Fig. 113 is seen to be several degrees below the horizon. If the latitude of the observer is assumed to be 52° N. then $QZ = 52°$ and $PZ = 38°$. Now, $ZN = 90°$, therefore $PN = 90° - 38°$—*i.e.*, 52°, so that the elevation of the celestial pole above the horizon is seen to equal the latitude in angular measure. Further, $PQ = 90°$ and $NQ_1 = PQ_1 - PN$, *i.e.* $= 38°$. Q_1✡ $= 23° 27'$ at the solstice, so that N✡, the depression of the sun below the horizon, will be $38° - 23° 27'$ or $14° 33'$. Now, as long as the sun is not more than 18° vertically below the horizon, astronomical twilight is said to exist, so that to an observer in latitude 52° N., astronomical night does not occur either on or about June 21. His day consists mostly of daylight, but partly of twilight. Nautical twilight is assumed to exist when the sun is not more than 12° below the horizon, and civil twilight when the sun is not more than 6° below the horizon. In these two cases night would exist for a small part of the twenty-four hours. When the observer increases his latitude he approaches the pole—*i.e.*, PZ (Fig. 113) becomes less, and, as N must move with Z, N✲ becomes less and finally vanishes when the sun appears on the horizon at midnight. There is then neither night nor twilight when P✲ is equal to or less than ZQ, or, in other words, when the observer's angular distance from the pole is equal to or less than the sun's declination. The same conditions, thus, now apply to the sun as we saw earlier applied to circumpolar stars.

STARS OF THE FIRST MAGNITUDE
ARRANGED IN ORDER OF BRIGHTNESS

Name of Star		*Magnitude*	*Distance in Light Years*
Sirius	α Canis Majoris	– 1·58	8
Canopus	α Carinæ	– 0·86	650
Rigel Kent	α Centauri	0·06	4
Vega	α Lyræ	0·14	23
Capella	α Aurigæ	0·21	43
Arcturus	α Boötes	0·24	38
Rigel	β Orionis	0·34	543
Procyon	α Canis Minoris	0·48	10
Achernar	α Eridani	0·60	71
	β Centauri	0·86	125
Altair	α Aquilæ	0·89	18
Betelguese	α Orionis	0·92	272
Acrux	α Crucis	1·05	210
Aldebaran	α Tauri	1·06	54
Pollux	β Geminorum	1·21	32
Spica	α Virginis	1·21	181
Antares	α Scorpii	1·22	172
Fomalhaut	α Piscis Australis	1·29	28
Deneb	α Cygni	1·33	1630
Regulus	α Leonis	1·34	71

General Exercises

1. What is meant by the term magnitude of a star?

2. How are stars named?

3. Name two pairs of pointers for locating the Pole Star.

4. What are the uses of star charts?

5. Name ten stars of navigational importance, and give reasons for choosing them.

6. How would you identify Arcturus, Fomalhaut, Capella, Achernar, Pollux, Regulus, Betelgeuse, Deneb?

7. Draw a diagram to illustrate the conditions under which a star is (*a*) circumpolar, (*b*) continually below the horizon.

8. Why does the period of daylight vary in length? When is the sun above and below the horizon for the same length of time?

9. What is meant by twilight? State the difference between astronomical, nautical, and civil twilight. Which controls lighting-up time?

10. How would a knowledge of stars, other than Polaris, help you to find your way?

CHAPTER XII

ASTRONOMICAL CO-ORDINATES

A simple theodolite—Personal co-ordinates—Azimuth—Altitude—Zenith—Prime vertical—Zenith distance—Celestial co-ordinates—Declination—Right ascension—Sidereal hour angle—Equidistant projection for the graphical solution of astronomical problems—How to use the personal co-ordinates for determining the celestial co-ordinates and so identifying a star—The values of 'R' for the first day of each month—How to locate a star by means of co-ordinates—Astronomical navigation tables—Mixed co-ordinates—Local hour angle—How to find the declination and altitude when hour angle and azimuth are known—How to find declination and azimuth when hour angle and altitude are known—How to find altitude and hour angle when declination and azimuth are known—True amplitude—How to find azimuth and hour angle when declination and altitude are known—How to find the latitude by means of co-ordinates—The stereographic projection.

It is one thing to identify a star when the night is dark and clear and all the 'leading lights' are visible, but quite another matter at twilight, the time when star altitudes at sea are usually taken because of the necessity for using the natural horizon. The paucity of visible stars at that time, however, enables approximate angles to be measured, which will lead to the solution if sufficient care is taken.

A Simple Theodolite. Provide yourself with some instrument for measuring horizontal and vertical angles. A home-made theodolite,

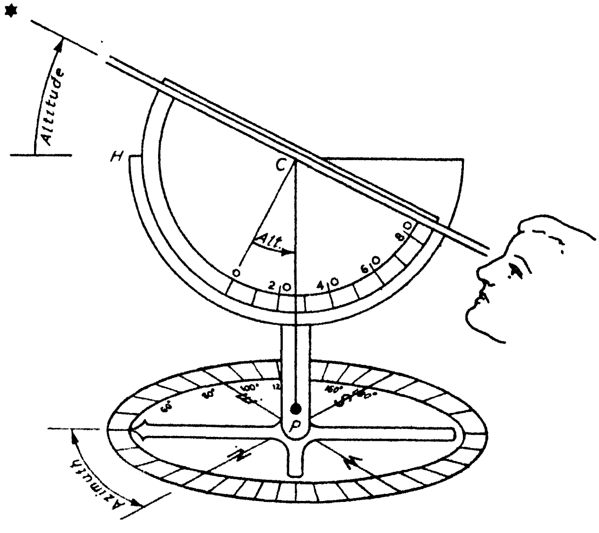


Fig. 114. A Theodolite for the Measurement of Personal Co-ordinates

similar to that illustrated by Fig. 114, will serve the purpose very well. All that is needed for its construction is a disc of wood about a foot in diameter around whose circumference degrees can be marked. Pivoted vertically in the centre of the disc is a rod of wood or metal, attached to the upper end of which is an inverted rotatable protractor fitted with a tube along its straight edge for sighting purposes. A plumb-line suspended from the centre of the protractor will indicate at the circumference the angle of elevation of the tube, and, therefore, the altitude of a celestial body in line with the tube. A pointer on the disc in the same vertical plane as the tube will indicate the horizontal angle between north and the direction of the body, provided the disc has been already orientated by turning it round until north on the disc marks the direction of the Pole Star.

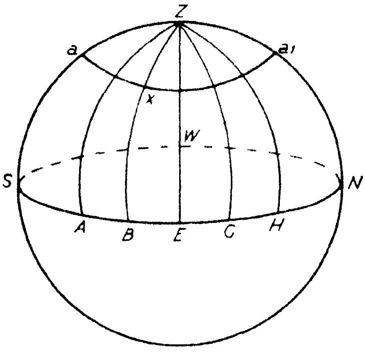


Fig. 115. Personal Co-ordinates

Personal Co-ordinates. The horizontal angle at the observer, between the vertical plane passing through the elevated pole and the vertical plane passing through the celestial object, is called the object's **azimuth.** It may be regarded as the first personal angular connexion between observer and object, or the first personal co-ordinate. Its value in Fig. 114 is seen to be N. 40° E. (040°). The vertical angle at the observer between the horizontal plane and the object is known as the object's **altitude,** and may be regarded as the second personal angular connexion between observer and object, or the second personal co-ordinate. In Fig. 114 this is indicated by angle HC✱, which is the angle of elevation of the sighting-tube measured by the number of degrees through which the zero-mark on the protractor has moved since being coincident with the plumb-line. This is seen to be about 27°.

$$\begin{array}{rcl} \ast CO = 90^\circ & = & PCH \\ \text{Subtract } HCO & & HCO \\ \hline \text{Then } HC\ast & = & OCP \\ \hline \end{array}$$

The altitude so obtained can be only approximate, but when the stars are widely spaced it should be accurate enough to combine with the azimuth for the purpose of ascertaining the celestial co-ordinates.

Fig. 115 illustrates how the personal co-ordinates may be represented on a sphere. NZS represents the observer's celestial meridian, Z his **zenith,** the point vertically overhead, and N and S the north and south points respectively of his celestial horizon which is represented by NESW. The arcs ZH, ZG, ZE, ZB, and ZA are 90° arcs of vertical great circles passing through Z, and, therefore, secondaries to the circle of the horizon. WZE, the vertical circle passing through the observer's east and west points, is known as the observer's **prime vertical.** To an observer north of the equator the azimuth of the star X, the first personal co-ordinate, is represented by the arc of the horizon NHGEB or its corresponding angle NZX at Z between the observer's meridian and the vertical circle passing through the star. To an observer in the southern hemisphere the azimuth would be represented by the arc SAB. The vertical arc BX represents the star's altitude, the second personal co-ordinate, and ZX (90° − BX) is, therefore, the angular distance of the star from the observer, and is known as the star's **zenith distance.** The arc axa_1 is an arc of a small circle parallel to the horizon, and therefore part of a circle of equal altitude.

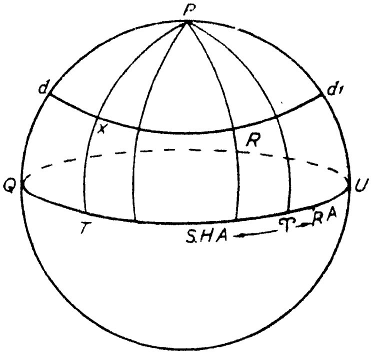


Fig. 116. CELESTIAL CO-ORDINATES

Celestial Co-ordinates. The celestial co-ordinates are **declination**—angular distance measured north or south from the celestial equator—and **Right Ascension** (R.A.)—angular distance measured east from the First Point of Aries up to 24 hours, as given in the *Nautical Almanac*—or its counterpart, **Sidereal Hour Angle** (S.H.A.)—angular distance measured west from the First Point of Aries up to 360°, as given in the *Air Almanac.*

Fig. 116 shows how the celestial co-ordinates may be represented on a sphere. The point P represents the north celestial pole and QTUR the **equinoctial**—the celestial great circle in the same plane as the earth's equator. ♈ represents the First Point of Aries, the point where the sun crosses the equinoctial in passing from south to north, and P♈ half of the celestial prime meridian. Right ascension (R.A.) is angular distance measured in time eastward from ♈, and sidereal hour angle (S.H.A.) is angular distance measured in degrees westward from ♈. One celestial co-ordinate of star X may be represented accordingly by either ♈T in terms of S.H.A. or ♈URQT in terms of R.A. The other celestial co-ordinate is then the star's declination—angular distance measured northward or southward from the equinoctial—and for star X is represented in Fig. 116 by TX. The arc dxd_1 represents half the star's circle of declination, a small circle parallel to the equinoctial and corresponding to a circle of latitude on the earth. PX, the **polar distance** (90° − TX), is the angular distance of the star from the pole.

The Equidistant Projection for the Graphical Solution of Astronomical Problems. The position of a celestial body may be referred to in terms of either pair of co-ordinates, and should one pair be known the other may be found—either graphically by combining the two sets of co-ordinates in one figure (an approximate method), or

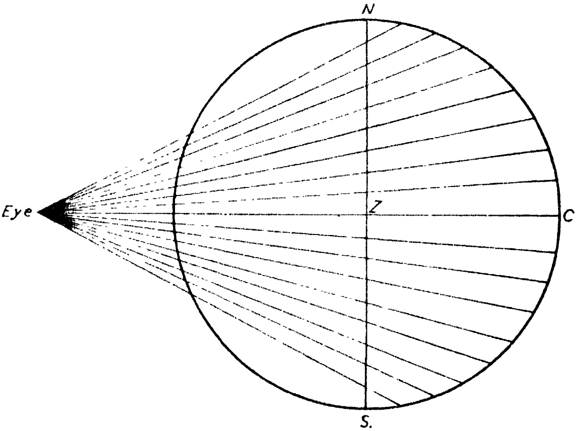


Fig. 117. Principle of the Equidistant Projection

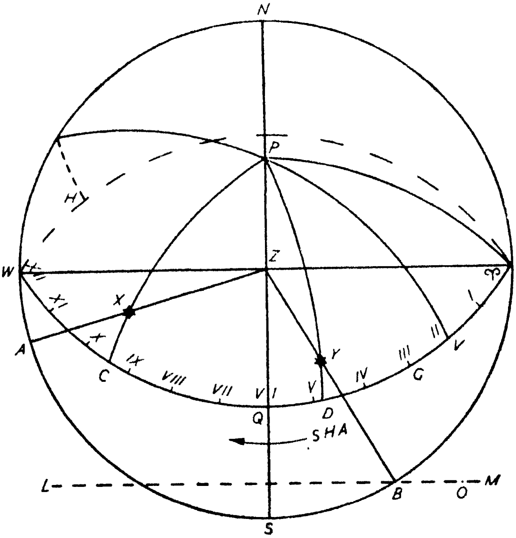


Fig. 118. What Star is it?

by calculation from a spherical triangle comprising all values, the accurate method.

We now proceed to illustrate the graphical method by means of a Figure drawn on the equidistant projection, the principle of which is illustrated in Fig. 117, where the point of projection is shown to be at a distance of $\frac{1}{\sqrt{2}}$ times the radius outside the circumference. The 10° divisions spaced along the arc NCS are projected as equally spaced divisions on to the primitive NZS, an important characteristic when simplicity of construction is desired. When NZS represents a meridian it may, therefore, be divided evenly to represent latitude or declination, and a convenient scale selected as radius for the horizon. Draw a circle with a radius of nine units, each of which represents 10°. Mark on it points to represent north, south, east, and west, as shown in Fig. 118. Join N–S to represent the observer's celestial meridian and E–W to represent the prime vertical. Call the point of intersection Z, and from it mark towards S sufficient units to correspond to the observer's latitude if in the northern hemisphere, or towards N if he is in the southern hemisphere, and mark the point Q. ZQ in Fig. 118 has been made to represent latitude 50° N. Draw an arc through EQW which will represent the equinoctial, the celestial equator. P representing the celestial pole is then marked 90° from Q, and the Figure is ready for showing the positions of the stars.

How to Use the Personal Co-ordinates for Determining the Celestial Co-ordinates and so Identifying a Star. Suppose you are not conversant with any of the stars. Let us take two of those we saw appearing when darkness fell as we sat by the camp-fire (Chapter XI). There was one which bore about N. 105° W. at an altitude of approximately 35° above the horizon. From Z, therefore, we draw a line representing a vertical circle making an angle of 105° with ZN to the westward. The point A at which this line intersects the circumference will represent the point on the horizon vertically below the star. By measuring 3½ scale units for 35° from A towards Z, a point is reached representing the position of the star it is desired to identify. This point is marked X. Then there was a star bearing 150° to the east of north at an altitude above the horizon of about 45°. Its position, Y in the diagram, is inserted similarly by means of its personal co-ordinates, NEB being the horizontal co-ordinate of 150° eastward from N, and BY the vertical co-ordinate of 45° above the horizontal plane. We now have to establish the value of their celestial co-ordinates. Let us begin by estimating their declinations, which are common to both almanacs. Angular distance north or south of the equinoctial must be measured along a celestial meridian, or hour circle, which it is therefore necessary to insert in the Figure. The centres of such hour circles will lie along a straight line, or locus, crossing the observer's celestial meridian at right angles through the central point of WPE. This line is marked LM in Fig. 118. The hour circle being the arc of a great circle must, when produced, bisect the horizon. Its centre is found by trial along LM so that the arc passes through P and the star's position. The centre of the hour circle passing through star X (Fig. 118) is located at O, and the hour circle is found to intersect the equinoctial at C. CX then represents the star's declination, and on being measured to scale shows a value of 1·85 units or 18·5° N. This is the first celestial co-ordinate of star X. Turning either to the *Nautical Almanac* or *Air Almanac* we find that the star of first magnitude with a declination nearest to this is Arcturus, but the evidence of identity is inconclusive as Aldebaran with a declination of 16·5° N. is near enough to create doubt. In order to establish the identity beyond doubt we must estimate the second celestial co-ordinate, the star's angular distance either east or west of ♈, the First Point of Aries. To do this we must first find the position of ♈ in the Figure. This may be obtained in two ways, according to whether the *Nautical Almanac* or the *Air Almanac* is used. The *Nautical Almanac* contains a value known as **'R,' the Right Ascension of Mean Midnight.** If, then, we insert in the figure the point at which the midnight meridian crosses the equinoctial, we can obtain the position of ♈ by applying R to the westward. The *Air Almanac* contains a value called the G.H.A.♈—Greenwich Hour Angle of Aries. If the point at which the Greenwich meridian crosses the equinoctial is marked in the Figure, the position of ♈ will be found by applying the G.H.A. ♈ to the westward. Let us see how this works in Fig. 118. The local mean time is 20.00 hrs., so that it is 20 hours since the midnight hour circle—*i.e.*, the mean sun's anti-meridian—coincided with the observer's meridian, or, in other words, it will be four hours before the midnight hour circle coincides with the observer's meridian again. Now, the midnight hour circle appears to move westward owing to the earth's rotation eastward, so that it will be placed correctly if put 4 hours eastward from Q. Because the projection is equidistant, we may divide QE and QW each into six equal parts to represent intervals of one hour, so that the midnight hour circle will pass through V, the position on the equinoctial diametrically opposite to H, the position of the mean sun. If the *Nautical Almanac* is not available, R for the date may be found approximately from the accompanying table by adding 4 minutes to R of the first day of each month for each day which has elapsed since then.

THE VALUES OF 'R' FOR THE FIRST DAY OF EACH MONTH

R, the Right Ascension of Mean Midnight, to the nearest minute, for noon G.M.T. on the first day of each month.

	hrs.	*mins.*		*hrs.*	*mins.*		*hrs.*	*mins.*
January 1	06	43	May 1	14	36	September 1	22	41
February 1	08	45	June 1	16	38	October 1	00	39
March 1	10	36	July 1	18	37	November 1	02	42
April 1	12	38	August 1	20	39	December 1	04	40

To obtain R approximately for intermediate dates, we add 4 minutes to the value on the first of the month for each day which has elapsed since then.

On August 20 R would be approximately 20 hours 40 minutes + 1 hour 20 minutes—*i.e.*, 22 hours 00 minutes. Now R is the angular distance in time of midnight east of ♈, so that the position of ♈ is placed 22 hours west of V or, taking the shorter arc, 2 hours east of V. This puts ♈ just 6 hours east of Q, and if QC be measured and added to 6 hours we shall have the angular distance of star X from ♈. ♈QC, however, is measured westward, and constitutes the sidereal hour angle of X. To find the star's right ascension we must measure eastward from ♈—*i.e.*, ♈HWC—which, of course, is 24 hours − ♈QC, $24 - 9\frac{1}{2} = 14\frac{1}{2}$ hours = R.A. of X. This corroborates the first assumption that X represents Arcturus, whose R.A. is seen to be 14 hours 13 minutes. If we assume the longitude to be that of Greenwich then the G.H.A.♈ will be represented by the arc QWH♈, which is 270°. This approximates very closely to the value of G.H.A.♈ found in the *Air Almanac* for 20.00 hrs. on August 20.

In any other longitude the difference in time between L.M.T. and G.M.T. would balance the different value of G.H.A. ♈ taken from the *Air Almanac* for the new G.M.T. Suppose the longitude were 45° W., then the Greenwich meridian would cross the equinoctial at G and the G.M.T. corresponding to the L.M.T. of 20.00 hrs. would be 23.00 hrs., and the G.H.A. ♈ for 23.00 G.M.T. is approximately 315°. Therefore, the L.H.A. ♈ would be $315° - 45° = 270°$, as shown in the diagram. ♈GC, the sidereal hour angle of X, is seen to measure $9\frac{1}{2}$ hours, or 142·5°, which is so close to 146·75°, the S.H.A. of Arcturus, as to leave little doubt about the identity. The only other stars with a S.H.A. approaching 142·5° are α and β Centauri, but as their declination is 60° S. they are never visible in latitude 50° N. The Roman numerals in Fig. 118 indicate sidereal hour angle.

The same technique may be used in establishing the identity of Y or of any other star whose personal co-ordinates have been measured. Now that the position of ♈ has been fixed, you need only measure the celestial co-ordinates DY and ♈HD or ♈D, the declination and right ascension or sidereal hour angle respectively. DY = 9°, ♈HD = 19 hours 15 minutes, ♈D = 64°. These values agree closely with those of Altair, whose identity is thereby established.

Exercise 1

Insert each of the following pairs of personal co-ordinates in an equidistant projection of the observer's celestial horizon for the given position, and by estimating the corresponding celestial co-ordinates identify the star to which they belong.

	L.M.T.			*Latitude*	*Longitude*	*Azimuth*	*Altitude*
	days	*hrs.*	*mins.*				
1. November	8	22	00	51° N.	00°	N. 110° E.	37°
2. April	24	03	00	20° N.	00°	N. 114° W.	26°
3. January	16	18	00	45° N.	45° W.	N. 62° W.	38°
4. June	1	04	00	24° S.	60° E.	S. 74° W.	34°
5. December	10	16	30	54° N.	15° W.	N. 90° W.	50°
6. May	30	21	40	10° S.	30° W.	S. 109° W.	27°
7. February	5	06	30	49° N.	150° W.	N. 104° E.	23°
8. August	12	23	20	50° S.	170° E.	S. 117° E.	58°
9. March	20	20	00	35° S.	90° W.	S. 115° W.	45°

How to Locate a Star by Means of Co-ordinates. When it is desired to find a particular star already known by name the reverse process of that described above may be followed—*i.e.*, the stars are inserted in the diagram by means of their declination and right ascension or sidereal hour angle, and the altitude and azimuth found by measurement. To illustrate the method, Fig. 119 has been drawn to show the positions of stars visible to an observer in latitude 50° N. six months before or after those shown in Fig. 118. Some of them, being circumpolar (see Fig. 113), would be visible on both occasions. The Figure is drawn in the same manner and to the same scale as Fig. 118, the only difference being in the method of inserting the positions of the stars. Whereas in the first case the personal co-ordinates (azimuth

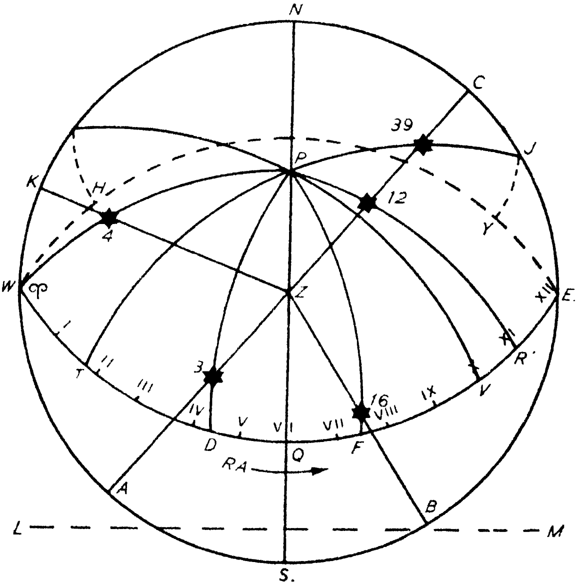


Fig. 119. Where is the Star?

and altitude) were used to discover the celestial co-ordinates (declination and right ascension or sidereal hour angle), the conditions are now reversed. On February 20 at 20.00 hrs. the position of the hour circle of mean midnight would be the same as shown in Fig. 118.—*i.e.*, four hours east of the observer's meridian. V marks the point where it would cross the equinoctial. The value of R on February 1 is 8 hours 45 minutes, and on February 20 8 hours 45 minutes + 1 hour 20 minutes—*i.e.*, 10 hours 05 minutes, or 10 hours approximately. V is therefore 10 hours east of ♈, so ♈ may be marked on the diagram 10 hours west of V. Stars useful for position line purposes at this time would be Aldebaran, Alpheratz, Dubhe, and Procyon, which are inserted in the diagram as follows. The R.A. of each star is taken from the *Nautical Almanac* or the S.H.A. from the *Air Almanac*. The points at which their hour circles would intersect the equinoctial are then marked in by measuring the star's R.A. along WQE eastward from ♈, or the star's S.H.A. along WQE westward from ♈. The R.A. of Aldebaran is 04 hours 33 minutes, which when measured eastward from ♈ gives point D on the equinoctial, as does the star's S.H.A. 292° when measured westward from ♈. The R.A. of Alpheratz is only 00 hours 05 minutes which cannot be indicated on so small a diagram. Its hour circle will therefore intersect the equinoctial approximately at ♈. The star's S.H.A. being 358° 39′ makes almost a complete circle when measured westward from ♈. The R.A. of Dubhe is 11 hours 00 minutes, which when measured eastward from ♈ gives R as the point where its hour circle intersects the equinoctial, the star's S.H.A. being 195°, which corresponds to 13 hours—*i.e.*, 24 hours — R.A. Lastly, Procyon has an R.A. of 7 hours 36 minutes measured eastward from ♈ or a S.H.A. of 246°, which place the hour circle of the star at F.

Now, the declination, the other celestial co-ordinate, must be measured along an hour circle, so those belonging to each of the above stars must now be inserted. When the locus of the centres has been added, as described earlier, the hour circles may be drawn to pass through P and the points of intersection on the equinoctial without much difficulty. The declination is then measured from the equinoctial along the hour circle in each case. Aldebaran has a declination of approximately 16½° N., which, when measured upward from D, reaches a point indicated by 3, the number being that by which Aldebaran is known in the *Air Almanac* and the *Astronomical Navigation Tables*.[1] Similarly, Alpheratz, with a declination of approximately 29° N. (measured from ♈ towards P), is indicated by 4; Dubhe with a declination of 62° N., and Procyon with a declination of 5½° N., are indicated by 12 and 16 respectively. To obtain the azimuth and altitude of the stars, join Z to each and produce to the horizon. The angle which each of these personal vertical circles makes with the observer's meridian, or the arc of the horizon intercepted between them, will be the star's azimuth. Measured from N, the direction of the elevated pole in each case, the azimuth of Aldebaran, angle NZ3, or the arc NWA, is found to be N. 140° W., or 220° when measured as a bearing clockwise from N. The azimuth of Alpheratz, angle NZ4, or the arc NK, is N. 69° W., or 291° measured clockwise from N. Similarly the azimuth of Dubhe and Procyon are found to be N. 41° E., and N. 149° E., respectively. The altitudes are obtained by measuring the distance along the vertical circle from horizon to star. For Aldebaran this is A 3, giving 5·2 units to scale, or 52°. K 4 gives 2·5 units, or an altitude of 25° for Alpheratz, and for Dubhe and Procyon the altitudes will be found from the diagram to be 48° and 42° respectively. Having thus found the personal co-ordinates it will now be possible to use them in locating the stars concerned. Should the star's local hour angle be over 6 hours east or west, the hour circle passing through the star will intersect the equinoctial on the reverse side of the diagram, but may be plotted on the obverse side by continuing its opposite hour circle. Instead of using the declination direct for finding the star's position, the angular distance of the star from the pole, its polar distance—*i.e.*, 90 — declination—is measured from the pole along the star's hour circle. The star Benetnasch has an R.A. of 13 hours 45 minutes, so that because the R.A.M. (right ascension of the observer's meridian—♈Q in our Figure) is only 6 hours the local hour angle of Benetnasch must be 7 hours 45 minutes east. To plot its position we must first draw in its opposite hour circle TP of R.A., 01 hours 45 minutes, and continue it across the pole to cut the horizon at J. The pecked arc indicates its continuation on the reverse side down to the equinoctial, which it intersects at Y. The declination is approximately 50° N., and therefore the polar distance 40° is measured to scale from P towards J, placing the star at the position marked 39—the star's identity number in the *Air Almanac*. Benetnasch is thus seen to lie on the same personal vertical circle as Dubhe, and its azimuth is therefore the same, but the altitude is found by measurement to be approximately 25°.

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[1] The A.N.T. solve the astronomical triangle and provide values of altitude and azimuth when latitude, declination, and hour angle are given. Each volume covers 5° of latitude, and stars are treated individually.

Exercise 2

Construct a diagram on the plane of the observer's meridian for each of the following positions, insert the celestial co-ordinates of the star named, and estimate its azimuth and altitude.

	L.M.T. *days*	*hrs.*	*mins.*	*Latitude*	*Longitude*	*Star*
1. July	17	21	00	52° N.	00°	Vega
2. October	6	00	00	20° S.	00°	Fomalhaut
3. September	26	01	00	38° N.	45° W.	Betelgeuse
4. June	30	03	00	30° S.	75° E.	Antares
5. April	12	01	30	45° N.	165° E.	Regulus
6. November	1	00	30	24° S.	15° W.	Sirius
7. May	21	02	00	60° N.	30° W.	Altair
8. February	13	23	30	10° S.	155° W.	Procyon
9. August	24	04	00	22° N.	65° E.	Deneb

Mixed Co-ordinates. Instead of both the personal co-ordinates or both the celestial co-ordinates being given we may have one of each and be asked to find the others. The following further four combinations are then possible when the latitude is known:

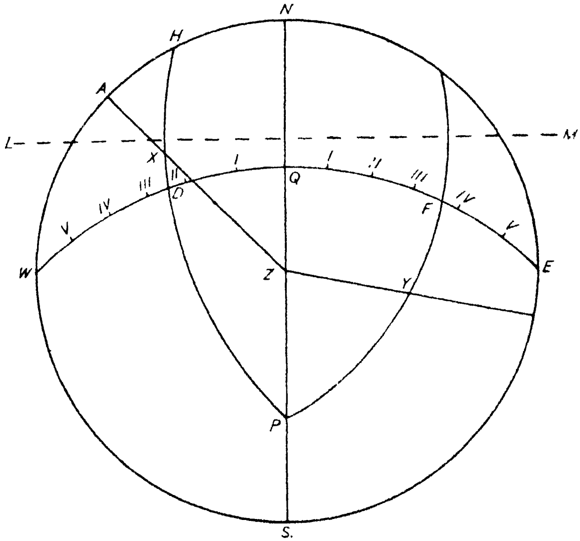


Fig. 120

(*a*) Given the local hour angle and azimuth, find declination and altitude.

(*b*) Given local hour angle and altitude, find declination and azimuth.

(*c*) Given declination and altitude, find local hour angle and azimuth. (Converse of (*a*).)

(*d*) Given declination and azimuth, find local hour angle and altitude. (Converse of (*b*).)

Local hour angle as a co-ordinate **is the arc of the equinoctial**

measured from the observer's meridian westward (**unless otherwise stated**) **to the hour circle passing through the celestial object.**

How to Find Declination and Altitude when Hour Angle and Azimuth are Known. When the local hour angle and the azimuth are given and the declination and altitude are required, the hour circle is first plotted through the point marking its value in hours along the equinoctial from Q. The azimuth measured at Z then provides the vertical circle passing through the object. The point at which these two intersect must be the position of the star, as shown in Fig. 120. The declination is then measured by proportion along the hour circle from the equinoctial to the point of intersection, and the altitude along the azimuth circle from the horizon to the point of intersection.

Example. In latitude 35° S. the hour angle of a star X was 2 hrs. 30 mins. (037½°) and its azimuth S. 135° W. What were the star's declination and altitude?

Fig. 120 is drawn on the equidistant projection, with NESW to represent the celestial sphere, NZS the observer's meridian, P the elevated celestial pole, and WQE the equinoctial. WQE is divided into 12 equal parts to represent hourly intervals up to 6 hours on each side of the meridian, and LM is the locus of the centres of all hour circles. The hour circle for 2 hours 30 minutes is inserted by drawing it from P to pass midway between the divisions marked II and III and continuing it to meet the horizon at H. Angle PZX, or arc SWA, represents the azimuth, and equals S. 135° W. The point of intersection between PH, the hour circle, and ZA, the azimuth circle, is the star's position. The declination, represented by DX, is found by measurement to be 13° N., and the altitude, represented by AX, 31°.

Exercise 3

Draw circles to represent an observer's celestial horizon on the equidistant projection, and from the following values of latitude, hour angle, and azimuth, find the values of altitude and declination in each case.

	Latitude	*Hour angle*			*Azimuth*	
		hrs.	*mins.*			
1.	50° S.	03	00	(45°)	S. 115° W.	(295°)
2.	30° N.	04	20	(65°)	N. 75° W.	(285°)
3.	12° S.	19	12	(288°)	S. 62° E.	(118°)
4.	45° N.	22	00	(330°)	N. 140° E.	(140°)
5.	40° S.	20	48	(312°)	S. 132° E.	(48°)

How to Find Declination and Azimuth when Hour Angle and Altitude are Known. When the local hour angle and altitude are given and the declination and azimuth required, the hour angle is drawn in as above and intersected by the circle of equal altitude whose radius is the zenith distance—*i.e.*, 90° − altitude. The point of intersection is the star's position and declination and azimuth may then be measured.

Example. Find the declination and azimuth of a star whose hour angle is 20 hrs. 15 mins. (303¾°) and whose altitude is 42°.

In Fig. 120 the hour circle is made to cut the equinoctial at F, which is 20 hrs. 15 mins. west, or 3 hrs. 45 mins. east of Q. The circle of equal altitude with the centre at Z and radius 48° intersects PF at Y, which is therefore the star's position. FY the declination and PZY the azimuth are found by measurement to be 29° S. and S. 80° E. respectively.

Exercise 4

Construct figures on the equidistant projection, and from the given value of latitude, hour angle, and altitude, find the values of azimuth and declination in each case:

	Latitude	*Hour angle*			*Altitude*
		hrs.	*mins.*		
1.	40° N.	21	00	(315°)	30°
2.	24° S.	02	20	(35°)	55°
3.	60° N.	05	32	(83°)	25°
4.	35° S.	19	40	(295°)	35°
5.	10° N.	22	00	(330°)	45°

How to Find Altitude and Hour Angle when Declination and Azimuth are Known. The treatment of this problem is somewhat different from what has been described for the earlier cases. We must now plot the circle of declination, and although stars appear to us to move round the pole on a perfectly circular path (as may be verified by a night photograph of the polar sky) we cannot plot it as such on our equidistant projection—unless approximately, when the star is circumpolar—without creating distortion, especially near the circumference. The method adopted in plotting circles of declination requires the use of tables giving the **true amplitude** of a celestial body—*i.e.*, arc of the horizon intercepted between the prime vertical and the vertical circle passing through the rising or setting body. The angle is measured from east or west towards north or south, whereas it will be remembered that azimuth is measured from north or south towards east or west, or sometimes, as in air navigation, from north clockwise to 360°. We need to know where the body will rise and set and where it will cross the observer's meridian, so that the arc of a circle joining these points will represent the star's route across the sky as nearly as possible on a simple projection. Because the star's declination is constant, the arc will also represent its circle of declination.

Example. In latitude 35° S. it is desired to find the altitude and hour angle of Sirius when its azimuth is S. 90° W.—*i.e.*, bearing 270°.

In Fig. 121 NESW represents the celestial horizon and NZS the observer's meridian on the equidistant projection. Q is marked 35° north of Z and WQE then represents the equinoctial. The declination of Sirius is approximately 17° S., which is measured to scale along the meridian to the southward of Q, and marked T. This is where Sirius crosses the meridian, or transits. The true amplitude taken from Norie's Tables is found to be 20°—*i.e.*, E. 20° S. at rising and W. 20° S. at setting. The points are marked R and G respectively, and the arc joining RTG represents a part of the circle of declination. The azimuth being S. 90° W. causes the vertical circle to pass through W, so Z is joined to W by a straight line, and the point X at which this intersects the circle of declination is the position of Sirius. The altitude represented by WX is found to measure 3·5 units, or 35° approximately. To obtain the hour angle we must draw in the hour circle of X, as described earlier, LM being the locus of all hour circles, and note where it intersects the equinoctial divided into intervals representing hours east and west of the observer's meridian. The hour circle is found to intersect the equinoctial at the point B 4 hours 12 minutes (063°) west.

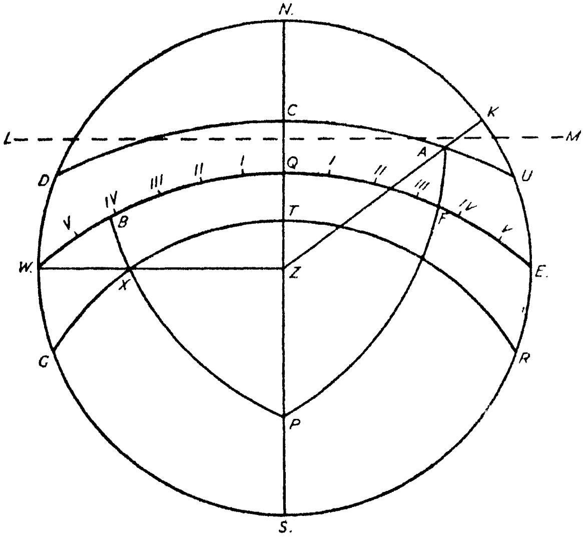


Fig. 121

Exercise 5

Enter the given values of latitude, declination, and azimuth on figures constructed on the equidistant projection, and estimate the corresponding values of hour angle and altitude in each case.

	Latitude	*Declination*	*Azimuth*
1.	15° N.	15° S.	N. 130° W. (230°)
2.	25° S.	20° N.	S. 128° E. (52°)
3.	50° N.	28° N.	N. 93° W. (267°)
4.	30° S.	20° S.	S. 80° W. (260°)
5.	00°	25° N.	N. 55° E. (55°)
6.	50° N.	00°	N. 150° E. (150°)

How to Find Azimuth and Hour Angle when Declination and Altitude are Known

Example. In latitude 35° S. the altitude of Arcturus was 16° to the east of the observer's meridian. What would be its azimuth and hour angle?

Using Fig. 121 C is put in to mark the point where Arcturus, with a declination of 19·5° N., would cross the observer's meridian. The true amplitude is found to be E. 23° N. and W. 23° N., the letters U and D indicating the points of rising and setting respectively. The arc joining UCD then represents the star's circle of declination. The altitude cannot be measured direct as the azimuth is not known, so the zenith distance, 90° − altitude, is taken as a radius, and with Z as centre is found to intersect UC at A. Z is then joined to A and continued to cut the horizon at K, KA representing the altitude. The azimuth, angle PZA, is then found to measure S. 126° E. or 054°, and when the hour circle is added it cuts the equinoctial at F midway between III and IV, indicating a local hour angle of 3½ hours east, or 307½°.

Exercise 6

Draw circles to represent an observer's celestial horizon on the equidistant projection, and from the following values of latitude, declination, and altitude estimate the values of hour angle and azimuth in each case.

	Latitude	*Declination*	*Altitude*
1.	54° N.	20° N.	30° E. of meridian
2.	20° S.	10° N.	40° W. of meridian
3.	35° N.	25° N.	20° W. of meridian
4.	14° N.	20° S.	35° E. of meridian
5.	45° S.	35° S.	45° E. of meridian

How to Find the Latitude by Means of Co-ordinates. When a celestial body is on the observer's meridian it must bear either north or south—*i.e.*, the azimuth is either 000° or 180°. The local hour angle is always zero when the body transits above the pole. If the altitude of a known body is obtained when on the meridian above the pole, the observer's latitude may be found quite simply by applying the declination to 90° − altitude.

Example. The true altitude of Aldebaran bearing south was 55°. What was the observer's latitude?

In Fig. 122 NZS represents the observer's meridian, and SX measured along it to represent the star's altitude of 55° to scale. The declination of Aldebaran is 16° 23′ N., so that Q the point at which the equinoctial crosses the meridian must lie 16° 23′ to the southward of X. The distance QZ must then represent the observer's latitude to scale. It is found by measurement to be 51° N.

When the star is on the meridian between the celestial pole and the horizon it is said to be below the pole. Its azimuth will then be 000° to an observer in the northern hemisphere or to an observer in the southern hemisphere. The local hour angle will be 12 hours (180°) in either case. It was explained in connexion with Fig. 113 that the altitude of the pole had the same value as the observer's latitude. To find the latitude from an observation of a star on the meridian below the pole, only the star and the pole need be inserted in the Figure.

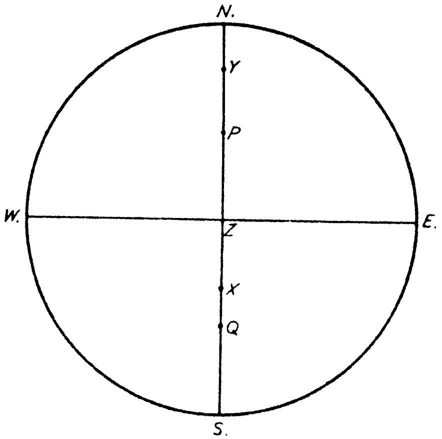


Fig. 122. LATITUDE BY ALTITUDE ON THE MERIDIAN

Example. The true meridian altitude of Dubhe below the pole was 23°. What was the observer's latitude?

In Fig. 122 NY represents the altitude to scale. Now, the declination of Dubhe is 62°, so that this star is always 28° from the pole. If, then, we measure YP to represent 28° to scale, NP will be found to measure 51° and this must be the observer's north latitude.

Exercise 7

Enter the following values on a figure representing an observer's celestial horizon on the equidistant projection, and determine the latitude.

	Hour angle hrs.	mins.		Altitude	Bearing	Declination
1.	00	00	(000°)	50°	180°	10° N.
2.	00	00	(000°)	35°	000°	25° N.
3.	00	00	(000°)	55°	180°	15° S.
4.	00	00	(000°)	80°	000°	30° S.
5.	00	00	(000°)	70°	180°	40° N.
6.	12	00	(180°)	15°	000°	50° N.
7.	12	00	(180°)	20°	180°	60° S.
8.	00	00	(000°)	40°	180°	50° S.

The Stereographic Projection. The problems introduced in this chapter may also be solved graphically by means of the stereographic projection, illustrated in Fig. 123, from which it will be seen that the point of projection is on the circumference.

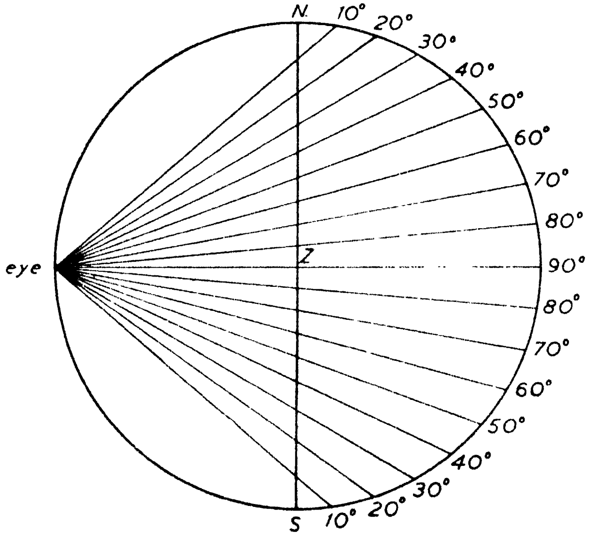


Fig. 123. PRINCIPLE OF THE STEREOGRAPHIC PROJECTION

The evenly spaced divisions on the circumference are not projected as evenly spaced divisions on the primitive NZS. In using this projection, therefore, it is advisable to construct a master copy to a suitable scale from which all measurements for other figures may be taken. The advantage of this projection lies in the fact that all circles on the sphere are projected on to the plane of the primitive either as straight lines or circles. Angles are represented correctly. It is often used to illustrate navigational problems solved by calculation.

General Exercises

1. What do you understand by the terms personal and celestial co-ordinates?

2. Define: zenith distance; First Point of Aries; prime vertical; equinoctial and polar distance.

3. From the given data construct figures to scale on the plane of the celestial horizon, and deduce the required values:

	Latitude	*L.H.A.*	*Declination*	*Bearing*	*Altitude*	
(*a*)	12° N.	—	—	270°	40°	
(*b*)	14° S.	—	—	157°	45°	
(*c*)	35° S.	318°	20° S.	—	—	
(*d*)	42° N.	335°	15° S.	—	—	
(*e*)	57° S.	023°	12° N.	—	—	
(*f*)	40° N.	030°	—	—	40°	
(*g*)	25° N.	—	10° N.	120°	—	
(*h*)	50° N.	060°	—	280°	—	
(*i*)	30° N.	—	20° N.	—	40°	
					Mer. alt.	*Zenith dist.*
(*j*)	—	—	20° S.	180°	—	50°
(*k*)	—	000°	10° N.	—	60°	—
(*l*)	—	—	35° N.	000°	20°	—
(*m*)	—	180°	20° N.	—	20°	—

4. From the following information construct figures similar to the above and identify the star concerned:

	Date	*Time (hrs.)*	*Latitude*	*Azimuth*	*Altitude*	*Longitude*
(*a*)	January 17	02.00 L.M.T.	30° N.	N. 130° W.	26°	—
(*b*)	May 7	23.12 G.M.T.	40° N.	N. 52° W.	26½°	45° W.
(*c*)	October 15	20.00 L.M.T.	10° S.	S. 32° E.	25°	—
(*d*)	October 15	03.00 L.M.T.	50° N.	N. 120° E.	35°	—
(*e*)	September 18	23.00 L.M.T.	35° S.	S. 127° W.	30°	—
(*f*)	May 10	22.30 L.M.T.	45° N.	N. 115° W.	36°	—
(*g*)	November 5	01.00 L.M.T.	30° S.	S. 100° E.	47°	—
(*h*)	July 8	04.00 L.M.T.	45° S.	S. 166° E.	15°	—
(*i*)	September 14	01.00 L.M.T.	35° N.	N. 93° E.	34°	—

5. Place the given stars on a scale drawing of the celestial horizon and deduce the azimuth and altitude in each case:

	Date	*Time (hrs.)*	*Star*	*Latitude*	*Longitude*
(*a*)	August 28	20.00 L.M.T.	Antares	20° N.	—
(*b*)	May 17	00.00 L.M.T.	Deneb	50° N.	—
(*c*)	April 23	20.20 G.M.T.	Regulus	20° S.	10° E.

CHAPTER XIII

THE ASTRONOMICAL TRIANGLE

Composition of the astronomical triangle—Stereographic projection of the triangle on to the plane of the rational horizon—Properties of a spherical triangle—The haversine formula for use in solving astronomical triangles—How to find an angle when all the sides are known—How to find a side when two sides and their included angle are known—How to calculate the times of theoretical sunrise and sunset—Azimuth at sunrise or sunset—Amplitude—Visible sunrise and sunset—The beginning and end of twilight—Solution of miscellaneous problems.

Composition of the Astronomical Triangle PZX. If Fig. 116 were superimposed on Fig. 115 and turned round to suit the latitude of Z, we should combine the personal and celestial co-ordinates and present a spherical triangle for solution. Fig. 124 shows how the completed arrangement would appear. P (Fig. 116) is first placed on Z (Fig. 115) and then turned so that Q approaches Z until QZ represents the given latitude to scale. The positions of X will then coincide, and the astronomical triangle PZX be completed. The three sides are PZ, the co-latitude (90° — latitude of Z), PX, the polar distance (90° — declination of X), and ZX, the zenith distance (90° — altitude of X). The angles are ZPX, the local easterly hour angle of X; PZX, the azimuth of X; and PXZ, the angle of position, not usually required in navigation.

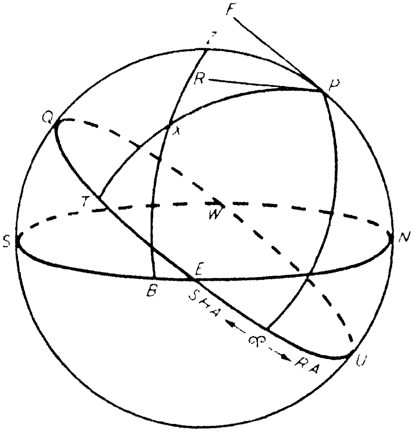


Fig. 124. The Spherical Triangle (i)

To derive the parts of the triangle from the co-ordinates we proceed as follows. Co-ordinate NB (Fig. 124) gives angle PZX, co-ordinate BX gives side ZX, co-ordinate ♈T gives angle ZPX if ♈Q is known or assumed, and co-ordinate TX gives side PX; then, if we know or assume QZ, the observer's latitude, PZ, is known. Five parts only are concerned in the astronomical triangle for ordinary navigational purposes, although six parts are often needed in great-circle sailing and flying.

Stereographic Projection of the Triangle on to the Plane of the Rational Horizon. Fig. 124, it will be noted, is constructed on the plane of the observer's meridian NPZQ, but sometimes a projection on the plane of the observer's horizon NESW is required.

Fig. 125 illustrates the development of this from the sphere by means of the stereographic projection, whereby NESW becomes the primitive plane and P is projected at P_1, Z at Z_1, and X at X_1. Fig. 126 illustrates the completed projection.

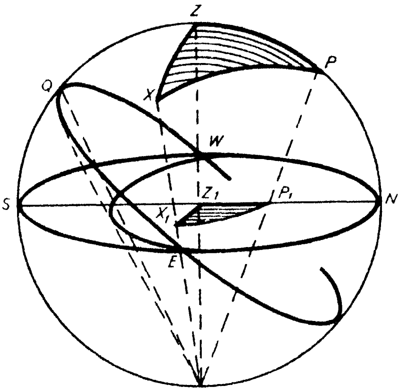


Fig. 125. Stereographic Projection of the Spherical Triangle on to the Plane of the Rational Horizon

Its production is facilitated by means of the stereographic projection card (Fig. 123), measurements from the circumference being used for angular distances round the celestial horizon, and measurements from the diameter for angular distances along the observer's celestial meridian. The spherical triangle for use in illustrating problems in astronomical navigation may be represented either by a projection on the plane of the observer's celestial meridian (Fig. 124) or on the plane of the observer's celestial (sometimes called rational) horizon (Fig. 126).

The type of projection to be used, either equidistant or stereographic, is usually a matter of choice.

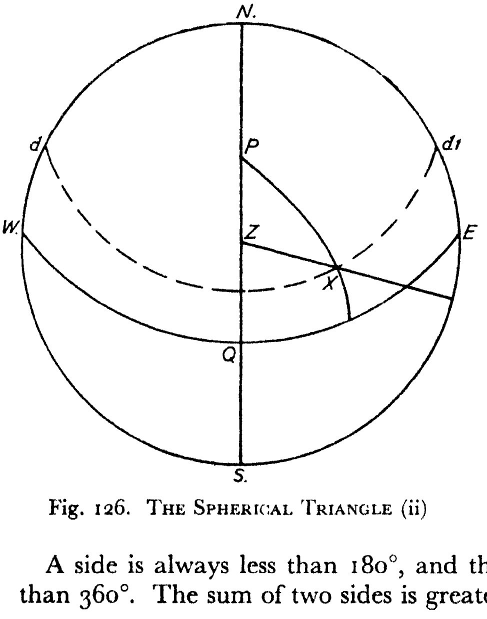


Fig. 126. The Spherical Triangle (ii)

Properties of a Spherical Triangle. Before going on to a solution of this triangle it will be necessary to point out where a spherical triangle differs from a plane triangle. Its sides are formed by the arcs of three great circles. The angles formed by the intersection of these arcs are determined by measuring the angle between the tangents to the arcs at their point of intersection.

In Fig. 124 the angle ZPX is measured by the angle made at P by the tangents FP and RP.

A side is always less than 180°, and the sum of the three sides less than 360°. The sum of two sides is greater than the third side.

An angle is always less than 180°, but the sum of the three angles must be more than 180° and less than 540°.

The Haversine Formula for Use in Solving Astronomical Triangles. When certain parts of a spherical triangle are known the remainder can be found by calculation, the simplest formula for use in all general cases being the haversine formula derived from the fundamental cosine formula (see Appendix, p. 395).

In ordinary astro-navigational problems two cases arise:

(i) Given three sides, find an angle.

(ii) Given two sides and the included angle find the remaining side.

Case (i) will arise when in Fig. 124 or 126 all the sides are given and angle ZPX or angle PZX is required.

Case (ii) will arise when sides PX and PZ together with angle ZPX are given and side ZX is required. This case might be varied by having any two sides and the included angle given. Should either of the two remaining angles be required it can then be found by using the formula for case (i).

Formula for case (i) using PZX triangle:

$$\text{Haversine ZPX} = \frac{\text{haversine ZX} - \text{haversine (PX} \sim \text{PZ)}}{\text{sine PX} \times \text{sine PZ}}$$

Formula for case (ii):

$$\text{Haversine ZX} = \text{haversine ZPX} \times \text{sine PX} \times \text{sine PZ} + \text{haversine (PX} \sim \text{PZ)}$$

(~ means difference.)

Only one of these need be committed to memory as the other can be found from it by transposition.

The haversine may be taken from tables for angles up to 180°, but where the sine or cosecant of an angle over 90° is required the same ratio of the supplement to the angle should be used.

The haversine $= \dfrac{1 - \text{cosine}}{2}$. As the cosine can never exceed $+1$ or be less than -1, the haversine is always positive. The sine and cosecant are positive for all angular values up to 180°.

Thus, as no side or angle of a spherical triangle can exceed 180° all ratios used in the haversine formula must have positive values.

How to Find an Angle when all the Sides are Known. This is case (i) of the solution by haversine formula, and the following general example will serve to illustrate the method to be adopted in handling it.

Example. Given PZ = 40°, PX = 60°, ZX = 30°. Find angle ZPX.

$$\begin{aligned}\text{Haversine ZPX} &= \frac{\text{haversine ZX} - \text{haversine (PX} \sim \text{PZ)}}{\text{sine PX} \times \text{sine PZ}}\\ &= [\text{haversine ZX} - \text{haversine (PX} \sim \text{PZ)}]\ \text{cosec. PX} \times \text{cosec. PZ.}\end{aligned}$$

The calculation has two stages. First find the natural haversines of the terms in the square brackets and subtract, then find the corresponding log. to this difference and add it to the log. cosecants of PX and PZ. This gives the log. haversine of ZPX.

ZX = 30°	natural haversine =	0·06699	
PX ~ PZ = 20°	natural haversine =	0·03015	subtract
	difference =	0·03684	
	log. diff. =	8·56630	add
PX = 60°	log. cosec. =	10·06247	
PZ = 40°	log. cosec. =	10·19193	
ZPX = 29° 48¾′	log. haversine =	8·82070	= sum

The next example shows how the same method may be applied to a problem of more specific character.

Example. To an observer in latitude 48° 30′ N., what would be the azimuth of Altair when its true altitude was 32° 19′ east of the meridian? Adapting Fig. 124 to this problem, we arrive at the following results for the values of the sides of triangle PZX.

Latitude QZ =	48° 30′	True altitude BX =	32° 19′
	90° 00′		90° 00′
co-latitude PZ =	41° 30′	Zenith distance ZX =	57° 41′

Declination of Altair from *Air Almanac* or *Nautical Almanac*.

TX =	8° 43′	PZ =	41° 30′
	90° 00′	ZX =	57° 41′
Polar distance PX =	81° 17′	PZ ~ ZX =	16° 11′

Using the formula, haversine PZX $= \dfrac{\text{haversine PX} - \text{haversine (PZ} \sim \text{ZX)}}{\text{sine PZ} \times \text{sine ZX}}$

PX = 81° 17′	natural haversine =	0·42423
PZ ~ ZX = 16° 11′	natural haversine =	0·01981
	difference =	0·40442
	log. diff. =	9·60683
PZ = 41° 30′	log. cosec. =	10·17874
ZX = 57° 41′	log. cosec. =	10·07309
PZX = 116° 23′	log. haversine =	9·85866 = sum

PZX is the angle measured from north, and corresponds to arc of the horizon NB, so that the azimuth is N. 116° 23′ E. and the bearing measuring clockwise from north is the same.

Exercise 1

Construct approximate triangles from the following values of three known sides, and find by calculation in each case the values of all their angles.

	PZ	*PX*	*ZX*
1.	45° 00′	73° 37′	50° 31′
2.	38° 20′	81° 17′	62° 36′
3.	72° 52′	44° 04′	42° 29′
4.	80° 35′	106° 38′	39° 54′
5.	66° 08′	97° 24′	70° 12′

Exercise 2

Construct approximate PZX triangles from the following values of latitude, declination, and altitude. Find by calculation the values of the hour angle (ZPX) and the azimuth (PZX) in each case.

	Latitude	*Declination*	*Altitude*
1.	52° 14′ N.	45° 04′ N.	38° 46′
2.	17° 42′ S.	26° 18′ S.	58° 11′
3.	32° 25′ N.	10° 51′ S.	23° 39′
4.	26° 04′ S.	12° 15′ N.	31° 26′
5.	45° 56′ N.	45° 56′ N.	40° 02′

How to Find a Side when Two Sides and their Included Angle are Known. This is case (ii) in the solution of a spherical triangle by means of the haversine formula, and the following general examples will serve to illustrate the method to be adopted in using it.

Example. In triangle PZX, given PZ = 40°, angle ZPX = 38°, PX = 75°, find side ZX.

Haversine ZX = haversine ZPX × sine PX × sine PZ + hav. (PX ~ PZ)

Arrange in a manner similar to that in example above, but this time the multiplication by logs is done first, then the natural values are added.

ZPX = 38°	log. haversine = 9·02528	add
PX = 75°	log. sine = 9·98494	
PZ = 40°	log. sine = 9·80807	
	sum = 8·81829	
	antilog. of sum = 0·06581	add
PX ~ PZ = 35° 00′	nat. haversine = 0·09042	
ZX = 46° 33¾′	nat. haversine = 0·15623 = sum	

The following example shows how the same method may be applied in the solution of specific navigational problems.

Example. In latitude 55° 10′ N. local sidereal time is 140° 20′. What would be the true altitude of Vega?

Adapting Fig. 124 once again, the following values for the triangle PZX can be deduced.

Latitude QZ = 55° 10′
90° 00′
Co-latitude PZ = 34° 50′

Local sidereal time = ♈Q = 140° 20′
Sidereal hour angle of Vega from *Air Almanac* = ♈T = 81° 16′
Easterly hour angle of Vega = QT = 59° 04′
Easterly hour angle of Vega ZPX = 59° 04′

Declination of Vega from *Air Almanac* = TX = 38° 44′ N.
90° 00′
Polar distance = PX = 51° 16′

The true altitude BX is found indirectly by first calculating the value of the co-altitude, or zenith distance ZX.

Using the formula:

Hav. ZX = hav. ZPX × sine PX × sine PZ + hav. (PX ∼ PZ)

ZPX = 59° 04′ log. haversine = 9·38557
PX = 51° 16′ log. sin. = 9·89213
PZ = 34° 50′ log. sin. = 9·75678

sum = 9·03448
antilog. of sum = 0·10827
PX ∼ PZ = 16° 26′ nat. haversine = 0·02043

ZX = 42° 02¾′ nat. haversine = 0·12870

90° 00′

BX = 47° 57¼′ = true altitude.

Up to now we have been dealing with a ready-made diagram whose sides and angles are less than 90°.

Exercise 3

From the following values of two sides and their included angle find, by calculation, the value of the third side.

	PZ	*ZPX*	*PX*
1.	52° 20′	45° 12′	61° 50′
2.	64° 29′	37° 44′	84° 38′
3.	81° 35′	25° 28′	51° 16′
4.	76° 09′	53° 50′	119° 56′
5.	37° 20′	60° 10′	37° 56′
6.	43° 45′	35° 36′	100° 51′

Exercise 4

Using the following values for latitude, declination, and hour angle, find the corresponding altitude in each case.

	Latitude	*Declination*	*Hour angle*
1.	48° 19′ N.	16° 23′ N.	34° 12′
2.	21° 38′ S.	26° 18′ S.	46° 27′
3.	54° 24′ N.	28° 46′ N.	29° 52′
4.	35° 12′ S.	57° 32′ S.	73° 14′
5.	16° 44′ N.	8° 16′ S.	55° 35′
6.	8° 10′ S.	7° 24′ N.	61° 46′

How to Calculate the Times of Theoretical Sunrise and Sunset. Navigational problems involving the solution of the spherical triangle are many and varied. The commonest already described are those used for finding positions or bearings, but the hour angle may be required for other purposes, such as finding the time of sunrise or sunset, and the duration of twilight. Fig. 127 is a representation of the conditions which arise when the sun is setting and after it has set. Z represents the observer's zenith, QZ his latitude 50° N., and P the elevated pole. NPZQS is, therefore, the observer's meridian, QWQ_1 part of the equinoctial, HKLD part of the circle of declination of 15° N., NWS the celestial horizon, and TLR the limits of nautical twilight. The sun will begin its P.M. journey at H, where it crosses the observer's meridian, and will reach the horizon at K, when theoretical sunset occurs and the zenith distance, ZK, is 90°. Since crossing the meridian a small change in declination may have occurred, so the declination at G.M.T. of local sunset will be necessary. Change in solar declination

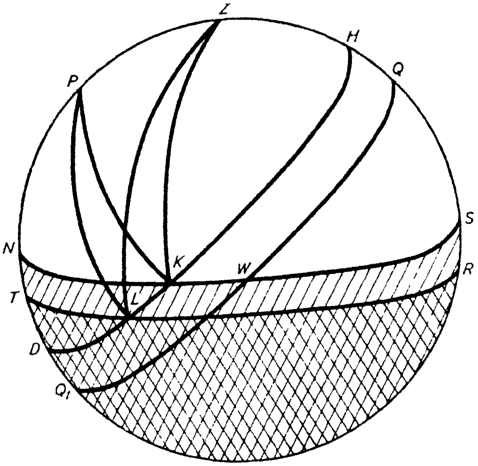


Fig. 127. SUNSET AND TWILIGHT

is maximum at equinoxes, when it reaches 1′ per hour, and nil at solstices, *when the sun stands*, *sol stitet.*

Now, in the spherical triangle PZK, three sides are known—*viz.*, PZ the observer's co-latitude, PK the sun's co-declination, or polar distance, and ZK the sun's zenith distance. To find the time of sunset it will be necessary to find the time which has elapsed since the sun crossed the meridian—*i.e.*, since noon. This, as we have seen before, is the hour angle, and is represented in the diagram by angle ZPK. To obtain it the haversine formula for the solution of spherical triangles may again be used, as in the following example illustrated by Fig. 127.

Example. What would be the local apparent time of sunset to an observer in latitude 50° N., assuming the sun's declination to be 15° N.?

$$\text{Hav. ZPK} = \frac{\text{hav. ZK} - \text{hav. (PZ} \sim \text{PK)}}{\sin \text{PZ} \times \sin \text{PK}}$$

ZK =	90° 00′	nat. hav. =	0·50000
PZ ~ PK	35° 00′	nat. hav. =	0·09042
		difference =	0·40958
		log. diff. =	9·61234
PZ =	40° 00′	log. cosec. =	10·19193
PK	75° 00′	log. cosec. =	10·01506
		log. hav. =	9·81933
ZPK =	7 hrs. 14 mins. 30 secs.		
	12 hrs. 00 mins. 00 secs.		
Sunset =	19 hrs. 14 mins. 30 secs. local apparent time.		

The time of sunrise is calculated in the same way as shown for sunset, except that when the angle ZPK has been found it must be subtracted from 12 hours to obtain the local apparent time which, as stated before, is always measured westward from midnight. The times of moonrise and moonset are found in a similar way, but in this case the hour angle must be added to the moon's R.A. in order to obtain the R.A.M. from which R is subtracted to obtain the L.M.T. This is explained in Chapter XIV.

Planets and stars are treated in the same way as the moon, but are rarely visible when rising or setting.

Azimuth at Sunrise or Sunset. The azimuth of a body when rising or setting may be calculated in the same way as required for the azimuth when the body is above the horizon. The calculation is simplified a little by the fact that the zenith distance at theoretical rising or setting of the body is 90°. Using the same values as given in the foregoing example, we may state the problem as follows.

Example. What would be the azimuth of the sun at sunset to an observer in latitude 50° N., assuming the declination to be 15° N.?

Using Fig. 127 to illustrate the problem, we have to find the value of angle PZK in triangle PZK, where PZ = 40°, PK = 75°, and ZK = 90°.

Applying the haversine formula:

$$\text{Hav.PZK} = \frac{\text{hav.PK} - \text{hav. (ZK} \sim \text{PZ)}}{\text{sine ZK} \times \text{sine PZ}}$$

we have the following development:

PK = 75°	nat. hav. =	0·37059
ZK ~ PZ = 50°	nat. hav. =	0·17861
	difference =	0·19198
	log. diff. =	9·28325
ZK = 90°	log. cosec. =	10·00000
PZ = 40°	log. cosec. =	10·19193
PZK = N. 66° 15¼′ W.	log. hav. =	9·47518

Amplitude. The result of the last example may be obtained more simply by calculating the sun's amplitude—the arc of the horizon measured from east or west northward or southward to the rising or setting body (WK, Fig. 127). The formula is as follows:

Sine true amplitude = secant latitude × sine declination.

Latitude	50° 00′ N.	log. sec. =	10·191933
Declination =	15° 00′ N.	log. sin.	9·412996
Amplitude (WK) =	W. 23° 44¾′ N.	log. sin. =	9·604929

N. or S. according to declination.

90° 00′

Azimuth (NK) = N. 66° 15¼′ W.

Exercise 5

Find the times of sunrise and sunset, and the azimuth or amplitude in each case under the following conditions.

	Latitude	*Declination*
1.	44° 17′ N.	5° 37′ N.
2.	16° 28′ N.	23° 10′ N.
3.	32° 40′ S.	17° 29′ S.
4.	27° 05′ N.	13° 44′ S.
5.	8° 37′ S.	21° 06′ N.
6.	52° 15′ N.	00° 00′
7.	00° 00′	23° 27′ S.

Visible Sunrise and Sunset. When comparing with the times of sunrise or sunset given in the *Almanacs* it must be remembered that they are given for the visible sunset or sunrise of the upper limb to an observer at sea-level, whereas we have calculated the time for the sun's centre on the celestial, or rational, horizon. The difference in zenith distance is 50′, 34′ of which are due to refraction and 16′ to semi-diameter. When the sun's upper limb is just on the visible horizon the sun's centre is actually 50′ below. When you see the sun's lower limb just touching the horizon the sun has actually set in theory, because the refraction of 34′ exceeds the sun's average diameter by 2′.

The Beginning and End of Twilight. The lower limit of twilight varies according to the authority. Civil twilight exists when the sun is between the horizon and 6° vertically below. At this lower limit the brightest stars are just visible, while terrestrial objects are easily discernible. Lighting-up times are controlled by it and the times given in the *Air Almanac* are for this limit. Nautical twilight exists when the sun is between the horizon and 12° vertically below, while astronomical twilight extends to 18° below. Times for civil and nautical twilight are given in the *Nautical Almanac*. Fig. 127 gives the limit for nautical twilight, and to calculate the time of the sun's crossing this line for the given latitude and declination it will be necessary to calculate angle ZPL, the hour angle. The triangle is, of course, an oblique spherical one, and the haversine formula must be used.

Example. At what local apparent time would nautical twilight end to an observer in latitude 50° N., assuming the sun's declination to be 15° N.?

$$\text{Hav. ZPL} = \frac{\text{hav. ZL} - \text{hav. (PL} \sim \text{PZ)}}{\text{sin. PL} \times \text{sin. PZ}}$$

ZL = 102°	nat. hav. = 0·60396
(PL ∼ PZ) = 35°	nat. hav. = 0·09042
	difference = 0·51354
	log. diff. = 9·71058
PL = 75°	log. cosec. = 10·01506
PZ = 40°	log. cosec. = 10·19193
ZPL = 8 hrs. 43 mins. 27 secs.	log. hav. = 9·91757

8 hrs. 43 mins. 27 secs. = hour angle at end of twilight
12 hrs. 00 mins. 00 secs.

20 hrs. 43 mins. 27 secs. = L.A.T. at end of twilight

The more obliquely the sun crosses the horizon the longer will twilight last. It will be shortest on the equator when the declination is nil, increasing with latitude and declination until, when the sum of latitude and declination in the same hemisphere exceeds 78°, the sun will not sink beyond 12° below the horizon and nautical twilight will be continuous from sunset to sunrise, as can be seen from Fig. 127. When latitude plus declination exceeds 90° there will be neither complete darkness nor twilight, as the sun will then be above the horizon the whole time—*i.e.*, will be visibly circumpolar.

When latitude and declination increase in opposite directions daylight will decrease, twilight will increase, and darkness will increase in length until, when the sum of latitude and declination exceeds 90°, the sun will no longer rise above the horizon and twilight will have reached its maximum, but will then decrease until, when latitude plus declination exceeds 102°, there will be neither daylight nor twilight in the observer's latitude.

Exercise 6

Calculate the local apparent times at which civil and nautical twilight begin and end under the following circumstances:

	Latitude	*Declination*		*Latitude*	*Declination*
1.	54° 20′ N.	16° 10′ N.	5.	51° 36′ N.	23° 17′ S.
2.	32° 44′ S.	21° 32′ S.	6.	66° 25′N.	22° 54′S.
3.	60° 30′ N.	20° 24′ N.	7.	50° 15′ S.	00° 00′
4.	43° 18′ S.	12° 48′ N.	8.	69° 00′ S.	23° 12′ N.

The Solution of Miscellaneous Problems. When solving a problem it is often not necessary to draw more than a token triangle, as in Fig. 128, which represents all the requirements of a problem such as the following.

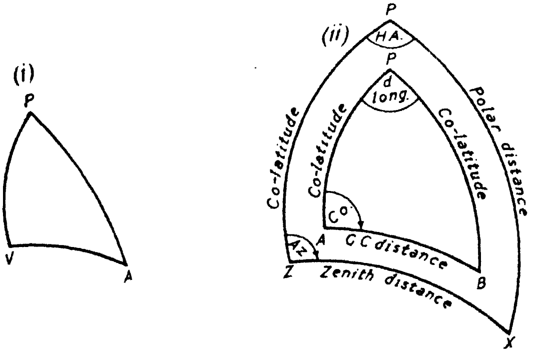


Fig. 128. SPHERICAL TRIANGLES COMPARED

Example. What is the angular distance and bearing of Vega from Altair?

In Fig. 128 (i) P represents the celestial north pole, A the position of Altair, and V the position of Vega. The sides PA and PV represent the polar distances of Altair and Vega respectively, and the angle APV represents the difference between the right ascensions or sidereal hour angles of the two stars.

To find the angular distance AV we use the haversine formula in the same way as when finding the zenith distance in the astronomical triangle PZX or distance along a great circle, Fig. 128 (ii).

$$\text{Hav. AV} = \text{hav. APV} \times \text{sin. PA} \times \text{sin. PV} + \text{hav. (PA} \sim \text{PV)}$$

Dec. of Altair =	8° 43′ N.	dec. of Vega =	38° 44′ N.
	90° 00′		90° 00′
PA =	81° 17′	PV =	51° 16′

	hrs.	*mins.*	*secs.*		
R.A. of Altair =	19	47	57	S.H.A. of Altair =	63° 01′
R.A. of Vega =	18	34	59	S.H.A. of Vega =	81° 15′
APV =	1	12	58	or APV =	18° 14′

APV = 1 hr. 12 mins. 58 secs.	log. hav. =	8·40015
PA = 81° 17′	log. sin. =	9·99496
PV = 51° 16′	log. sin. =	9·89213
	sum =	8·28724
	antilog. of sum =	0·01938
PA ∼ PV = 30° 01′	nat. hav. =	0·06706
AV = 34° 11¾′	nat. hav. =	0·08644

Angular distance of Vega from Altair = 34° 11¾′

The bearing of Vega from Altair is given by the angle PAV, the calculation of which is analogous to determining the azimuth in the PZX triangle, or a course when maintaining a great-circle route.

$$\text{Hav. PAV} = \frac{\text{hav. PV} - \text{hav. (PA} \sim \text{AV)}}{\text{sin. PA} \times \text{sin. AV}}$$

PV = 51° 16′	nat. hav. =	0·18715
(PA ∼ AV) = 47° 5¼′	nat. hav. =	0·15956
	difference =	0·02759
	log. diff. =	8·44067
PA = 81° 17′	log. cosec. =	10·00505
AV = 34° 11¾′	log. cosec. =	10·25025
PAV = 25° 45′	log. hav. =	8·69597

Bearing of Vega from Altair = N. 25° 45′ W. or 334° 15′

Fig. 128 (ii) shows how the astronomical triangle and the great-circle triangle may be compared.

General Exercises

1. In a spherical triangle PZX, PZ = 50°, PX = 75°, ZX = 45°. Find the angles ZPX and PZX by calculation.

2. In a spherical triangle PZX, PZ = 40°, PX = 60°, ZPX = 50°. Find the side ZX.

3. Find the local hour angle and azimuth of Betelgeuse, whose altitude is 30° west of the meridian of an observer in latitude 45° N.

4. Find the zenith distance, altitude, and azimuth of Spica, whose hour angle is 3 hours to an observer in latitude 30° S.

5. What would be the amplitude of the sun (dec. 20° N.), and the apparent time of theoretical sunset to an observer in latitude 40° N.? Find the duration of civil and nautical twilight.

6. Find the declination of the moon when it sets theoretically bearing 300° T. to an observer in latitude 52° N.

7. On June 10 at evening twilight, in latitude 52° 10′ N. longitude 00° 00′, G.M.T. 20 hrs. 35 mins., a star with an approximate altitude of 57° bore 170° T. Establish by calculation the identity of the star.

8. On July 18 at 21 hrs. 25 mins. G.M.T., in latitude 47° 15′ N. longitude 17° 40′ W. by dead reckoning, an unidentified star with an approximate altitude of 28° 25′ bore 110° T. Establish by calculation the identity of the star.

9. On March 12 at 01 hr. 12 mins. 44 secs. G.M.T. an observer A (45° 17′ N. 71° 15′ W.) and an observer B (20° 34′ S. 43° 11′ W.) find simultaneous altitudes of a star to the eastward to be 31° 13′ and 42° 27′ respectively. Identify the star.

10. Find the altitude and azimuth of Procyon when Capella is on the meridian of an observer in latitude 42° 20′ N.

11. Find the altitude and azimuth of Aldebaran when Regulus has an altitude of 30° 10′ east of the meridian to an observer in latitude 50° N.

12. In what latitude would the true altitude of Arcturus be 35° 12′ when the true altitude of Altair was 40° 05′?

CHAPTER XIV

TIME

Apparent solar day—Mean solar day—Astronomical day—Civil day—Local mean time—Greenwich mean time—Zone time—The chronometer—Equation of time—The 'E' correction—The sun-dial—Sidereal time—Sidereal day—The month—Lunar day—Perigee—Apogee—Lunation—The year—Tropical year—Precession of the equinoxes—Sidereal year—The seasons—*Abridged Nautical Almanac*—Meridian passage of the moon—Meridian passage of planets—*Air Almanac*—Hour angle—Sidereal hour angle—Greenwich hour angle of Aries—Local Hour Angle—Time equations.

LIKE space time has no limits, no perceptible beginning or ending, and our measure of it is not noticeably connected with the whole. That we are able to measure intervals is a well-known fact, and we rely upon the repetition of obvious events for our standards. Artificial means of recording abstract intervals have been in existence for many centuries. The clepsydra, or water clock, was common during the early Mediterranean civilization, and consisted of a cone-shaped vessel through which water was allowed to flow into another vessel at a regular rate, the amount of water so collected being a measure of the elapsed interval, a principle which was elaborated upon later, especially in Egypt. Sand-glasses, which still exist in the kitchen as egg-timers, were used by many of the ancients to measure intervals, the sand performing the same function as water in the clepsydra, although not in such quantity. In the early monasteries a candle burning at a regular rate marked the intervals and regulated the ringing of the bells for matins and vespers, thus providing the clock (French, *cloche*, bell) for all within sound range. For maintaining regular watches at sea the striking of the ship's bell was regulated by a half-hourly sand-glass, so that 'eight bells' would be struck every four hours, the day being thus divided into six watches. It has never been difficult to accept the day as an interval of time, and the shadow of some stone or monolith cast by the sun determined for the ancients the part of the day.

Apparent Solar Day. As the sun climbs into the sky shadows become shorter, until a stage is reached at which they are at their shortest for that particular revolution of the sun. The sun is then at its highest and on the observer's meridian. This is apparent noon. In latitudes north of the tropic of Cancer the sun will always bear south when on the meridian, south of the tropic of Capricorn it will bear north, but between the tropics it will bear either north or south when on the meridian, according to the time of year. When the sun crosses an observer's meridian it is said to transit. The interval between two successive transits of the true sun across the same meridian constitutes an **apparent solar day.** Such intervals between transits of the true sun are, however, unequal, so that no two consecutive apparent solar days have the same length. This means that their corresponding hours, minutes, and seconds cannot have the same length. Apparent time, although obtained by direct observation of the sun, is thus unsuitable for accurate comparisons.

Mean Solar Day. In order to avoid where possible the use of irregular time intervals, the true sun has been replaced for accurate time-keeping purposes by an imaginary sun which moves along the equinoctial at a regular rate, and thereby manufactures days whose length is the mean of the 365¼ apparent solar days provided in the course of a year by the true sun. The interval which elapses between any two successive transits of this mean sun across the same meridian is a **mean solar day.** It is divided into 24 hours containing 60 minutes of 60 seconds, and fixes the time which all ordinary clocks and watches are designed to keep.

The Astronomical Day. It has been found more convenient for tabulation purposes, as in almanacs, to reckon the day from one mean midnight to the next, and make the mean solar hours concurrent from 0 to 24. This is referred to as an **astronomical day.** The hours and minutes are grouped together in a four-figure notation such as 05.42 and 17.42 or 05 hrs. 42 mins. and 17 hrs. 42 mins.

The Civil Day. For civil needs the day begins and ends at mean midnight, but is divided into two equal parts of 12 mean solar hours each. In the first half the hours are numbered from 0 hrs. to 12 hrs. A.M. (*ante meridiem*), the sun having not yet reached the meridian, and in the other half the hours numbered from 0 hrs. to 12 hrs. P.M. (*post meridiem*), the sun having crossed the meridian. We thus find times given as 5.42 A.M. and 5.42 P.M.

Local Mean Time (L.M.T.). Owing to the eastward rotation of the earth, the mean sun travels westward along the equinoctial and completes one revolution round the earth—*i.e.*, 360° in 24 hours. The mean sun thus covers 15° of longitude per hour, so that one hour after crossing an observer's meridian the mean sun will be 15°-west of it, two hours after crossing the meridian the sun will be 30° west of it, and so on. Arc of the equinoctial measured westward from the observer's meridian to the mean sun's centre is known, therefore, as **hour angle of the mean sun.** It corresponds to the angle at the pole measured westward between the observer's meridian and the hour circle passing through the centre of the mean sun. As, however, the astronomical day begins at midnight and not at noon, when the sun is on the meridian, local mean time at any instant is the local hour angle of the mean sun $\pm$ 12 hours.

Greenwich Mean Time (G.M.T.). This is the local mean time for the meridian of Greenwich. The mean day begins at Greenwich when the mean sun is over the 180th meridian—*i.e.*, at Greenwich mean midnight, 12 hours before the mean sun reaches Greenwich meridian at mean noon. The importance of G.M.T. to navigators is very great. The positions of celestial bodies are recorded in almanacs for the use of navigators, and, as celestial bodies are constantly on the move relative to the earth, their positions must be given for definite and regular instants of time. Time of day depends on longitude, and as the standard meridian for navigational purposes is that of Greenwich, it is reasonable that G.M.T. be selected as the standard time of reference. Most details in almanacs are given, therefore, at regular intervals of G.M.T.

When combined with the date G.M.T. is referred to as the Greenwich date (G.D.).

Zone Time (Z.T.). To avoid frequent and inconvenient alterations of the clock due to change of longitude when going eastward or westward, the world is divided into time-zones of one hour or 15° each. Zone 0, which has Greenwich meridian as its central meridian, maintains G.M.T. and extends from longitude 7½° E. to longitude 7½° W. Each other zone maintains the time of its central meridian, which is situated at an exact number of hours from Greenwich meridian. The time kept in each zone thus differs from G.M.T. by multiples of an hour, and is fast or slow according to whether the zone is in east or west longitude. The zones are, therefore, numbered according to this difference. Zone (+ 1) extends from 7½° W. to 22½° W., zone (− 1) from 7½° E. to 22½° E. and so on up to (+ 12) or (− 12), as shown in Fig. 5, the sign being an indication of how these hours should be applied to zone time in order to obtain G.M.T. Zone limits must sometimes be modified to accommodate national boundaries and geographical features.

The following examples will illustrate how the Greenwich date is obtained from zone time:

Zone time 14.28 (+ 3) 10 May = G.D. 17.28 10 May.
Zone time 03.16 (− 8) 24 June = G.D. 19.16 23 June.
Zone time 21.45 (+ 9) 31 March = G.D. 06.45 1 April.

Exercise 1

1. In what time zones would you expect the following longitudes to be: 72° 15′ W.; 113° 28′ E.; 158° 47′ W.; 175° 40′ E.?

2. What Greenwich date corresponds to the following zone times:

1. Zone time 07.38 (− 5) 16 September;
2. Zone time 15.22 (− 9) 5 February;
3. Zone time 04.36 (− 6) 1 May;
4. Zone time 12.55 (+ 7) 11 November;
5. Zone time 18.15 (+ 11) 12 January;
6. Zone time 22.50 (+ 5) 31 December.

The Chronometer. Most of the data such as declination, R.A., and G.H.A.♈ required in navigational astronomy are taken from one of the almanacs, which, however, can be of little use unless the correct Greenwich mean time of observation is known. This means that the navigator must be provided with an accurate time-keeper. Before 1765 this was not available, and difficulty was experienced in calculating longitude correctly, the method then employed depending on eclipses of the sun and moon and Jupiter's satellites. Prizes were offered by different governments for a solution of what seemed an impasse, and when Sir Cloudesley Shovel's squadron was wrecked on the Scilly Isles the Commissioners for the Longitude in 1714 offered £20,000 to the man who could invent a method sufficiently accurate to determine the longitude to within 30′ on a voyage across the Atlantic.

John Harrison, a carpenter, finally produced a timepiece, his fourth, which fulfilled the conditions on a voyage to the West Indies and back, and eventually received his reward by instalments. He invented the compensating pendulum and balance wheel. The original instruments are now housed at the National Maritime Museum, Greenwich.

The modern chronometer leaves little to be desired in the way of accuracy, and, although it might gain or lose a second or two per day regularly, wireless time-signals heard daily all over the world have clinched the matter firmly. Problems on chronometer correction still, however, appear in examination papers, and typical examples are given below. Firstly, it must be remembered that the chronometer registers time up to 12 hours only, and to find the Greenwich date (G.D.) it is necessary to know the approximate local time and longitude.

Example. On March 10 at about 06.00 hrs. local mean time in longitude D.R. 100° E., the chronometer showed 11 hrs. 19 mins. 35 secs. What was the Greenwich date?

Approximate L.M.T.	06 hrs. 00 mins. 00 secs.
D.R. longitude	6 hrs. 40 mins. 00 secs. East
Estimated G.M.T.	23 hrs. 20 mins. 00 secs.

The Greenwich date was, therefore, March 9, 23 hrs. 19 mins. 35 secs. if the chronometer were correct.

Example. Supposing the chronometer in the above example to have been 1 min. 05 secs. fast of Greenwich mean noon on December 19 but 0 mins. 16 secs. slow of Greenwich mean noon on February 12, what would be the Greenwich mean time when L.M.T. was 06 hrs. 00 mins. 00 secs. in longitude 100° E.?

December 19 . . . . . 1 min. 05 secs. fast
February 21 . . . . . 0 min. 16 secs. slow

in 54 days . . . . . 1 min. 21 secs. lost—*i.e.*, 1·5 secs. daily.
February 12 . . . . . 0 min. 16 secs. slow
25 days @ 1·5 secs. = 37·5 secs. lost

∴ Chron. on March 9 = 0 min. 53·5 secs. slow
Time by Chron. March 9 = 23 hrs. 19 mins. 35 secs.

Correct G.M.T. March 9 = 23 hrs. 20 mins. 28·5 secs.

Ships carry at least two chronometers for purpose of comparison.

Example. Chronometer A is 1 min. 29 secs. fast of chronometer B and gaining 1·5 secs. daily. 30 days later A is 1 min. 44 secs. fast of B. What is B's daily rate?

A 1 min. 29 secs. fast of B
30 days @ 1·5 secs. = 45 secs. gained

A 2 mins. 14 secs. fast of B if B gaining at A's rate
but A 1 min. 44 secs. fast of B

B gained 30 secs. on A—*i.e.*, 1 sec. daily.
B's rate = A's rate + 1 sec. daily
= 1·5 + 1·0 = 2·5 secs. daily.

Exercise 2

1. When the L.M.T. on April 4 was 09.20 in longitude 162° 45′ W. by D.R., a corrected chronometer showed 8 hrs. 12 mins. 35 secs. What was the Greenwich date?

2. In longitude by D.R. 144° 16′ E. a chronometer, which was 3 mins. 12 secs. fast on G.M.T. showed 7 hrs. 42 mins. 36 secs. What was the Greenwich date if the L.M.T. was June 15 5 hrs. 16 mins. 28 secs.?

3. A chronometer losing 2·4 seconds daily was found to be 9 mins. 53 secs. fast of Greenwich mean noon on April 8. What would be the G.M.T. when the chronometer showed 4 hrs. 17 mins. 48 secs. in the afternoon of July 4, the D.R. longitude being 18° 30′ W.?

4. A chronometer was found to be 5 mins. 14 secs. slow of Greenwich mean noon on May 10 and 3 mins. 19 secs. slow of Greenwich mean noon on August 14. What would be the correct G.M.T. when the chronometer showed 5 hrs. 42 mins. 26 secs. on November 20?

5. On January 10 chronometer A was 21 mins. 20 secs. fast of G.M.T. and losing 2·1 seconds daily. Chronometer B was 5 mins. 16 secs. slow of A on January 10. If B were 4 mins. 26 secs. slow of A on February 19 what would be B's daily rate?

6. Chronometer A was 5 mins. 25 secs. fast of G.M.T., chronometer B was 3 mins. 14 secs. fast of A, and chronometer C was 7 mins. 52 secs. slow of B. Compare C with A.

Equation of Time. It is impossible to reconcile artificial time-keepers with the apparent solar day. The reason for this is that the apparent, or true, sun—the one we see—does not cross a stationary observer's meridian at regular intervals, due to two causes. Firstly, the path which the sun appears to follow over the earth's surface in the course of a year is inclined to the earth's equator at an angle of about 23½°. In Fig. 129, ♈Q_1♎Q represents the equinoctial, ♈ ♋ ♎ ♑ the ecliptic, and P the pole of the equinoctial. Suppose the true sun to move at a uniform speed along the ecliptic, how would its positions en route compare with those of the imaginary mean sun moving at a uniform speed along the equinoctial? At ♈ and ♎ their positions coincide, also when one is at ♋ and ♑ the other will be at Q_1 and Q respectively—*i.e.*, on the same hour circle—but at intermediate positions they will not be on the same hour circle, and one will be ahead of the other. Consider the case half-way between ♈ and ♋ when the true sun is at T and ♈T = 45°. The triangle ♈TR is a right-angled spherical triangle whose hypotenuse ♈T is greater than ♈R, the true sun's right ascension, which must, therefore, be less than 45°. The mean sun moving at a uniform speed along ♈Q_1 will be 45° from ♈—*i.e.*, at M—so that the mean sun will be ahead of the true sun in terms of R.A., but the true sun will precede the mean sun across an observer's meridian because the apparent diurnal movement of the sun is from east to west. It is necessary, therefore, to subtract the difference from the true sun's westerly hour angle in order to find the mean sun's hour angle. The amount of difference varies according to the position of T along ♈♋, increasing from zero at ♈ to a maximum of 10 minutes a little beyond half-way, then decreasing to zero again at ♋. In the next quadrant—*i.e.*, from ♋ to ♎—the true sun is ahead of the mean sun in terms of R.A., and so crosses an observer's meridian behind the mean sun, the correction being then additive to hour angle of the apparent sun in order to obtain the mean sun's hour angle. The sign is different in alternate quadrants.

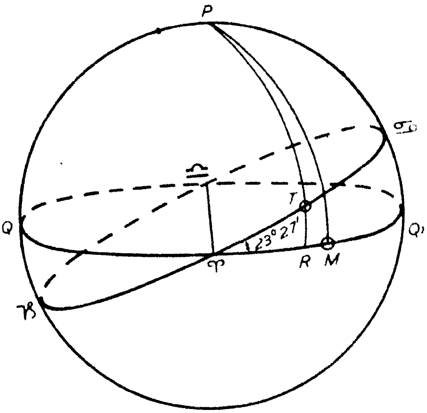


Fig. 129. The Equation of Time (i)

It would be a comparatively easy matter to calculate the correction if the difference were due to this one cause alone, but, as shown in Chapter X, the earth moves along its orbit at a rate depending on its proximity to the sun, which makes the sun appear to move along the ecliptic at a varying speed. This is fastest on January 3 when the earth is nearest the sun, in perihelion, and slowest on July 3 when it is farthest away, in aphelion. The first correction then needs adjusting for this further complication, so that, instead of the two hour angles being the same in amount at equinoxes and solstices, they agree on April 16, June 14, September 1, and December 25, as shown in Fig. 130, a graph illustrating the relationship between apparent time and mean time throughout the year. This combined correction is known as the **Equation of Time.** It is generally the excess of mean time over apparent time, and the sign + or − given to it applies to apparent time unless otherwise stated. The value of the equation of time does not now appear in the *Abridged Nautical Almanac.* It has been supplanted by the 'E' correction.

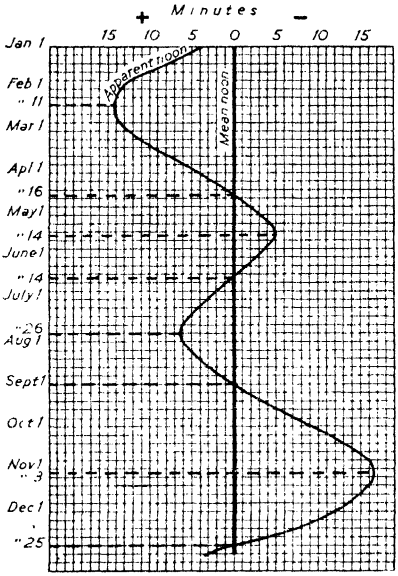


Fig. 130. Equation of Time (ii)

The 'E' Correction. In order to deduce the local time from an observation of the sun, the hour angle must first be calculated from a knowledge of other parts in the astronomical triangle. Hour angle is the interval which has elapsed since the sun's transit at noon, whereas the time of day is the interval which has elapsed since midnight, the difference being 12 hours. If then we require local apparent time we need only add 12 hours to the hour angle of the true sun (H.A.T.S.) rejecting 24 hours if the sum exceeds 24. If local mean time were required, we could first add 12 hours to the hour angle and then apply the equation of time. As this involves more work than is necessary, 12 hours and the equation of time are combined into a correction known as 'E.' Now, the equation of time is the excess of mean time over apparent time—*i.e.*, Eq. T. = local mean time − local apparent time

or, L.A.T. + Eq. T. = L.M.T.
i.e., H.A.T.S. − 12 hrs. + Eq. T. = L.M.T.
i.e., H.A.T.S. − (12 hrs. − Eq. T.) = L.M.T.
i.e., H.A.T.S. − E = L.M.T.

E is thus 12 hours minus the equation of time, and must be subtracted from the H.A.T.S. in order to obtain L.M.T. or, conversely, added to L.M.T. in order to obtain H.A.T.S., subtracting 24 hours if necessary. E will be less than or greater than 12 hours according as the equation of time is + or −.

The Sun-dial. Since the advent of reliable mechanical time-keepers the sun-dial has had little vogue except as a garden curiosity. Nevertheless, when correctly constructed it is an accurate instrument for recording time as indicated by the true sun. Now, although you may be familiar with the appearance and purpose of a sun-dial, you are probably not aware of the conditions under which it will give reliable readings.

Firstly, it is valid only in the latitude for which it was constructed, and secondly, the divisions representing equal intervals of time must, to be perfectly accurate, be found by calculation, as they cannot be represented by equal-spaced markings. To construct a horizontal sun-dial, select a metal sector equal in radius to that of the chosen dial and having an angle equivalent to the local latitude. Set it up vertically on the dial, centre to centre, so that the upper edge of the sector, or style, as it is called, points in the direction of the elevated celestial pole. When the sun is on the meridian—*i.e.*, bearing true south from us—there will be only a line as shadow, but the farther the sun is from the meridian the bigger will be the shadow. The sizes of the angular divisions on the dial can be calculated from a formula for the solution of right-angled spherical triangles. A simpler but less accurate way of making the divisions, however, would be to note when there was least shadow, the sun bearing south, and then to wait for an hour measured by an accurate clock, and make a mark indicating 13·00 hrs. at the point reached by the shadow, followed by hourly intervals by similar marks indicating the subsequent hours. Intervals could then be marked off on the A.M. side corresponding to those on the P.M. side.

The following formula provides the mathematical solution:
Tan. dial angle = tan. hour angle × sine latitude, the dial angle being marked off at the centre.

Example. Graduate a horizontal sun-dial for London (Latitude 51° 30′ N.).

For 11.00 hrs. and 13.00 hrs.: tan. dial angle = tan. 15° × sin. 51° 30′.

$$\begin{aligned} \text{Log. tan. } 15^\circ\ 00' &= 9{\cdot}4280052 \\ \text{Log. sin. } 51^\circ\ 30' &= 9{\cdot}893544 \\ \hline \text{Log. tan. dial angle} &= 9{\cdot}321596 \\ \therefore \qquad \text{dial angle} &= 11^\circ\ 50\tfrac{1}{2}' \end{aligned}$$

For 10.00 hrs. and 14.00 hrs.:

$$\begin{aligned}
\text{Tan. dial angle} &= \text{tan. } 30^\circ \times \text{sin. } 51^\circ\ 30' \\
\text{Log. tan. } 30^\circ\ 00' &= 9{\cdot}761439 \\
\text{Log. sin. } 51^\circ\ 30' &= 9{\cdot}893544 \\
\hline
\text{Log. tan. dial angle} &= 9{\cdot}654983 \\
\therefore \quad \text{Dial angle} &= 24^\circ\ 19' \\
\hline
\end{aligned}$$

The dial angles are not, as you see, merely multiples of each other.

A vertical sun-dial can be similarly constructed. The style must still point to the north pole of the sky, and if the wall runs east and west the formula for the dial angle would be:

$$\text{Tan. dial angle} = \text{tan. hour angle} \times \text{cos. latitude.}$$

Sidereal Time (S.T.). The rotatory movement of the earth causes not only the sun but every other celestial body to cross an observer's meridian. As already explained, the interval which elapses between two successive transits of the true sun is known as an apparent solar day, and of the mean sun a mean solar day. Similarly, the interval which elapses between two successive transits of a star across the same meridian is a **sidereal day.**

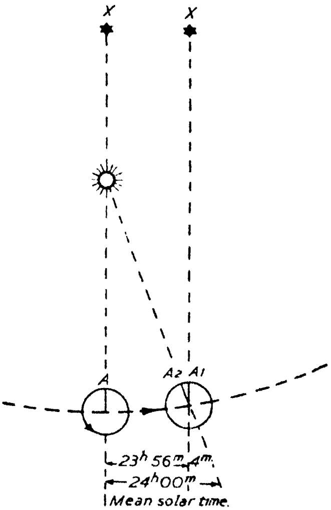


Fig. 131. The Day

Each planet would also produce its particular day. Because stars, owing to their great distances from the earth, appear fixed relative to it and each other, the regular rotatory speed of the earth makes each star produce days of the same and equal length. This circumstance would provide an admirable 'mean' day if the stars kept pace with the sun, but such is not the case, the sidereal day being 3 mins. 56·55 secs., or approximately four minutes of mean solar time shorter than a mean solar day. In the course of a complete revolution of the earth about the sun, the earth makes one more rotation relative to a selected star than it does relative to the sun. The reason for this is that the earth rotates in the same direction as it revolves round the sun and so loses one rotation relative to the sun in the course of a year. To prove this to yourself, ask one of your friends to represent the sun and sit on a chair in the middle of a room, and another representing a star to stand in a corner, while you, the earth, rotating with arm outstretched, make one revolution round your seated friend. If both friends count the number of times your arm moves past them, it will be found that the star in the corner has scored one more than the sun in the centre. In Fig. 131 an observer on the earth at A is seen to have both the sun and star X on his meridian at the same time. When the earth has made one rotation it will have moved along its orbit and star X, whose direction will not have changed owing to its vast distance from both sun and earth, is again on A's meridian, and the time which has elapsed since the preceding transit constitutes a sidereal day. The sun, however, is not yet on the meridian, and a further four minutes of mean solar time must elapse to complete the solar day. This difference multiplied by 365¼ approximates very closely to the difference of one day per year mentioned above. Like the solar day the sidereal day is divided into 24 equal hours containing 60 minutes of 60 seconds each. Because the stars bear a constant relation to the First Point of Aries, the instant of its transit across an observer's meridian is accepted as the beginning of the sidereal day for that observer, and its angular distance as it moves westward from him, measured in hours, minutes, and seconds, constitutes the sidereal time for that meridian.

Owing to the sidereal day being four minutes shorter than a mean solar day the stars rise four minutes earlier every 24 hours, and it is this fact which makes it inconvenient to use a sidereal day for civil purposes, despite its regularity. Man's activities, especially in agriculture, are still dependent to a large extent on light provided by the sun, and the alternating periods of light and darkness fulfil man's requirements of work and rest, making their dual combination the natural division into which the year should be divided.

The Month. The interval which elapses between two successive transits of the moon across the same meridian is a **lunar day,** and provides a division much more irregular than that created by the apparent sun, for, although the plane of the moon's orbit is inclined to that of the equinoctial at an angle only about 5° different from that of the sun's, the difference between her distances from the earth at **perigee** (when nearest) and **apogee** (when farthest away) is proportionately greater to the average distance than is the case with the sun. There is, therefore, a greater difference between the moon's maximum and minimum orbital speeds; and because the moon passes through all her phases in one lunation, the change in her declination and right ascension is rapid, so great care is necessary when dealing with this fickle lady. In Fig. 132 the moon, sun, and star are all shown on the meridian of an observer at A. In 27⅓ mean solar days the moon will have made one revolution round the earth and be once more together with the star on the same meridian, but owing to the movement of the earth eastward along its orbit, the moon and the sun will not be together on the same meridian for another $2\frac{1}{6}$ mean solar days, thus completing one **lunation,** or lunar month, of $29\frac{1}{2}$ mean solar days—*i.e.*, from new moon to new moon. To the man in the moon the earth has made only $28\frac{1}{2}$ rotations, due to the fact that the moon goes round the earth in the same direction as the earth rotates, thus reducing the number of rotations relative to some external body by one, as we saw in the case of the earth's revolution round the sun. If $28\frac{1}{2}$ lunar days are equivalent to $29\frac{1}{2}$ mean solar days the lunar day must be longer, and works out at 24 hours $50\frac{1}{2}$ minutes of mean solar time. In terms of apparent motion the moon thus drops behind the mean sun at the rate of $50\frac{1}{2}$ minutes a day; but this rate is not regular, as can be seen by comparing the intervals between successive transits of the moon, the irregularity being due, as explained earlier, to the varying orbital speed of the moon.

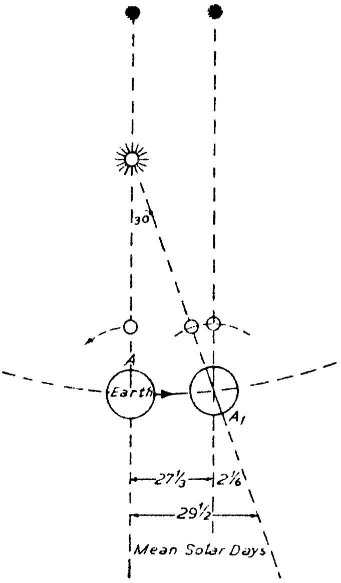


Fig. 132. The Month

This interval is of little use except in the computation of tides, on which the moon has the greater influence, as explained in Chapter IV, although as long ago as 432 B.C. an Athenian astronomer, named Meton, discovered that 235 lunations are very nearly equal to the duration of 19 solar years.

The Year. A solar year can be regarded as the time taken for the apparent sun to move round from vernal equinox back to vernal equinox again. This is known as the **tropical year,** and is not quite equal to the time taken by the earth to go completely round the sun owing to a slight movement westward of the First Point of Aries known as **precession of the equinoxes.** The time occupied by a complete revolution of the earth round the sun is a **sidereal year,** and would be the time taken by the earth to move round from a particular star back to that star again. It will thus be seen that the tropical year is slightly shorter than the sidereal year owing to the First Point of Aries moving very slightly, about $50''$ annually, to meet the earth. In Fig. 133 we have the earth represented in different positions along its orbit, so that on March 21, the Vernal Equinox, the sun is projected from the earth on to the First Point of Aries ♈. As the earth moves round eastward, the sun will be projected on to first ♉ Taurus the bull, then ♊ Gemini the twins, and so on until the circuit is almost completed, when it will be found that ♈ has moved to ♈$_1$, and the sun will be back again in Aries slightly sooner than would have been the case had ♈ not moved.

The tropical year of 365 days 5 hours 48 minutes 45·5 seconds is the year adopted for civil use owing to its being the basis of the seasons.

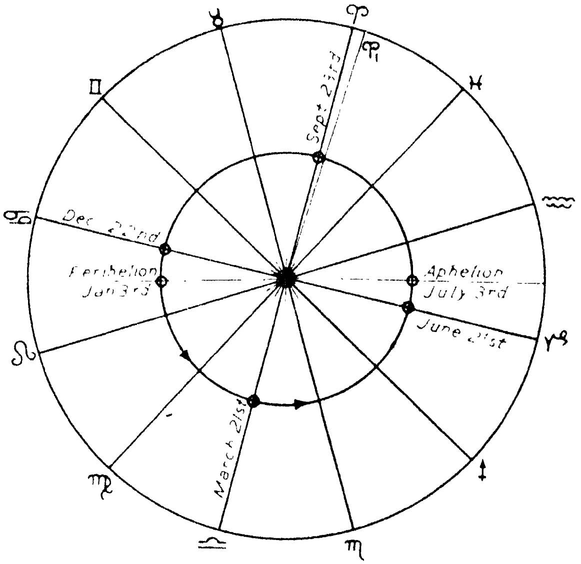


Fig. 133. The Year

From the time of Julius Cæsar the year was taken to be exactly 365¼ days, and a leap year occurred every fourth year. This made the year longer than it actually was and the vernal equinox fell back until, in 1582, it occurred on March 11 instead of on March 21. On the advice of the astronomer Clavius, Pope Gregory XIII ordered that the date should be advanced by ten days—*i.e.*, from October 5 to October 15. The change was immediately brought into force by most Catholic countries, but the Greek Church and most Protestant countries refused to recognize it, and not until 1752 was the reformed calendar adopted in England and then in the teeth of popular opposition. The new calendar allowed an extra day to every year divisible by four with the exception of the centuries, which could only count as leap years if the number of centuries was divisible by four—*e.g.*, 1900 was not a leap year, but 2000 will be. The 'Gregorian Correction' is so accurate that it will be only one day out in 3200 years.

Another year, about five minutes longer than the sidereal year and little used in calculations, is that known as the anomalistic year, and is the period which elapses between the earth's moving from perihelion back to perihelion again. This is indicated in Fig. 133, where it will be noticed that the sun takes longer to pass from ♈ via ♋ to ♎ than from ♎ via ♑ to ♈—*i.e.*, the sun is north of the equinoctial for a longer time than south of it, due to the earth's slower orbital motion when in aphelion than in perihelion. The earth, however, is nearer the sun during the southern summer than during the northern, giving rise to probable higher temperatures while south, although there are mitigating factors.

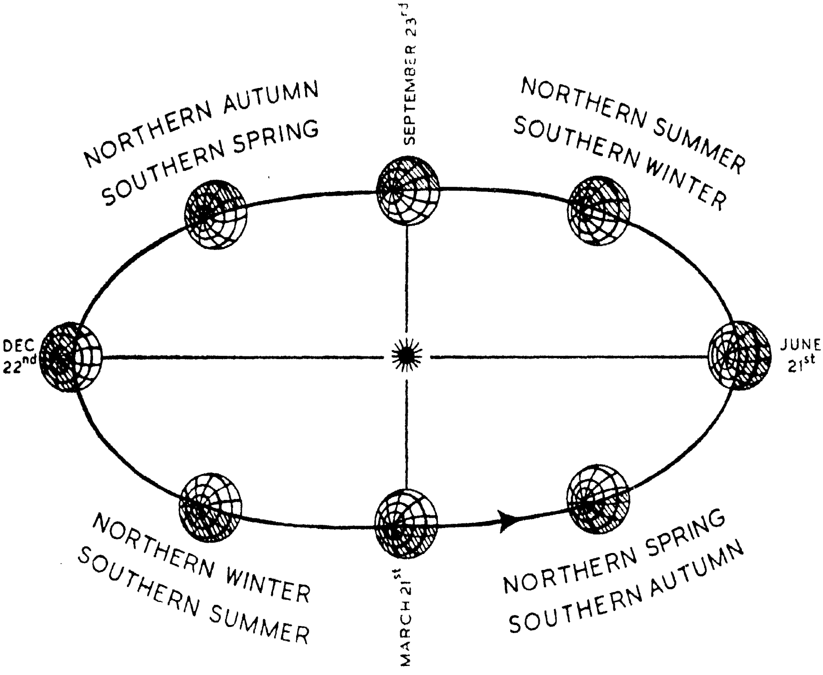


Fig. 134. The Seasons

The Seasons. The circumstance causing the sun to move from one hemisphere to the other and back in the course of a year is that due to the earth's axis maintaining a constant angle of 66° 33′ with the plane of the earth's orbit in its passage round the sun. In Fig. 134 the earth is shown in four main and four intermediate positions relative to the sun. The main positions are, when the sun is crossing the equator from south to north, about March 21, the vernal equinox; at its most northerly declination, about June 21, the northern solstice; crossing the equator from north to south, about September 23, the autumnal equinox; and at its most southerly declination, about December 22, the southern solstice. The period from March 21 to June 21 is known to us as Spring, from June 21 to September 23 as Summer, September 23 to December 22 as Autumn, and December 22 to March 21 as Winter.

The maximum and minimum declinations of the sun are not the same on corresponding dates in succeeding years owing to the year not containing an integral number of days, so that equinoxes and solstices do not always occur on the same date every year but, as can be verified by referring to the *Abridged Nautical Almanac*, the declination of the sun will agree on corresponding dates every fourth year.

The "Abridged Nautical Almanac." This is a publication designed primarily for the use of navigators in surface craft. It contains all the astronomical data required for the computation of position at sea by observation of the sun, moon, stars or planets, together with such tables as those giving time of sunrise and sunset, moonrise and moonset, and the beginning and ending of twilight. It has a separate section for each month. On p. I of the month will be found the names and magnitudes of suitable navigational stars, together with the R.A. and declination of each. On p. II, as shown in the extract at the end of this volume (p. 381), will be found for 00 hrs. G.M.T. of each day the sun's semi-diameter and the moon's semi-diameter and horizontal parallax. The time of the moon's meridian passage at Greenwich for each day both above and below the pole is also given. As already explained, the lunar day exceeds 24 mean solar hours, and so the time of crossing the Greenwich meridian is later on each succeeding day. The difference is given in minutes between the times of transit for each day. The two asterisks found in place of a time on January 13 indicate that the moon did not cross the Greenwich meridian on that date. It crossed at 23 hrs. 41 mins. on the 12th, but because the interval before the next transit was 24 hrs. 54 mins. it could not cross again until 00 hrs. 35 mins. on the 14th.

SUN

On pp. III–VI of each month are given the particulars relating to the sun—*i.e.*, the values of R, declination, and E for every even-numbered hour of G.M.T. See the extracts covering January 1–8 and July 2–9 (pp. 382 and 385).

The value of R increases at the regular rate of approximately 20 seconds per 2 hourly interval—*i.e.*, 10 seconds per hour—so it is a comparatively simple matter to interpolate for values required between the given times. An interpolation table is nevertheless provided with the almanac. The declination and E do not change regularly, they sometimes increase and sometimes decrease with time, but the change in two hours is small, especially for E, and mental interpolation for intermediate times should not prove difficult. If calculation must be resorted to, the following example will show how the working should be arranged.

Example. What would be the value of R, declination, and E for the sun at 17 hrs. 35 mins. 00 secs. G.M.T. on January 4?

The time 17 hrs. 35 mins. falls between 16 hrs. and 18 hrs. By tabulating the values of R, dec., and E for these times we can find the change in values per hour. If we then multiply this by the interval in hours and tenths which has elapsed from 16 hrs. to the given time, we shall find the corrections to apply to the values of R, dec., and E at 16 hrs. The results are tabulated below.

		R.		*Declination*		*E.*	
	hrs.	*mins.*	*secs.*		*hrs.*	*mins.*	*secs.*
Values at 16 hrs.	06	55	28·2	22° 42·9′ S.	11	54	56·1
Values at 18 hrs.	06	55	47·9	22° 42·4′ S.	11	54	53·8
Change from 16 hrs. to 18 hrs.			+ 19·7	− 0·5′			− 2·3
Changes per hour			+ 9·85	− 0·25′			− 1·15
Interval since 16 hrs.			1·6	1·6			1·6
Correction to nearest tenth			+ 15·8	− 0·4′			− 1·8
Values at 16 hrs.	06	55	28·2	22° 42·9′ S.	11	54	56·1
Values at 17 hrs. 35 mins.	06	55	44·0	22° 42·5′ S.	11	54	54·3

MOON

Pages VII–X of each month contain the R.A. and declination of the moon for the even-numbered hours of G.M.T. Both elements change rapidly, as may be seen in the extract covering January 25–31, but, whereas the R.A. always increases, the declination sometimes increases and sometimes decreases. The change in value per 2-hourly interval is tabulated between the given quantities, and the values of R.A. and declination for intermediate times must be calculated carefully. The method is similar to that adopted for the sun's elements, as the following example shows, except that the interval must, to ensure accuracy, be taken to the nearest tenth of a minute in the case of the R.A.

Example. What would be the values of the moon's R.A. and dec. at 15 hrs. 27 mins. 48 secs. G.M.T. on January 31?

The time 15 hrs. 27 mins. 48 secs. falls between 14 hrs. and 16 hrs. Owing to the rapid change in the moon's R.A., we must find what it amounts to per minute and multiply this by the minutes and tenths in the interval before applying it to the value for 14 hrs.

The rate of change of declination varies considerably from day to day, but as it never amounts to more than about 12′ per hour, we shall be quite safe in working to the nearest minute of time in the interval.

	R.A. hrs.	*mins.*	*secs.*	*Declination*
Values at 14 hrs.	23	55	42	0° 04·9′ S.
Rate of change per hour		+	114·5	− 9·5′
Rate of change per minute		+	1·91	− 0·16′
Interval in minutes			87·8	87·8
Correction to nearest tenth		+ 2	47·6	− 14·0′
Values at 14 hrs.	23	55	42	0° 04·9′ S.
Values at 15 hrs. 27 mins. 48 secs.	23	58	29·6	0° 09·1′ N.

It will be noted that the declination changes hemisphere between 14 hrs. and 16 hrs., hence the change of description from S. to N.

Planets

Pages XI and XII of each month contain the R.A. and declination of the four navigational planets Venus, Mars, Jupiter, and Saturn for 00 hrs. G.M.T. of each day, as well as the G.M.T. of their crossing the meridian at Greenwich. On referring to the extract for the planets you will see that the rate of change in R.A. and declination is given per 24 hours. To find their values at times other than 00 hrs. it will be necessary to multiply the daily rate of change by the proportion of day which has elapsed since 00 hrs.

Example. What would be the R.A. and declination of Venus on January 20 at 07 hrs. 40 mins. 10 secs. G.M.T.?

The R.A. changes 326 seconds during the 24 hours. The proportionate part of the day up to the time of observation is $\frac{460}{1440}$ working to the nearest minute, or $\frac{23}{72}$. This when multiplied by 326 gives 232 seconds as the correction to add to the R.A. for 00 hrs. The correction for declination is $\frac{23}{72} \times 1{\cdot}9 = 0{\cdot}6$ decrease. The tabulated values are then as follows:

	R.A. *hrs.*	*mins.*	*secs.*	*Declination*
Value for 00 hrs. January 20	18	33	08	22° 58·5′ S.
Correction for 7 hrs. 40 mins. 10 secs.		+ 3	52	− 0·6′
Values for 7 hrs. 40 mins. 10 secs.	18	37	00	22° 57·9′ S.

Pole Star

An observer's latitude may be found direct by applying three tabulated corrections to the true altitude of Polaris. Tables for this purpose are found in the *Nautical Almanac* after the details of each month. Table I contains by far the most important correction, which is obtained by entering the table with the local sidereal time of observation to the nearest two minutes.

The second correction is obtained by entering Table II with the local sidereal time and the altitude, while Table III must be entered with the local sidereal time and the date, for the third correction. A Table for finding the azimuth of the Pole Star is also given. It is entered with either the local sidereal time and the latitude or the local hour angle and the latitude. The azimuth, however, never exceeds 2° up to latitude 60°.

The *Nautical Almanac* made its first appearance in 1767. The degree of accuracy of the quantities now given is comparable with that obtainable in all sextant observations.

Exercise 3

Using the given extracts from the *Nautical Almanac*, find the correct values of R, declination, and E for the sun at the following Greenwich mean times:

		hrs.	*mins.*	*secs.*
1.	January 1	09	54	00
2.	January 7	02	17	44
3.	July 3	17	25	09
4.	July 5	23	41	10
5.	July 8	00	38	52

Find the moon's R.A. and declination on

		hrs.	*mins.*	*secs.*
6.	January 25	05	11	16
7.	January 28	23	19	38
8.	January 31	14	53	20

Find the R.A. and declination of Venus on

		hrs.	*mins.*	*secs.*
9.	January 2	14	21	12
10.	January 25	06	42	45

Find the R.A. and declination of Mars on

		hrs.	*mins.*	*secs.*
11.	January 5	03	05	16
12.	January 23	19	32	24

Meridian Passage of the Moon. We have already noted that the moon crosses Greenwich meridian at a later time each day. In fact, it crosses every meridian at a later time on succeeding days. This is due to the lunar day for any meridian being always longer than the solar day for that meridian. The moon is continually retarding on, or following behind, the sun. If, then, the moon crosses Greenwich meridian on January 10 at 21 hrs. 56 mins. L.M.T. it cannot cross any other meridian, say 60° W. at the same L.M.T., as by the time the moon reaches 60° W. it will have fallen still farther behind the sun and the L.M.T. of transit will be later. The difference between the L.M.T. of transit at Greenwich and at any other longitude will depend upon two factors:

(i) the full amount of retardation made by the moon on the sun in a complete revolution of the earth—*i.e.*, the difference in time between one transit at Greenwich and the next—*viz.*, 52 minutes between January 10 and 11;

(ii) the proportion which the difference of longitude from Greenwich —*i.e.*, the longitude—bears to the complete revolution of 360°, *viz.*, $\frac{60°}{360°}$ or $\frac{1}{6}$ in the case of 60° W.

By the time the moon reaches 60° W., therefore, it will already have fallen still farther behind the sun by $\frac{1}{6}$ of the total amount—*i.e.*, $\frac{52}{6}$ or $8\frac{2}{3}$ minutes—so that the L.M.T. of transit in 60° W. on January 10 will be

21 hrs. 56 mins. + $8\frac{2}{3}$ mins. = 22 hrs. 05 mins. to the nearest minute.

When in west longitude we must always select from the *Nautical Almanac* the transit difference which lies between the current date and the following day, and the proportionate part of it due to longitude is then added to the time of transit at Greenwich on the current date.

When in east longitude the transit difference to choose is that which lies between the current date and the preceding day, as the moon has not yet reached Greenwich meridian. The proportionate part due to longitude is then subtracted from the time of transit at Greenwich on the current date.

Example. What would be the L.M.T. of the moon's meridian passage at Melbourne (long. 145° E.) on January 29?

Difference between transits at Greenwich on 29 and 28 = 48 mins.
Proportionate value for 145° = $\frac{145 \times 48}{360}$ = 19 mins.
L.M.T. of transit at Greenwich on January 29 = 13 hrs. 48 mins.

L.M.T. of transit at Melbourne on January 29 = 13 hrs. 29 mins.

Exercise 4

What would be the L.M.T. of moon's transit on the following dates in the longitudes indicated?

	Date	*Longitude*
1.	January 1	165° 00′ W.
2.	January 9	105° 00′ E.
3.	January 19	97° 20′ W.
4.	January 25	72° 50′ E.
5.	January 13	160° 00′ W.

Meridian Passage of Planets. You will have noticed in the *Nautical Almanac* for January that Venus crosses Greenwich meridian later on succeeding days, whereas Mars crosses earlier. Both Jupiter and Saturn also cross earlier.

The times however differ only slightly from day to day, but, as with the moon, a correction for longitude should be applied to the time of transit at Greenwich if the L.M.T. of transit at a place not on Greenwich meridian is required accurately. The correction will be a proportionate part of the difference in transit times for the date in question, and the preceding one if in east longitude, but the following one if in west longitude. Whether the correction is additive or subtractive will depend on whether the planet is crossing Greenwich meridian earlier or later on successive days.

In west longitude planet gaining, correction −
In west longitude planet losing, correction +
In east longitude planet gaining, correction +
In east longitude planet losing, correction −

Think this out, but remember the planet almost keeps pace with the sun.

Example. Find the L.M.T. and G.M.T. of the meridian passage of Venus on January 11 at New Orleans (long. 90° W.).

As New Orleans is in west longitude the difference between the times of passage on 11 and 12 must be found.

	hrs.	*mins.*	
Mer. pass. of Venus at Greenwich on January 11 =	10	24	G.M.T.
Mer. pass. of Venus at Greenwich on January 12 =	10	26	G.M.T.
Difference =		02	lost
Proportionate part of difference = $\frac{90}{360} \times 2$ =		00½	lost
Mer. pass. of Venus at Greenwich on January 11 =	10	24	G.M.T.
Mer. pass. of Venus at New Orleans on January 11 =	10	24½	L.M.T.
Longitude in time =	6	00	W.
Mer. pass. of Venus at New Orleans on January 11 =	16	24½	G.M.T.

The maximum difference between the times of meridian passage of any planet on successive days is 4 minutes, so that usually the proportionate part for longitude is negligible in practice and may be ignored. The G.M.T. of meridian passage at Greenwich can then be taken as the L.M.T. of passage across any other meridian.

Exercise 5

What will be the L.M.T. of meridian passage of:

1. Venus on January 15 in longitude 120° W.
2. Venus on January 26 in longitude 145° E.
3. Mars on January 1 in longitude 160° W.
4. Mars on January 17 in longitude 110° E.

"Air Almanac" The *Air Almanac* is issued every four months now. On the inside of the front cover are given the magnitude, sidereal hour angle (S.H.A.), and declination of fifty stars. The first twenty-two, given in block letters, are of chief importance and will be found in the Tables (p. 381). Here we have a contrast with the *Nautical Almanac*. Sidereal hour angle of heavenly bodies is given instead of right ascension, and here lies the chief difference between nautical and aeronautical practice in the treatment of astronomical problems.

On the pages for each day will be found, for every 10 minutes of Greenwich mean time, the Greenwich hour angle of First Point of Aries (G.H.A. ♈), the Greenwich hour angle of the sun, and the Greenwich hour angle and declination of the moon.

For values at intermediate times mental interpolation may be resorted to if no table of proportional parts is available. It will be noticed on inspection that the rate of change in G.H.A.♈ or G.H.A. of sun is almost 2° 30′ per 10-minute interval of G.M.T.—*i.e.*, 15′ per minute, or 1′ per 4 seconds of time. To be perfectly accurate, the change in any interval of less than 10 minutes can be found by proportion.

Example. What would be the G.H.A.♈ and G.H.A. of sun on July 9 at 10 hrs. 17 mins. 40 secs. G.M.T.?

We must first find the change from 10 hrs. 10 mins. to 10 hrs. 20 mins.

	G.M.T. *hrs.*	*mins.*	*secs.*	*G.H.A.* ♈	*G.H.A. of sun*
July 9	10	10	00	79° 27′	331° 15′
July 9	10	20	00	81° 58′	333° 45′
Change in		10	00	2° 31′	2° 30′
Change in $7\frac{2}{3}$ mins. $= \frac{23}{30} \times 151' =$				1° 56′	
Change in $7\frac{2}{3}$ mins. $= \frac{23}{30} \times 150' =$					1° 55′
July 9	10	10	00	= 79° 27′	331° 15′
July 9	10	17	40	= 81° 23′	333° 10′

The G.H.A. of moon could be found in a similar way. Its rate of change per 10-minute interval is about 2° 24′.

The sun's declination and the G.H.A. and declination of three of the navigational planets are given for each hour, while the mean S.H.A. and mean R.A. of each planet are given for the day, together with their time of meridian passage and the sun's semi-diameter. The G.H.A. of one of the planets for a time other than at the integral hour could be found by proportion, as already explained in the case of the G.H.A.♈ and G.H.A. of sun.

Example. What would be the G.H.A. of Jupiter on July 9 at 09 hrs. 37 mins. 30 secs.

We must first find the rate of change from 09 hrs. to 10 hrs. and then the proportionate part for 37 mins. 30 secs.

	hrs.	*mins.*	*secs.*		
G.H.A. Jupiter at	09	00	00 G.M.T.	=	353° 33′
G.H.A. Jupiter at	10	00	00 G.M.T.	=	8° 35′
Change in 1 hour				=	15° 02′
Change in ⅝ hour = ⅝ × 902				=	9° 24′
G.H.A. Jupiter at	09	00	00	=	353° 33′
					362° 57′
					360° 00′
G.H.A. Jupiter at	09	37	30	=	2° 57′

Tables covering the G.M.T. of sunrise, sunset, twilight, moonrise, and moonset are given for certain latitudes, together with the G.M.T. of the moon's meridian passage. These values, with the exception of those for the moon's phenomena, will be the approximate L.M.T. of occurrence in any longitude. For the moon's values a Difference Table is given, so that allowance may be made for the longer time occupied by the moon between successive risings and settings. The table really provides the half-difference for use with longitudes up to 180°.

A Pole Star table giving the correction to apply to an observed altitude of Polaris in order to find the latitude is also given. The table is entered with the L.H.A.♈ and the correction so found is known as 'Q'. Various interpolations and correction tables are also given.

The best way of learning what the almanacs contain is by using their data, and ample opportunity for this will be provided.

Exercise 6

Find the G.H.A.♈ and G.H.A. of sun at the following Greenwich mean times:

		hrs.	*mins.*	*secs.*		*hrs.*	*mins.*	*secs.*		*hrs.*	*mins.*	*secs.*
1. January 4	(*a*)	00	16	24;	(*b*)	05	55	12;	(*c*)	17	08	40
July 9	(*d*)	02	44	50;	(*e*)	04	57	10;	(*f*)	23	32	15

Find the G.H.A. of moon for

		hrs.	*mins.*	*secs.*		*hrs.*	*mins.*	*secs.*
2. January 4	(*a*)	08	29	06;	(*b*)	17	27	10
July 9	(*c*)	00	15	10;	(*d*)	20	58	25

Find the G.H.A. of

		hrs.	*mins.*	*secs.*			*hrs.*	*mins.*	*secs.*
3. Venus for January 4	(*a*)	06	24	00	and July 9	(*b*)	13	50	18
Mars for January 4	(*c*)	08	55	00	and July 9	(*d*)	17	29	10
Jupiter for January 4	(*e*)	15	32	30	and July 9	(*f*)	22	32	06

Hour Angle. Fig. 135 provides a representation of the earth (pepw) and the celestial sphere (PEPW) under a model set of conditions. X represents the position of a star, g the projected position of Greenwich, and Z the observer's zenith. Corresponding meridians and hour circles are marked on earth and celestial sphere. WA♈BCE represents the equinoctial in the plane of the equator wabce. ♈, the First Point of Aries, connects all celestial bodies; it is the astronomical linchpin. Arc of the equinoctial measured westward from ♈ is known as **Sidereal Hour Angle** (S.H.A.), which may also be defined as the angle at the pole between the hour circle passing through ♈ and the hour circle passing through a celestial body, measured westward. ♈A or ♈PA thus represents the sidereal hour angle of the star X. Its

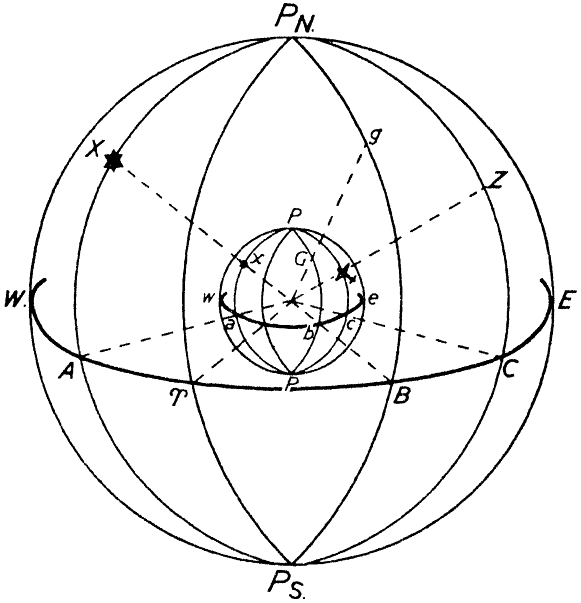


Fig. 135. Hour Angle

value for any navigational star is a fixed quantity found inside the front cover of the *Air Almanac.*

B♈ represents the **Greenwich Hour Angle of Aries** (G.H.A.♈), arc of the equinoctial measured westward from the meridian of Greenwich to the First Point of Aries. It forms the connexion between the earth and the celestial sphere, a sort of Jacob's ladder connecting earth and heaven. If you imagine the earth rotating eastward, it is easy to see that G and g and B all move eastward, so increasing the angle between B and ♈. That is why the G.H.A.♈, as already noted, always increases at a regular rate.

It will now be seen that to find the G.H.A. of a star X (Fig. 135) we need only find the angular measurement of the arc B♈A—*i.e.*, we add the value of G.H.A.♈ and S.H.A. of the star, both of which are obtained from the *Air Almanac.* Before turning to the Astronomical Navigation Tables, by which the triangle PZX is solved, we must find the **Local Hour Angle** (L.H.A.) of the star, the angle at the pole between the observer's celestial meridian and the star's hour circle, measured westward, or arc of the equinoctial measured westward from the observer's celestial meridian to the star's hour circle. In Fig. 135 C is the point at which the observer's celestial meridian intersects the equinoctial, and A the point at which the star's hour circle intersects the equinoctial. CB♈A is, therefore, the L.H.A. of the star. The difference between this and B♈A, the G.H.A. of the star is, of course, BC which corresponds to bc, the longitude. In this case the longitude is east, and we must add it to the G.H.A. of the star in order to obtain the L.H.A. If, however, the longitude had been west, C would have been on the west side of B, and we should have to subtract it. Formula:

$$\text{L.H.A.}^{*} = \text{S.H.A.}^{*} + \text{G.H.A.♈}\begin{Bmatrix} + \text{E. longitude} \\ - \text{W. longitude} \end{Bmatrix}$$

Example. Find the L.H.A. of Vega on January 4 at 20 hrs. 32 mins. 30 secs. G.M.T. to an observer in longitude 25° 40′ E.

S.H.A. Vega = 81° 16′
G.H.A.♈ for 20 hrs. 30 mins. = 51° 33′
Increment for 2 mins. 30 secs. = 38′
Longitude = 25° 40′ E.

L.H.A. Vega = 159° 07′

The Greenwich hour angle of the sun, moon or planets is given direct, so one need apply only the longitude to them in order to obtain their L.H.A.

Example. What would be the L.H.A. of the sun on July 9 at 12 hrs. 25 mins. 40 secs. G.M.T. to an observer in longitude 44° 18′ W.?

G.H.A. of sun for 12 hrs. 20 mins. = 112° 03′
Increment for 5 mins. 40 secs. = 1° 25′

G.H.A. of sun for 12 hrs. 25 mins. 40 secs. = 113° 28′
Longitude = 44° 18′ W.

L.H.A. of sun = 69° 10′

Exercise 7

Find the L.H.A. of the following celestial bodies:

	G.M.T.			*Longitude*
	hrs.	*mins.*	*secs.*	
Canopus January 4	22	15	20	64° 20′ E.
Betelgeuse January 4	01	42	15	95° 42′ W.
Regulus January 4	05	53	12	25° 17′ E.
Sun January 4	07	39	40	29° 19′ E.
Moon January 4	18	21	06	75° 52′ W.
Altair July 9	17	44	11	175° 48′ W.
Deneb July 9	20	02	35	139° 56′ E.
Achernar July 9	03	16	54	31° 15′ W.
Venus July 9	15	32	28	104° 25′ W.
Jupiter July 9	08	07	46	92° 45′ E.

Time Equations. It must be remembered that all time is measured from the observer westward and is the interval which has elapsed since the hour circle marking the commencement of the day (mean, apparent, sidereal), crossed the observer's meridian moving westward. Hour angle also is measured westward unless explicitly stated otherwise, and is the elapsed interval since the particular body concerned crossed the observer's meridian moving westward. Thus there is a difference of 12 hours between the mean sun's hour angle at a certain place and astronomical mean time which starts from midnight. The sidereal hour angle of ♈ is, however, the same as sidereal time because both are the measure of the time which has elapsed since ♈ crossed the observer's meridian moving westward. Right ascension is always measured from ♈ eastward up to 24 hours. All celestial bodies of navigational importance are connected to ♈ by angular measurement eastward or westward from it, and thereby through it to each other, either in terms of R.A., as in the *Nautical Almanac*, or in terms of H.A., as in the *Air Almanac*, so that when two angles are known another can be found by means of time equations. These equations, however, often prove puzzling unless a diagram is drawn and the given values marked on it in the correct direction.

There are two usual starting-points—*i.e.*, (i) the meridian of the observer and (ii) the hour circle of the First Point of Aries. Fig. 136 represents a set of conditions existing at 15·00 hours L.M.T. on November 3 to an observer in latitude 45° N. longitude 20° W. The Equation of Time, or excess of mean time over apparent time, was then at its maximum negative value (— 16 mins. 23 secs.), making E 12 hrs. 16 mins. 23 secs. R was 03 hrs. 53 mins. 12 secs., and the sun's declination 15° 05′ S. To clarify particulars the positions of the observer and Greenwich have been projected on to the celestial sphere at Z and G respectively. Starting from the observer's meridian and measuring from D westward along the equinoctial:

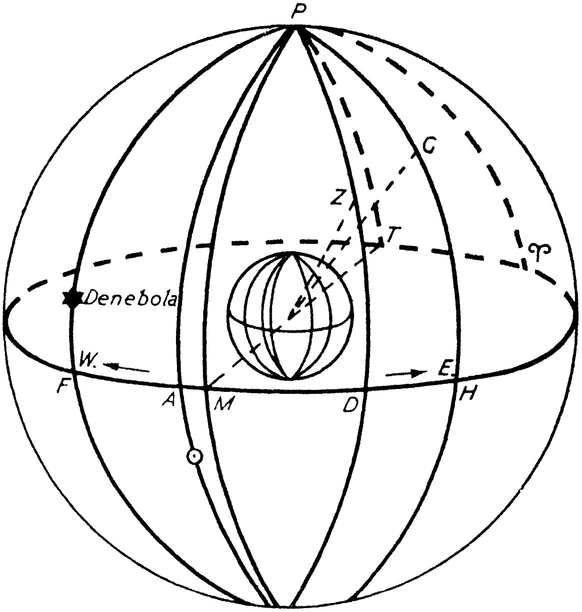


Fig. 136. Time and Right Ascension (i)

Arc DM represents the local hour angle of the mean sun.

Arc DA represents the local hour angle of the apparent sun.

Arc DF represents the local hour angle of the star Denebola.

Arc DFT represents the local hour angle of mean midnight, *i.e.*, arc DFT represents local mean time (L.M.T.).

Arc DFT♈ represents the local hour angle of ♈, *i.e.*, arc DFT♈ represents local sidereal time (L.S.T.).

Arc HFT represents the Greenwich hour angle of mean midnight, *i.e.*, arc HFT represents Greenwich mean time.

Arc AM represents the equation of time (negative to apparent time).

Arc THMA is a measure of E, *i.e.*, arc THMA is 12 hours (THM) — equation of time (AM).

Arc ♈T represents R, *i.e.*, arc ♈T represents the R.A. of mean midnight.

Arc ♈TF represents the right ascension of Denebola.

Arc ♈TA represents the right ascension of the apparent sun.

Arc ♈TM represents the right ascension of the mean sun (R.A.M.S.).

Arc ♈TH represents the right ascension of the Greenwich meridian.

Arc ♈TD represents the right ascension of the observer's meridian (R.A.M.).

Arc ♈FD, the right ascension of the observer's meridian, is equal to DF♈, the local sidereal time, but is measured in the opposite direction. Similarly the right ascension of Greenwich meridian is equal to the Greenwich H.A. of ♈, but opposite in direction. A satisfactory definition of any of the above would be: arc of the equinoctial measured westward or eastward as the case might be, from one meridian to the other.

Fig. 136 could be used for solving the time equations, but once a knowledge of the terms has been acquired a less elaborate diagram may be used. The details of Fig. 136 are reproduced in Fig. 137 on the plane of the equinoctial, and the following relationships should be studied carefully in conjunction with it.

L.H.A. of a body = R.A. of observer (R.A.M.) − R.A. of body (I)

I.e., for a star *: DMF = ♈TFMD − ♈TF (Fig. 137)

or,

for the mean sun: DM = ♈TFMD − ♈TFM

Now, DM = ♈TFMD − ♈T − TFM

∴ DMFT − MFT = ♈TFMD − ♈T − TFM

I.e., local mean time − 12 hours = R.A. of observer − R − 12 hours.

Transposing: local mean time + R = R.A. of observer.

Briefly: L.M.T. + R = R.A.M. . . . . . . (II)

Note: The 12 hours involved is subtracted from local mean time or from the R.A. of the body if possible (*i.e.*, if over 12 hours), but otherwise added.

Substituting in (I)

L.H.A. of body = local mean time + R − R.A. of body . (III)

which can be verified in Fig. 137 by using the star*.

Viz., DMF = DMFT + ♈T − ♈TF

It has already been explained that:

L.H.A. of apparent sun = local apparent time ± 12 hours . (IV)

Now, as shown in Fig. 137

DM − DA = AM (which is negative).

I.e., H.A. of mean sun − H.A. of apparent sun = equation of time (in this case minus from apparent time).

∴ H.A. of mean sun + 12 hours − (H.A. apparent sun + 12 hours) = equation of time.

I.e., L.M.T. − L.A.T. = equation of time . (V)

or L.A.T. = L.M.T. − equation of time.

Substituting in (IV)

L.H.A. apparent sun = L.M.T. − equation of time ± 12 hours.
Now 12 hours − equation
of time = E
∴ L.H.A. apparent sun
(H.A.T.S.) = L.M.T. + E
or,
H.A.T.S. − E = L.M.T. . . . . . . (VI)

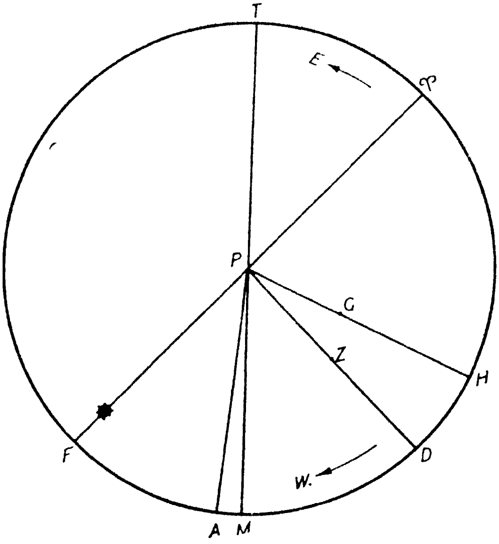


Fig. 137. Time and Right Ascension (ii)

The apparent sun is often referred to as the true sun (T.S.).

Longitude is obtained by comparing L.M.T. with G.M.T., the difference giving the angular distance of the observer east or west of Greenwich. In Fig. 137 G.M.T. is represented by HDMFT and L.M.T. by DMFT, the difference being HD, measured westward from Greenwich, and so we arrive at the aid to memory if not to grammar:

Greenwich time best, longitude west.
Greenwich time least, longitude east.

At this stage there is no need to commit any of the above time equations to memory, they should be solved by means of a figure whose angles need not be drawn with perfect accuracy, although this aids as a check to calculation. In working the navigational problems of the next chapter it will be necessary to have equations II and VI at your finger tips.

General Exercises

1. Graduate a horizontal sun-dial for Edinburgh (55° 57′ N.).

2. What is the Equation of Time? Explain its cause and variability.

3. Account for the disparity in length between a solar day and a sidereal day.

4. What is a lunation?

5. Explain the difference between a tropical year, a sidereal year, and an anomalistic year.

6. Draw a diagram to illustrate the cause of the seasons.

7. Draw suitable diagrams and prove the following:

(*a*) R.A.M. = L.M.T. + R = ✱ H.A. + ✱ R.A.
(*b*) L.M.T. = H.A.T.S. − E.
(*c*) ✱ S.H.A. + G.H.A. ♈ − W. long. = ✱ L.H.A.
(*d*) G.H.A.T.S. = G.M.T. − 12 hrs. + Eq.T.
(*e*) R.A.T.S. = R − E.
(*f*) Long. = G.H.A. ♈ ~ L.S.T.
(*g*) L.M.T. ✱ transit = ✱ R.A. − R.
(*h*) ✱ H.A. + ✱ R.A. − R = H.A.T.S. − E.
(*i*) Long. = G.M.T. + R − ✱ R.A. − ✱ H.A.

8. At what local mean time on January 4 will the stars Aldebaran, Rigel, Capella, Sirius, Vega, Regulus, Canopus, and α Centauri cross the Greenwich meridian? Will they then be visible to an observer at Greenwich (51° 29′ N.)? When will Capella, being circumpolar, cross the meridian below the pole?

9. Find the G.M.T. of the meridian passage, or transit, of Venus on January 25 at San Francisco (122° W.), and of Mars on January 2 at Wellington, N.Z. (176° 35′ E.).

10. Find the G.M.T. of the upper meridian passage of the moon on January 30 at Colombo (80° 00′ E.).

11. Find the G.M.T. of the upper meridian passage of the moon on January 23 at Montreal (73° 36′ W.).

12. A chronometer was 12 mins. 09 secs. fast of Greenwich mean noon on July 4. At Greenwich mean noon on September 2 it was 11 mins. 49 secs. fast. What would be the correct G.M.T. on December 1 at about 7 hrs. 40 mins. P.M., when the chronometer showed 6 hrs. 22 mins. 20 secs. in an approximate longitude of 145° W.?

13. At 9.40 P.M. the hour angle of Arcturus was 2 hrs. 27 mins. to an observer in longitude 53° W. What was the date?

14. On April 1 chronometer A was 2 mins. 25 secs. fast of chronometer B. On May 12 A was 1 min. 54 secs. fast of B. If B's daily rate was 1 sec. losing what was A's daily rate?

15. Chronometer A was 4 mins. 15 secs. fast of G.M.T. Chronometer B was 9 mins. 32 secs. slow on A. Chronometer C was 16 mins. 24 secs. fast on B. Compare C with A.

16. On July 7 in long. 25° 56′ W., when H.A.T.S. was 2 hrs. 40 mins. 20 secs., the time by chronometer was 4 hrs. 49 mins. 50 secs. What was the error of chronometer?

17. Draw a diagram to illustrate any connexion that exists between E and R as found in the *Nautical Almanac*. Is the difference between their angular values constant? If not, why not?

CHAPTER XV

ASTRONOMICAL POSITION LINES

Historical—Calculation of latitude—Calculation of longitude—Circle of position—Geographical position—Position line by longitude and azimuth—The use of a figure—Plotting a position line—The value of a single position line—Second position line from a simultaneous celestial observation—Plotting a fix from simultaneous observations—A running fix—Celestial and terrestrial position lines compared—Marcq St Hilaire method of position lines—Intercept—Nautical example—How the position line is plotted—The second observation—Fixing the position—Aeronautical example—Plotting position arcs—Coincidence of position line with parallel of latitude—Coincidence of position line with meridian—The sextant—Correction of observed altitudes—Dip—Semi-diameter—Parallax—Refraction—Looking ahead.

Historical. It is just over a hundred years since Captain Sumner, Master of an American merchant vessel, published an account of a discovery he had made some six years earlier. He writes,

> Having sailed from Charleston, S.C., on 25th November, 1837, bound for Greenock, a series of heavy gales from the westward promised a quick passage. After passing the Azores, the wind prevailed from the south, with thick weather: after passing 21° W. no observation was had until near the land, but soundings were had not far, as was supposed, from the edge of the bank. The weather was now more boisterous and very thick and wind still southerly. Arriving about midnight 17th December within 40′ by D.R. of Tuskar Rock, the wind hauled S.E. true making the Irish coast a lee shore. The ship was then kept close to the wind . . . . At about 10 A.M. (18th) an altitude of the sun was observed and the chronometer time noted, but having run so far without observations it was evident that the latitude by dead reckoning was liable to error and could not be entirely relied upon. However, longitude by chronometer was determined, using an uncertain D.R. latitude, and the position fixed accordingly. A second latitude was then assumed 10′ to the north and a second position obtained, and again a third position by means of a third latitude 10′ still further north. On pricking off these three positions it was discovered that the three points were disposed in a straight line lying E.N.E. and W.S.W., and that when this line was produced in direction E.N.E. it also passed through the Smalls Light. The conclusion arrived at was that the observed altitude must have happened at all three points, at the Smalls Light and at the ship at the same instant. The deduction followed that though the absolute position of the ship was doubtful, yet the true bearing of the Smalls Light was certain, provided the chronometer was correct. The ship was, therefore, kept on E.N.E., and in less than an hour the Smalls Light was made bearing E. by N. ½ N. and close aboard. The latitude turned out to be 8′ in error.

This discovery laid the foundation of what has become the modern method of astronomical navigation by position lines.

The Calculation of Latitude. In Sumner's day the custom was to find the latitude and longitude from separate observations of celestial bodies, the latitude being found when the body was on the observer's meridian and consequently bore north or south, and the longitude, when the body was not near the meridian but bearing as near to east or west as possible. The latitude could be obtained with tolerable accuracy, if the G.M.T. were known, by applying the sun's meridian zenith distance to its declination. In Fig. 138 PZQSP$_1$ represents the observer's meridian, and SX represents the true meridian altitude of the sun above the horizon NES, found by sextant angle, and ZX (90° – SX), the zenith distance. By applying ZX to QX (the declination of the sun), QZ can be obtained, and this represents the latitude of the observer. The G.M.T. is required to find the sun's correct declination, but in the case of star observations even the G.M.T. is not necessary, as the celestial positions of stars are almost fixed. (General Exercises, No. VII.)

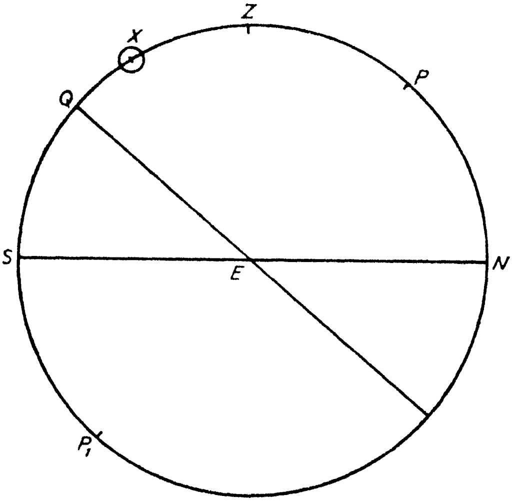


Fig. 138. LATITUDE BY MERIDIAN ALTITUDE

The Calculation of Longitude. The calculating of the longitude was a more complicated problem, necessitating the solution of the PZX spherical triangle, as described in Chapter XIII. Longitude has been defined as angular distance measured east or west from the Greenwich meridian, and can be measured in time as well as in arc. If, then, the Greenwich time is known and the observer's time calculated, the difference between them will give the longitude in time, which can then be readily converted into arc. The observer's time can be deduced from the hour angle ZPX (Fig. 124) by means of the time equation, H.A.T.S. – E = L.M.T. When the sun is east of the observer's meridian, as in Fig. 124, its westerly hour angle (H.A.T.S.) is found by subtracting angle ZPX from 24 hours. If it is P.M., the sun is shown west of the observer's meridian and ZPX *is* its westerly hour angle (H.A.T.S.). The difference between G.M.T. and L.M.T. gives the longitude.

Circle of Position. Notice that because Sumner's latitude was in error he distrusted his calculated longitude. What he meant, of course, was that the side PZ was in error, and hence the hour angle ZPX also. In his anxiety to find the limits of his error he calculated the hour angles corresponding to possible errors in his first PZ of 10′ and 20′. He naturally plotted the consequent positions on a chart and found the three of them to lie in a straight line which, when produced, passed through the Smalls Light, so that if he had used the co-latitude of the Smalls Light in his hour angle calculation, he would have obtained the longitude of that point. This implied that the sun's altitude must have been the same at all four positions at the same instant. We know that, if Sumner had continued to vary the co-latitude, or PZ in further calculations for hour angle and longitude, the line joining the positions so found would not have continued in a straight line but formed an arc along which the sun's altitude, and hence its co-altitude, or zenith distance as it is more often called, would be the same at the same instant. If the zenith distance be regarded as the radius of the arc, it follows that the sun's position must be at the centre of a circle along which the sun's altitude must have been the same at the same instant. This is a circle of position. Sumner's line was such a small part of it that it appeared straight.

Geographical Position. If you were to suspend a plumb-line and cause the lead to circulate, it would trace out circles of varying radii comparable with circles of position. The angle between the plumb-line and the horizontal would also vary, and would compare with a celestial body's altitude, being least when the circle was greatest, and at its maximum—*i.e.*, 90°, when the circle had shrunk to a mere point. At this last rather rare stage in practical astronomical navigation there can be only one possible position for the observer —*viz.*, at that point on the earth where a line joining the object's centre to the earth's centre would penetrate the earth's surface. This spot, x in Fig. 135, is known as the object's **geographical position,** or sub-solar, sub-stellar, sub-lunar or sub-planetary point, according to the type of body observed, and to the observer at x the object would for the instant be directly overhead. To plot this position on the chart would be a comparatively easy matter, as the observer's latitude ax would correspond to the object's declination AX, and his longitude if west, to the object's Greenwich hour angle BA, or if east to 24 hours —the G.H.A. (*e.g.*, 24 hours – G.H.A. of Z = BC). If the circle were quite small it might be possible to plot part of it satisfactorily on the same large-scale chart as the object's geographical position, particularly in low altitudes where the scale is more uniform, by using its zenith distance in minutes of arc as the radius of the position circle in nautical miles (see Fig. 146).

Position Line by Longitude and Azimuth. The position circle in itself, however, would not determine the observer's position. Some means of establishing his particular point on the circle is necessary, and this is given by the reversed bearing of the object—*i.e.*, the bearing of the observer from the object. The position at which this line cuts the circle must obviously be the observer's position, and it should be remembered that the radius meets the circle, or tangent to it, at right angles. When the circle is large and the plotting of any part of it on the same chart as the object's geographical position is impossible, a small part of the arc will coincide for practical purposes with the tangent. Hence we speak of position lines. This is by far the most common case in practice. The radius of the position circle generally exceeds the dimensions of the chart, and yet an accurate means of establishing connexion between the observer and the object's geographical position must be found. Sumner discovered that by assuming himself to be in different latitudes one true altitude of the sun would provide a series of longitudes, which, when plotted with their respective latitudes, gave a locus of positions lying in a straight line. But Sumner did not know that this straight line was at right angles to the true bearing, or azimuth, of the sun, otherwise the laborious method, known as Sumner's Method, of calculating two separate positions for each altitude and joining them would have been discarded long before it was. A quicker way and one still extant, is to use the D.R. latitude with the object's altitude and declination to solve the PZX triangle for the hour angle ZPX, and so deduce a longitude. Then find the azimuth, angle PZX, a line at right angles to which, when drawn through the position given by the D.R. latitude and calculated longitude, would be the position line.

Example. On July 9 at 12 hrs. 24 mins. 08 secs. G.M.T. in latitude by D.R. 51° 10′ N. the true altitude of the sun's centre above the celestial horizon was 56° 27′ east of the meridian. Required, the longitude and the direction in which to draw the position line.

We must first deduce the values of the sides in the PZX triangle before we can proceed to its solution for the hour angle ZPX.

True altitude = $56^\circ\ 27'$	latitude = $51^\circ\ 10'$ N.	declination at 12·4 hrs. G.M.T. = $22^\circ\ 22\tfrac{1}{2}'$ N.
$90^\circ\ 00'$	$90^\circ\ 00'$	$90^\circ\ 00'$
$ZX = 33^\circ\ 33'$	$PZ = 38^\circ\ 50'$	$PX = 67^\circ\ 37\tfrac{1}{2}'$
		$PZ = 38^\circ\ 50'$
		$PX \sim PZ = 28^\circ\ 47\tfrac{1}{2}'$

The Use of a Figure. An approximate solution may be obtained by means of a scale drawing illustrating the problem, as explained in Chapter XII, but it must be remembered that no projection can represent a spherical triangle satisfactorily and, as one degree of arc represents 60 nautical miles, accuracy according to navigational standards is unattainable by this means. Such a method is valuable, however, in that it serves a very useful purpose as a demonstration of the principles of the problem and provides a basis of reference from

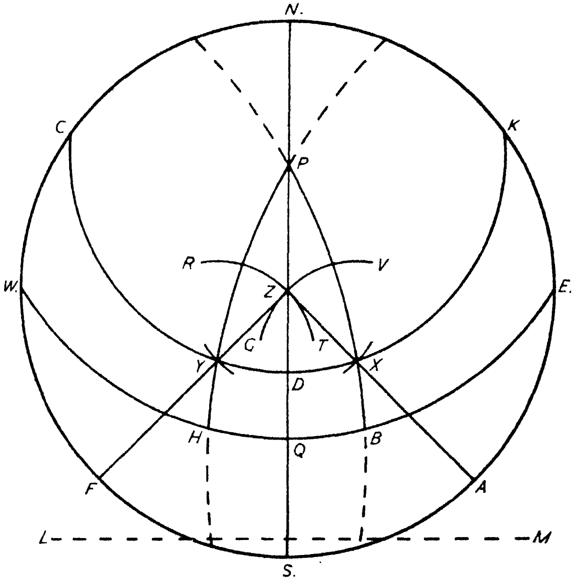


Fig. 139. The Longitude Method of Position Lines

which the mathematical solution may be approached. In Fig. 139 NESW represents the observer's celestial, or rational, horizon, Z the observer's zenith, NZS his meridian, and ZQ his latitude, 51° 10′ N., drawn to scale on the equidistant projection. The semi great circle passing through Q and the E and W points of the horizon then represents the equinoctial, QD represents the sun's declination 22° 22½′ N. to scale, EK and WC the sun's amplitude when rising and setting respectively, and KDC the circle of declination. To obtain the sun's position on the circle of declination, or path for the day, its zenith distance 33° 33′ is taken to scale and used as a radius with Z as centre. The point of intersection of the two circles will indicate the position of the sun and is marked X. The vertical arc ZX produced meets the horizon at a point A, so that AX, being complementary to ZX, represents the sun's true altitude. To complete the PZX triangle, P is joined to X by an hour circle whose centre will lie on LM, the locus of the points P and X, and must be found by trial. When drawn it should be produced to meet the equinoctial at B, as the arc QB will then be a measure of the sun's easterly hour angle ZPX. On measuring, QB is found to be about a quarter of QE, a quarter of 6 hours —*i.e.*, 1 hr. 30 mins.—but it will be found by calculation to be 1 hr. 28 mins. 47 secs. The azimuth PZX is found from the drawing to measure N. 136° E., but by calculation is nearly N. 141° E. It can be seen, therefore, that although the degree of accuracy is almost as good as can be expected from a plane drawing of a sphere, it is not accurate enough to compare favourably with results obtainable by calculation. The arc GZV in the diagram represents a portion of the position circle whose radius is XZ.

We now proceed to calculate the sun's hour angle and azimuth from the triangle PZX, and so deduce the longitude and position line. The calculation of hour angle and longitude:

The sun's local hour angle ZPX is found from the haversine formula.

$$\text{Haversine ZPX} = \frac{\text{haversine ZX} - \text{haversine (PX} \sim \text{PZ)}}{\text{sine PX} \times \text{sine PZ}}$$

which is developed as follows:

ZX = 33° 33′	nat. hav. = 0·08330
(PX ∼ PZ) = 28° 47½′	nat. hav. = 0·06181
	difference = 0·02149
	log. diff. = 8·33219
PX = 67° 37½′	log. cosec. = 10·03399
PZ = 38° 50′	log. cosec. = 10·20269
ZPX = 1 hr. 28 mins. 47 secs. E.	log. hav. = 8·56887

24 hrs. — E.H.A. = H.A.T.S.

	hrs.	*mins.*	*secs.*	
H.A.T.S. =	22	31	13	H.A.T.S. − E = L.M.T.
E at 12·4 hrs. G.M.T. =	11	54	59	
L.M.T. July 9 =	10	36	14	
G.M.T. July 9 =	12	24	08	Greenwich time best, longitude west.
Long. in time =	1	47	54 W.	1 hour = 15°
Longitude =	26° 58½′ W.			

(General Exercises No. VIII.)

Calculation of the Azimuth:

Having found one angle the azimuth PZX may be found by means of the sine formula

$$\frac{\text{sine PZX}}{\text{sine PX}} = \frac{\text{sine ZPX}}{\text{sine ZX}}$$

I.e., sine PZX = sine PX × sine ZPX × cosec. ZX
but, because the sine of an angle reaches its maximum value at 90° and is positive up to 180°, mistakes may arise due to ambiguity, especially when the azimuth is near 90°. The haversine formula should, therefore, be used throughout to avoid confusion.

$$\text{Haversine PZX} = \frac{\text{haversine PX} - \text{haversine (PZ} \sim \text{ZX)}}{\text{sine PZ} \times \text{sine ZX}}$$

This is arranged for calculation as follows:

PX = 67° 37½′	nat. hav. = 0·30967	PZ = 38° 50′
PZ ~ ZX = 5° 17′	nat. hav. = 0·00212	ZX = 33° 33′
	difference = 0·30755	PZ ~ ZX = 5° 17′
	log. diff. = 9·48792	
PZ = 38° 50′	log. cosec. = 10·20269	
ZX = 33° 33′	log. cosec. = 10·25754	
PZX = 140° 48′	log. hav. = 9·94815	

The angle at the observer between the elevated pole and the object east of the meridian is 140° 48′.
True azimuth = N. 140° 48′ E. and the position line will run at right angles through Z in the direction of N. 51° E.—S. 51° W. to the nearest degree. (General Exercises, No. X.)

Plotting a Position Line. The next step is to plot the position line on either a Mercator chart showing the D.R. latitude or, if that is not available, on squared or plain paper. The chart supplied with this volume includes latitude 51°, and would be suitable for plotting the position lines of this problem. Such charts may be used to solve problems coming within the limits of their latitudes in either hemisphere, although for the southern hemisphere the chart should be disposed so that the latitude graduations increase downward and not upward as in the northern hemisphere. As the longitude scale is constant, any degree of longitude may be ascribed to a particular meridian to suit the demands of the problem. So in this case longitude 2° E. on the chart may be marked 26° W., longitude 1° E. 27° W. and so on, always taking care to let west longitude increase to the left and east longitude to the right. Charts, however, have a habit of wearing out and recourse may have to be had to a home-made edition. This will provide no difficulties if the technique of construction, explained in Chapter III, has been learned. Fig. 140 illustrates the working of the foregoing example on such a chart, and the method of solving the problem should be followed carefully. The point A indicates the position given by the D.R. latitude 51° 10′ N., and the longitude, found by observation of the sun, 26° 58½′ W. A✱ represents the sun's azimuth N. 141° E. and at right angles to it N. 51° E. – S. 51° W., PAL, marked by single arrows, is the position line along which the observer's actual position must be.

When using squared paper we first select a point situated centrally or, if necessary, in such a position that the position line may, after plotting, be carried forward in the direction in which the vessel is moving. The side of each small square on the paper represents a minute of latitude, or a nautical mile, so that departure, distance made good east or west, must be converted into difference of longitude and vice versa, as explained in Chapter VII. The longitude by observation of the sun, combined with the latitude by D.R., is the position A through which the position line is plotted, as shown in Fig. 141.

The Value of a Single Position Line. One position line, however, is limited in its usefulness. It may be produced to meet a coastline or pass close to some navigational landmark, when a course may be steered along it to make a landfall, or a course may be set to reach a line parallel to it through the destination. A position on it may even be established by soundings, and it may serve as a check on the ground speed, but otherwise it must be combined either immediately or within a reasonable period of time with a second position line, obtained by another observation of a celestial object, by a radio bearing or by a visual bearing of a terrestrial object.

Second Position Line from a Simultaneous Celestial Observation. To fix the vessel's position in the above example a second position line was necessary, so, as Jupiter was suitably placed, a simultaneous observation was taken to ascertain its true altitude. This was found to be 56° 57′ west of the observer's meridian, and a second longitude was obtained for use with the D.R. latitude to determine the position through which to draw the second position line provided by the observation of Jupiter. On the scale drawing (Fig. 139) the procedure is the same as for the sun. The declination and altitude are approximately the same as those of the sun, and their circles intersect at Y, whose hour angle is found to measure 1 hr. 30 mins. west, and azimuth N. 136° W. The same formulæ and method are used to calculate the hour angle and longitude as in the first half of the example and need not be repeated, the calculation only being given below.

True altitude = 56° 57′
90° 00′
ZY = 33° 03′

latitude = 51° 10′ N.
90° 00′
PZ = 38° 50′

Declination = 21° 16′ N.
90° 00′
PY = 68° 44′
PZ = 38° 50′
PY ~ PZ = 29° 54′

ZY = 33° 03′	nat. hav. = 0·08090
(PY ~ PZ) = 29° 54′	nat. hav. = 0·06655
	difference = 0·01435
	log. diff. = 8·15674
PY = 68° 44′	log. cosec. = 10·03063
PZ = 38° 50′	log. cosec. = 10·20269
ZPY = 1 hr. 12 mins. 07 secs. W.	log. hav. = 8·39006
H.A. = 01 hr. 12 mins. 07 secs.	
R.A. = 04 hrs. 33 mins. 32 secs.	
R.A.M. = 05 hrs. 45 mins. 39 secs.	H.A. + R.A. = R.A.M.
R. = 19 hrs. 08 mins. 12 secs.	
L.M.T. = 10 hrs. 37 mins. 27 secs.	R.A.M. − R = L.M.T.
G.M.T. = 12 hrs. 24 mins. 08 secs.	
	L.M.T. ~ G.M.T. = longitude.
Long. in time = 1 hr. 46 mins. 41 secs. W.	
Longitude = 26° 40¼′ W.	

(General Exercises, No. IX)

For the azimuth:

PY = 68° 44′	nat. hav. = 0·31865
PZ ~ ZY = 5° 47′	nat. hav. = 0·00255
	difference = 0·31610
	log. diff. = 9·49982
PZ = 38° 50′	log. cosec. = 10·20269
ZY = 33° 03′	log. cosec. = 10·26331
PZY = 148° 04′	log. hav. = 9·96582

True azimuth = N. 148° W.
Position line = N. 58° W.—S. 58° E.

(General Exercises, No. XI.)

Plotting a Fix from Simultaneous Observations. The position thus provided by the D.R. latitude and the longitude from an observation of Jupiter, 26° 40¼′ W., is plotted at B in Fig. 140. The azimuth of Jupiter N. 148° W. (212°) is marked B*, and the position line drawn through B at right angles to it. As the observations were taken simultaneously the vessel or aircraft must be on both position lines at the same time—*i.e.*, where they intersect, the position marked C in the diagram. This is found to be latitude 51° 14′ N. longitude 26° 50′ W. In order to place position B correctly in Fig. 141, however, the difference of longitude between positions A and B—*i.e.*, 18¼′ E.—must be converted into nautical miles, as explained in Chapter VII, and measured to scale (in terms of squares) from A along their mutual line of latitude. This distance eastward, or departure, is found to be 11½ nautical miles. The corresponding azimuth and position line are then drawn through it. The point C at which

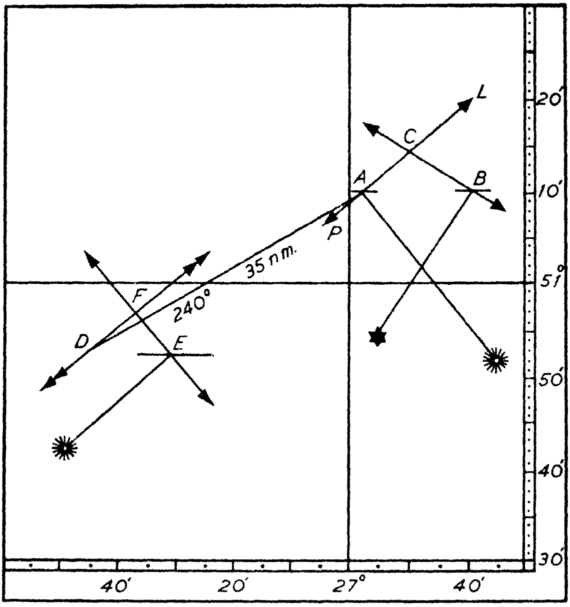


Fig. 140. Fix from Simultaneous Observations Combined with Running Fix

the two position lines cut is the actual position of the ship at 12 hrs. 24 mins. 08 secs. G.M.T. This is found from the diagram to be 4 nautical miles—*i.e.*, 4 minutes of latitude north of A—making the latitude of C 51° 14′ N., and 5 nautical miles east of A, corresponding to a difference of longitude of 8½′, and making the longitude of C 26° 50′ W., the same result as on the chart (Fig. 140). The correct L.M.T. at which the observations were taken is found by applying the longitude in time to the G.M.T. as follows:

G.M.T. =	12 hrs. 24 mins. 08 secs.
Longitude in time =	1 hr. 47 mins. 20 secs. W.
Correct L.M.T. =	10 hrs. 36 mins. 48 secs.

(General Exercises, No. XIII)

A Running Fix. If neither Jupiter nor any other body had been suitably placed for obtaining a second position line at the time of the first observation and only the sun had been available, a second observation of the sun could have been taken after an interval of time sufficient to allow a change in its azimuth of about 60°. The D.R. latitude for use with the second altitude would be found by carrying forward the first position line along the vessel's effective course, or aircraft's track, for a distance corresponding to that covered by the vessel or aircraft in the interval. This is shown in Fig. 140, where the first position line obtained by an observation of the sun is that used in the above example. Suppose the vessel to be making good a course of 240° T. at a speed of 10 knots, then, if the second observation were taken at 15 hrs. 53 mins. 49 secs. G.M.T., the true altitude being 53° 20′ west of the meridian, she would have covered a distance of 35 nautical miles in the interval. The point D, reached by measuring this distance from A in the direction 240° T., is that through which the first position line PL is transferred and the D.R. position, whose latitude is used in calculating the second longitude. The latitude of this point D is found to be 50° 52′ to the nearest minute, which is accurate enough for our purpose, as generally the latitude used may be within 30′ of the real latitude without an appreciable

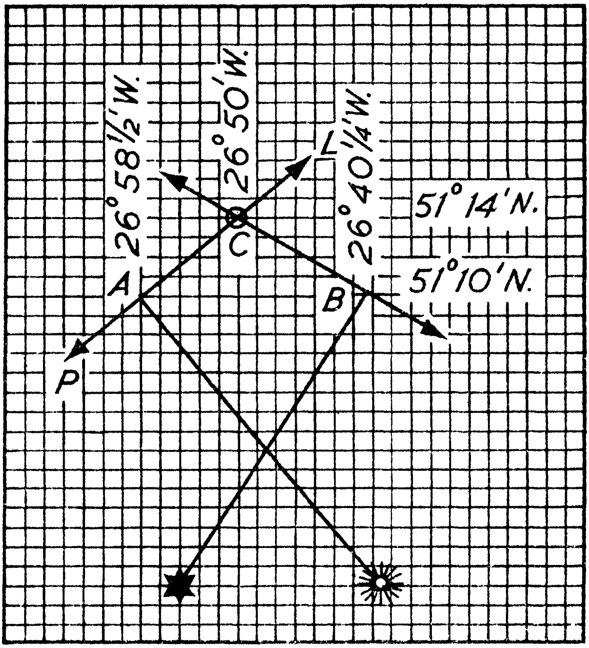


Fig. 141. Fix from Simultaneous Observations

error appearing in the final result. Tabulating the known parts, we have for calculating the hour angle and longitude:

True altitude = 53° 20′	latitude = 50° 52′ N.	declination = 22° 21·5′ N.
90° 00′	90° 00′	90° 00′
ZX = 36° 40′	PZ = 39° 08′	PX = 67° 38·5′
		PZ = 39° 08′
		PX ∼ PZ = 28° 30·5′

ZX = 36° 40′	nat. hav. = 0·09894
PX ∼ PZ = 28° 30½′	nat. hav. = 0·06063
	difference = 0·03831
	log. diff. = 8·58333
PX = 67° 38½′	log. cosec. = 10·03394
PZ = 39° 08′	log. cosec. = 10·19988
ZPX = 1 hr. 58 mins. 45 secs. W.	log. hav. = 8·81715

H.A.T.S. = 1 hr. 58 mins. 45 secs.
E = 11 hrs. 54 mins. 58 secs.

L.M.T. = 14 hrs. 03 mins. 47 secs.
G.M.T. = 15 hrs. 53 mins. 49 secs.

Long. = 1 hr. 50 mins. 02 secs. W.
Long. = 27° 30½′ W.

For calculating the azimuth and position line we use the same method and arrangement as in the previous example.

PX = 67° 38·5′	nat. hav. = 0·30980	PZ = 39° 08′
PZ ∼ ZX = 2° 28′	nat. hav. = 0·00046	ZX = 36° 40′
	difference = 0·30934	PZ ∼ ZX = 2° 28′
	log. diff. = 9·49044	
PZ = 39° 08′	log. cosec. = 10·19988	
ZX = 36° 40′	log. cosec. = 10·22391	
PZX = 129° 54½′	log. hav. = 9·91423	

True azimuth = N. 129° 54·5′ W.
Position line = N. 40° W.—S. 40° E.

The position obtained from the second observation of the sun is indicated by E in Fig. 140, and the azimuth by E✸, at right angles to which the second position line is drawn through E. This position line, and the transferred position line marked by two arrows at each end, intersect at F, which marks the position of the vessel at the time of the second observation, and is found to give latitude 50° 57′ N. longitude 27° 37½′ W.

Fig. 142 shows how the same details are worked out on squared paper. The difference between the longitude found by the second observation of the sun and the longitude of A is 32′, corresponding to 20 nautical miles. The point through which the second position line should be drawn is situated on the same line of latitude as D and 20 squares to the left, or westward, of A. It is marked E, and the position line is drawn through it in the direction at right angles to the true azimuth—*i.e.*, N. 40° W.—S. 40° E. The first position line through A is then transferred, according to the track and distance covered between observations, and drawn parallel through D. Where the transferred position line and the second position line intersect at F is the position of the vessel at the time of taking the second observation. From Fig. 142 the difference of latitude between A and F is 13′ of latitude, making the latitude of F 50° 57′ N. and the departure 24·5 nautical miles west, corresponding to a difference of longitude of 39′, making the longitude of F 27° 37·5′ W. This agrees with the result already found. (General Exercises, No. XIV.)

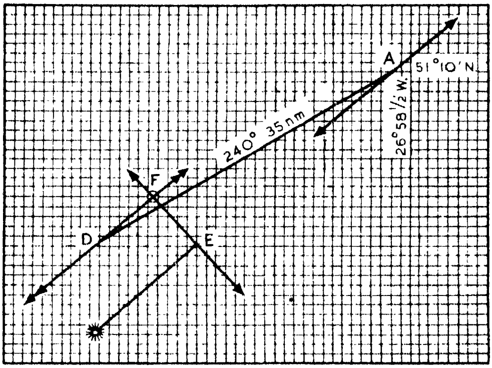


Fig. 142. The Running Fix

Separate observations of two bodies differing sufficiently in azimuth will provide a running fix when simultaneous observations are not possible.

Celestial and Terrestrial Position Lines Compared. It will be noticed that the foregoing method of transferring a position line is the same as that described in Chapter VIII for the running fix. The differences are that one deals with position lines found by observations of *celestial* bodies, the distance covered during the interval being comparatively large, while the other deals with position lines found by observations of *terrestrial* objects, the distance run between them being small. The technique, however, is the same.

The considerable distance of an observer from the geographical position of a celestial body constitutes an obstacle to finding the position from one observation. As explained in Chapter VIII, when a bearing is taken of a terrestrial object and the reciprocal to the true bearing drawn through the object a position line is obtained, along which the actual position is found by measuring the vertical angle subtended by the object and hence ascertaining its distance from the observer. The distance of an observer from a celestial body's geographical position is the same as that between the observer's zenith and the celestial body itself, both being the complement of the body's altitude above the celestial horizon as found by sextant angle. But, although a corrected compass bearing of a terrestrial object suffices to provide a position line, the bearing of a celestial body, as calculated by azimuth, would have to be plotted with minute exactness to avoid errors liable to introduction over the great distance existing between the body's geographical position and the observer. Such precision in the drawing of angles is not possible on a chart, and, even if it were, the size of chart required to cover the distances involved would be enormous except at a scale too small to be practicable. It must be remembered that when a body's true altitude is 45° its geographical position is 2700 nautical miles away from the observer—*i.e.*, $\frac{1}{8}$ the earth's circumference.

Marcq St Hilaire Method of Position Lines. As a direct method of finding the position is not possible, the indirect method of determining and combining position lines by means of assuming one or more approximate values still persists. But an advance in technique was made when Captain Marcq St Hilaire, of the French Navy, developed a method in which not only was the latitude assumed but also the longitude. This meant that in the PZX triangle, two sides, the observer's co-latitude and the object's polar distance, were regarded as known, together with the included angle, the hour angle, computed from the assumed longitude. From these data the zenith distance was calculated and compared with the zenith distance obtained from actual observation of the object's altitude. As might be expected a difference generally occurs, which is a measure of the error in the assumed position, and constitutes the difference in length between the radius of the position circle passing through the assumed position and the radius of the position circle passing through the actual position. This difference is known as an **intercept.** If the true zenith distance by observation is greater than that by calculation the observer must be farther away than assumed, if, on the other hand, the true zenith distance is less, he must be nearer than assumed. The difference in minutes of arc is, therefore, measured in nautical miles from the assumed position along the great circle joining it to the object—*i.e.*, along the azimuth circle or line of bearing whose direction angle PZX, is calculated at the same time as the zenith distance. Strictly speaking the straight line drawn on a chart or on squared paper to represent the object's bearing is not an arc of a great circle but a rhumb line. The distance is so small, however, that for practical purposes the two may be taken as coincident. It may also be asked whether the azimuth would be the same at both assumed and actual positions, the answer being that, provided the intercept is small—not more than about 60'—no appreciable change in direction can occur at moderate altitudes. When the D.R. position is used the intercept is often quite small, no more than 10', but if a short method by tabulated values is preferred, necessitating the assumed latitude and hour angle to be an integral degree, as is customary in the R.A.F., the intercept may be as much as 40'. The position circle is always a small circle.

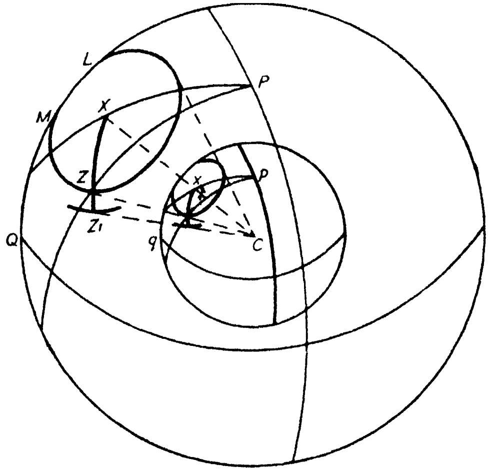


Fig. 143. The Position Circle

Graphical Illustration. The foregoing explanation is clearly illustrated in Fig. 143, in which the outer circle represents the celestial sphere and the inner concentric circle, the earth, of which C is the centre. X represents the body whose geographical position is at the point x, where the line joining C to X penetrates the earth's surface. P is the elevated celestial pole and PX the polar distance. Z represents the observer's assumed position projected on to the celestial sphere, and PZ its co-latitude. The angle ZPX will then be the body's local hour angle. ZX, the zenith distance from the assumed position, is calculated from the haversine formula using PX, PZ, and ZPX as known parts. XZ is then the radius of the position circle LZM, and in order to establish the direction of X from Z the angle PZX, the azimuth, is also calculated by means of the haversine formula. Supposing the true zenith distance by observation to be greater than the calculated zenith distance, the actual position of the observer would be along XZ produced at a distance in nautical miles from Z equal to the amount of intercept in minutes of great-circle arc—*i.e.*, the difference between the calculated and true zenith distances or their complements the corresponding altitudes. The circle described with the true zenith distance as radius, would be the observer's actual position circle, a portion of which is shown passing through Z_1. A very small portion would be almost identical with a straight line and would be plotted as such on a chart or squared paper, at right angles to the azimuth. As in the longitude method, two such position lines would be required to obtain a fix. (General Exercises, No. XVI.)

Nautical Example. On January 4 at 11 hrs. 28 mins. 52 secs. G.M.T. in a position by D.R. of latitude 19° 40′ S. longitude 53° 58′ E., the sun's true altitude west of the observer's meridian was 47° 56′.

Calculate the zenith distance and the azimuth, and hence find the intercept and the direction of the position line.

The haversine formula for finding the third side of a spherical triangle when the other sides and their included angle are known must be used to calculate the zenith distance (side ZX). It is given in Chapter XIII, and is also the same as that required for calculating distance along a great-circle track.

Viz., hav. ZX = hav. ZPX × sin. PX × sin. PZ + hav. (PX $\sim$ PZ).

Before proceeding to the calculation we must first deduce the values of the co-latitude (side PZ), the polar distance (side PX), and their included angle, the local hour angle of the sun (H.A.T.S.). Both the *Nautical Almanac* and the *Air Almanac* are used for finding the hour angle.

D.R. latitude = 19° 40′ S.
90° 00′
PZ = 70° 20′

dec. at 11 hrs. 28 mins. 52 secs.
G.M.T. = 22° 44′ S.
90° 00′
PX = 67° 16′
PZ = 70° 20′
PX $\sim$ PZ = 3° 04′

Nautical Almanac

	days	*hrs.*	*mins.*	*secs.*
G.M.T. =	4	11	28	52
Long. E =		3	35	52
L.M.T. =	4	15	04	44
E at 11 hrs. 28 mins. 52 secs. =		11	55	01
H.A.T.S. =		02	59	45

Air Almanac

G.H.A. of sun for 11 hrs. 28 mins. 52 secs. = 350° 58′
Long. E = 53° 58′
L.H.A. of sun = 404° 56′
L.H.A. of sun = 44° 56′
L.H.A. of sun 02 hrs. 59 mins. 44 secs.

(General Exercises, No. XVIII*a*)

The working is then set out as follows:

ZPX =	44° 56′	log. hav. =	9·16537		
PX =	67° 16′	log. sin. =	9·96488		
PZ =	70° 20′	log. sin. =	9·97390		
		sum =	9·10415		
		antilog. of sum =	0·12710		
PX ~ PZ =	3° 04′	nat. hav. =	0·00072	True alt. =	47° 56′
ZX =	41° 53¾′	nat. hav. =	0·12782		90° 00′
True zen. dist. =	42° 04′			True zen. dist. =	42° 04′
Intercept =	10¼′ away				

For the azimuth, the haversine formula as for the longitude method is used.

$$\text{Hav. PZX} = \frac{\text{hav. PX} - \text{hav. (PZ} \sim \text{ZX)}}{\text{sine PZ} \times \text{sine ZX}}$$

PX =	67° 16′	nat. hav. =	0·30678	PZ =	70° 20′
PZ ~ ZX =	28° 26¼′	nat. hav. =	0·06033	ZX =	41° 53¾′
		difference =	0·24645	PZ ~ ZX =	28° 26¼′
		log. diff. =	0·39173		
PZ =	70° 20′	log. cosec. =	10·02610		
ZX =	41° 53¾′	log. cosec. =	10·17537		
PZX =	77° 31′	log. hav. =	9·59320		

True azimuth = S. 77·5° W. (257·5°)
Position line runs S. 12·5° E.—N. 12·5° W. (167·5°—347·5°)

How the Position Line is Plotted. The D.R. or assumed position is now plotted on a chart (see Fig. 144) or on squared or plain paper (see Fig. 145), and the azimuth drawn through it. The intercept is measured from the assumed position *away*—*i.e.*, in the reciprocal direction to the azimuth, and a line through this terminal point at right angles to the azimuth gives the position line.

The Second Observation. As no other celestial body was available for a simultaneous observation with the sun, a second position line had to be obtained at a later time. The sun had not changed sufficiently in azimuth before it became too low to be useful, and finally set. A star was therefore sought which would provide a position line as nearly at right angles as possible to the first one. Alpheratz, although not the brightest star available, was the one most suitably placed.

Example. At 15 hrs. 31 mins. 36 secs. G.M.T. on the same day, when the vessel had covered a distance of 46 n.m. along a course 235° T. (S. 55° W.) since the first position line was determined, the true altitude of Alpheratz was found to be 35° 27′.

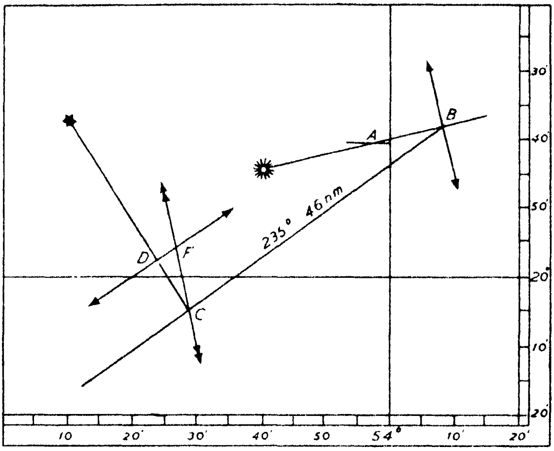


Fig. 144

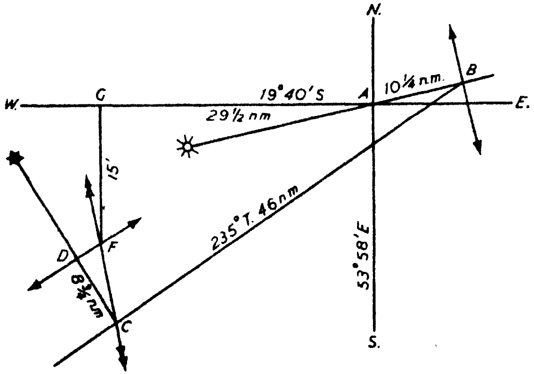


Fig. 145

Plotting the course and distance run from position B (Fig. 144) the D.R. position at the time of observing Alpheratz was found to be 20° 04′ S. longitude 53° 29′ E. (position C in Fig. 144).

The sun's position line through B is now transferred through C, and marked with two arrows at each end to indicate that it has been brought forward.

Using the haversine formula as before, the calculation to determine the second position line is as follows:

Latitude =	20° 04′ S.	declination =	26° 55′ N.
	90° 00′		90° 00′
PZ =	69° 56′	PX =	116° 55′
		PZ =	69° 56′
		PX ~ PZ =	46° 59′

Nautical Almanac

	hrs.	*mins.*	*secs.*
G.M.T. =	15	31	36
long. E =	3	33	56
L.M.T. =	19	05	32
R for 15 hrs. 31 mins. 36 secs. =	06	55	23
R.A.M. =	02	00	55
R.A. Alpheratz =	00	05	20
L.H.A. Alpheratz =	01	55	35

Air Almanac

S.H.A. Alpheratz =	358° 40′
G.H.A. ♈ =	336° 45′
G.H.A. Alpheratz =	695° 25′
long. E =	53° 29′
L.H.A. Alpheratz =	748° 54′
less =	720° 00′
L.H.A. Alpheratz =	28° 54′
	01 hr. 55 mins. 36 secs.

(General Exercises, No. XVIII*b*)

To calculate the zenith distance:

$$\text{Hav. ZX} = \text{hav. ZPX} \times \text{sin. PX} \times \text{sin. PZ} + \text{hav. (PX} \sim \text{PZ)}$$

ZPX =	28° 54′	log. hav. =	8·79426		
PX =	116° 55′	log. sin. =	9·95020		
PZ =	69° 56′	log. sin. =	9·97280		
		sum =	8·71726		
		antilog. of sum =	0·05215		
PX ~ PZ =	46° 59′	nat. hav. =	0·15889	True alt. =	35° 27′
ZX =	54° 41¾′	nat. hav. =	0·21104		90° 00′
True z.d. =	54° 33′			True zen. dist. =	54° 33′
Intercept =	8¾′ towards				

To calculate the azimuth:

$$\text{Hav. PZX} = \frac{\text{hav. PX} - \text{hav. (PZ} \sim \text{ZX)}}{\text{sin. PZ} \times \text{sin. ZX}}$$

PX = 116° 55′	nat. hav. =	0·72635	PZ = 69° 56′
PZ ~ ZX = 15° 14¼′	nat. hav. =	0·01758	ZX = 54° 41¾′
	difference =	0·70877	PZ ~ ZX = 15° 14¼′
	log. diff. =	9·85050	
PZ = 69° 56′	log. cosec. =	10·02720	
ZX = 54° 41¾′	log. cosec. =	10·08826	
PZX = 148° 08′	log. hav. =	9·96596	

True azimuth = S. 148° W. (328°)
Position line runs — N. 58° E.—S. 58° W. (058°—238°)

Fixing the Position. The star's azimuth is now drawn in from C and 8¾ nautical miles measured along it to D. A line through D at right angles to the azimuth will represent the second position line, and its point of intersection F with the first position line transferred, will give the position at the time of taking the second observation. This is found to be latitude 19° 55′ S. longitude 53° 26½′ E. The same result would have been achieved by plotting on plain paper as shown in Fig. 145, in which A, being the D.R. position, is used as the reference point, NAS representing its meridian and WAE its line of latitude. A convenient distant scale is 10 nautical miles to one inch. When B has been plotted by marking the intercept of 10¼ nautical miles away, along the reciprocal of the sun's azimuth, and the position line drawn in, the direction of BC is obtained by making the angle ABC correspond to the difference between the sun's azimuth and the course—*i.e.*, 257½° − 235° = 22½° = ABC. The distance of 46 nautical miles is then measured from B in the direction 235° T. to establish the position of C, the assumed position for the second observation, and through which the first position line is transferred. The azimuth of Alpheratz is 328°, so that CD makes an angle of 93° with BC produced and provides the direction along which the intercept of 8¾ nautical miles is measured. The latitude and longitude of F, the point where the position lines intersect, are found by measuring the difference of latitude FG and the departure AG. These are 15 nautical miles and 29½ nautical miles respectively, making the latitude of F 19° 40′ S. + 15′ = 19° 55′ S. and the longitude 53° 58′ E. − 31½′ = 53° 26½′ E. This result is the same as that obtained by plotting on the chart in Fig. 144. (General Exercises, Nos. XIX, XX, and XXIII.)

Aeronautical Example. On July 9 at 22 hrs. 48 mins. 03 secs. G.M.T. the navigator of an aircraft, in a position by D.R. latitude 53° 35′ N. longitude 03° 50′ E., found the true altitude of Arcturus to be 33° 25′. At 22 hrs. 52 mins. 31 secs. he found the true altitude of Dubhe to be 38° 32′. Calculate the intercept and azimuth in each case, and find by plotting the position of the aircraft at the time of taking the second observation, assuming the aircraft to be flying at a ground speed of 180 knots along a track of 300° T.

Arcturus: G.M.T. 22 hrs. 48 mins. 03 secs.

S.H.A.✱ = 146° 45′ from the *Air Almanac.*
G.H.A. ♈ = 269° 29′ from the *Air Almanac* for G.M.T. of observation.

G.H.A.✱ = 416° 14′
less 360° 00′

G.H.A.✱ = 56° 14′
long. E = 3° 50′

L.H.A.✱ = 60° 04′ = ZPX Hour angle always west unless otherwise stated.

(General Exercises, No. XVIII*b*)

In the triangle PZX, to find ZX:

Hav. ZX = hav. ZPX × sin. PX × sin. PZ + hav. (PX ∼ PZ)

D.R. latitude =	53° 35′ N.	declination =	19° 29′ N.
	90° 00′		90° 00′
PZ =	36° 25′	PX =	70° 31′
		PZ =	36° 25′
		PX ∼ PZ =	34° 06′

In air navigation the intercept is usually found by taking the difference between the true and calculated altitudes. When the true altitude is **greater** the intercept is **towards**; when the true altitude is **less** the intercept is **away.** The calculated altitude is the complement of the calculated zenith distance.

ZPX =	60° 04′	log. hav. =	9·39881
PX =	70° 31′	log. sin. =	9·97439
PZ =	36° 25′	log. sin. =	9·77353
		sum =	9·14673
		antilog. of sum =	0·14019
PX ∼ PZ =	34° 06′	nat. hav. =	0·08597
ZX =	56° 47½′	nat. hav. =	0·22616
	90° 00′		
Calculated altitude =	33° 12½′		
True altitude =	33° 25′		
Intercept =	12½′ towards		

For the azimuth:

$$\text{Hav. PZX} = \frac{\text{hav. PX} - \text{hav. (PZ} \sim \text{ZX)}}{\text{sin. PZ} \times \text{sin. ZX}}$$

PX =	70° 31′	nat. hav. =	0·33323	PZ = 36° 25′
PZ ~ ZX =	20° 22½′	nat. hav. =	0·03128	ZX = 56° 47½′
		difference =	0·30195	PZ ~ ZX = 20° 22½′
		log. diff. =	9·47994	
PZ =	36° 25′	log. cosec. =	10·22647	
ZX =	56° 47½′	log. cosec. =	10·07744	
PZX =	102° 28′	log. hav. =	9·78385	

True azimuth = N. 102° 28′ W. or, 257½°

The D.R. position at 22 hrs. 48 mins. 03 secs. G.M.T. is now plotted on the example chart at the end of the book, and marked C. The azimuth of Arcturus 257° is drawn in and 12½′ (nautical miles) marked along it in the direction of A. This point, marked T, is that through which the first position line is drawn, and by moving it forward along a track of 300° T. for a distance of 13½ n.m., corresponding to the distance covered in 4 mins. 28 secs. at 180 knots, we arrive at point P, which may then be used as the assumed position for the second observation. As the actual position is not known to a few miles, however, any approximate position may be taken as the assumed one, and C may be found more convenient because the position of P need not then be taken off. The calculated altitudes, and hence the intercepts, found in each case would, of course, be different but this does not affect the final result materially. The azimuth would be approximately the same. C has, therefore, been used in the example as the assumed position for finding the position line from both observations—*i.e.*, of Dubhe as well as of Arcturus.

The assumed position when using Astronomical Navigation Tables is that which takes the latitude as the nearest complete degree to the D.R. latitude, and adjusts the longitude to make the L.H.A. a complete number of degrees.

Dubhe: G.M.T. 22 hrs. 52 mins. 31 secs.
S.H.A.✱ = 194° 59′ from the *Air Almanac.*
G.H.A.♈ = 270° 37′ from the *Air Almanac* for G.M.T. of observation.

G.H.A.✱ = 465° 36′
less 360° 00′

G.H.A.✱ = 105° 36′
long. E = 3° 50′

L.H.A.✱ = 109° 26′ = ZPX

In the triangle PZX, to find ZX:

$$\text{hav. ZX} = \text{hav. ZPX} \times \text{sin. PX} \times \text{sin. PZ} + \text{hav. (PX} \sim \text{PZ)}$$

D.R. latitude =	53° 35′ N.	declination =	62° 04′ N.
	90° 00′		90° 00′
PZ =	36° 25′	PX =	27° 56′
		PZ =	36° 25′
		PZ ∼ PX =	8° 29′
ZPX =	109° 26′	log. hav. =	9·82371
PX =	27° 56′	log. sin. =	9·67066
PZ =	36° 25′	log. sin. =	9·77353
		sum =	9·26790
		antilog. of sum =	0·18531
PX ∼ PZ =	8° 29′	nat. hav. =	0·00547
ZX =	51° 48′	nat. hav. =	0·19078
	90° 00′		
Calculated altitude =	38° 12′		
True altitude =	38° 32′		
Intercept =	20′ towards.		

For the azimuth:

$$\text{Hav. PZX} = \frac{\text{hav. PX} - \text{hav. (PZ} \sim \text{ZX)}}{\text{sin. PZ} \times \text{sin. ZX}}$$

PX =	27° 56′	nat. hav. =	0·05825	PZ =	36° 25′
PZ ∼ ZX =	15° 23′	nat. hav. =	0·01791	ZX =	51° 48′
		difference =	0·04034	PZ ∼ ZX =	15° 23′
		log. diff. =	8·60573		
PZ =	36° 25′	log. cosec. =	10·22647		
ZX =	51° 48′	log. cosec. =	10·10466		
PZX =	34° 12′	log. hav. =	8·93686		

True azimuth = N. 34° 12′ W., or 326°

The line CD on the chart represents part of the azimuth circle of Dubhe, and CK represents the intercept. The second position line is drawn through K at right angles to the azimuth and intersects the transferred first position line in F, which gives the position of the aircraft at 22 hrs. 52 mins. 31 secs. G.M.T. as 53° 42′ N. 3° 09′ E.

(General Exercises, Nos. XIX, XX, and XXI)

Plotting Position Arcs. Fig. 146 illustrates the following problem and shows how position arcs may be plotted direct on to a chart by using the star's geographical position as centre. The altitudes concerned are necessarily high and difficult to measure accurately, but provided conditions for observations are good the result may have considerable value.

Example. On January 3 at 21 hrs. 56 mins. 03 secs. G.M.T. the navigator of an aircraft in a position by D.R. 46° 45′ N. 5° 00′ E., found the true altitude of Capella to be 89° 00′. At 21 hrs. 59 mins. 02 secs. G.M.T. he observed the true altitude to be 89° 15½′ southward and at 22 hrs. 04 mins. 00 secs. it was 88° 40½′, the G.S. in the interval being 180 knots along a track 060° T.

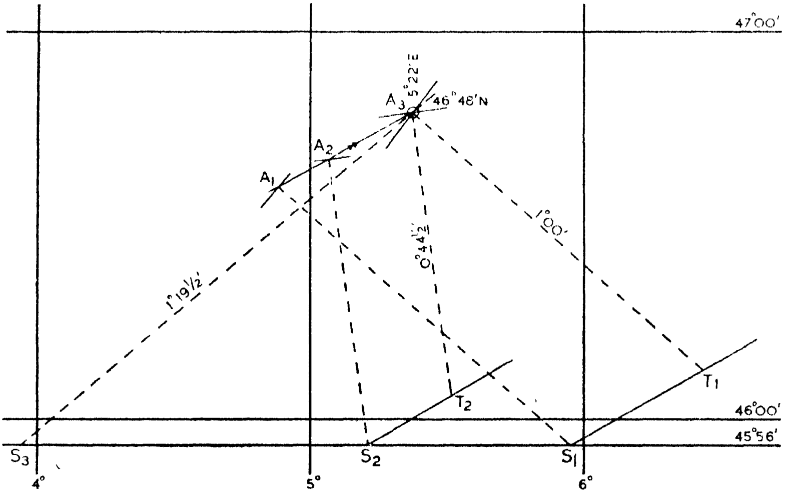


Fig. 146. Position Arcs

The declination of Capella is 45° 56′ and, in Fig. 146, S_1, S_2, and S_3 represent the successive positions of the star in its apparent movement along its circle of declination and are fixed as follows:

	S_1	S_2	S_3
G.M.T.	21 hrs. 56 mins. 03 secs.	21 hrs. 59 mins. 02 secs.	22 hrs. 04 mins. 00 secs.
G.H.A. ♈	72° 08′	72° 53′	74° 08′
S.H.A. ✱	281° 55′	281° 55′	281° 55′
G.H.A. ✱	354° 03′	354° 48′	356° 03′
	360° 00′	360° 00′	360° 00′
Long. E.	5° 57′	5° 12′	3° 57′

With centre S_1 and radius equal to the zenith distance found at the first observation—*i.e.*, 60′—an arc is drawn in the vicinity of the

D.R. position of the aircraft, which aircraft must at that time have been situated somewhere along this arc. Similarly arcs with radii $44\frac{1}{2}'$ and $79\frac{1}{2}'$ are drawn about their respective centres S_2 and S_3, in the neighbourhood of the aircraft's D.R. position at 21 hrs. 59 mins. 02 secs. G.M.T. and 22 hrs. 04 mins. 00 secs. G.M.T. These separate position arcs may then be combined in a running fix, to find a final position at the time of taking a third observation by transferring the first or second arc in the direction of the aircraft's track for a distance corresponding to the ground speed and the interval of time which has elapsed since that particular observation was taken. The transfer is effected by moving the centre S_1 or S_2 in the direction of the track to T_1 or T_2 respectively and redrawing the arc. The place at which the transferred first or second arc intersects the third arc will be the position of the aircraft at the time of taking the third observation. This is seen to be the point A_3 in latitude 46° 48′ N. longitude 5° 22′ E., and the actual positions at the earlier observations may be found by plotting the reversed track from A_3 and noting where it cuts the other arcs. These are marked A_2 and A_1 respectively.

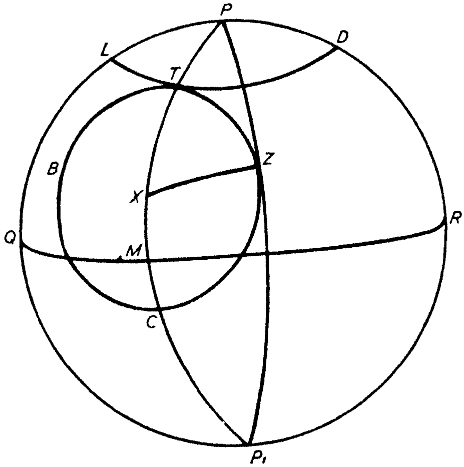


Fig. 147. LATITUDE OR LONGITUDE FROM ONE OBSERVATION

Coincidence of Position Line with Parallel of Latitude. Should the bearing of a celestial object happen to be 000° or 180° at the time of measuring its altitude, the resulting position line would coincide for practical purposes with a line of latitude, although both lines are, except in the case of the equator, arcs of small circles which will almost certainly differ in size and can actually coincide at only one point. This is seen from Fig. 147, in which BTZC represents a circle of position coinciding with a parallel of latitude LTD at T. The latitude, angular distance north or south of the equator, is then represented by MT, and is found by adding the celestial body's declination MX to its meridional zenith distance TX. To an observer at C, who also would have the celestial body on his meridian, the latitude would be found by subtracting the declination MX from the zenith distance CX.

Coincidence of Position Line with Meridian. The position line obtained by observing the altitude of a body bearing east or west will run north and south, and, although the circle of position, being a small circle, cannot coincide with a meridian, a small portion of each at their point of contact may be taken to coincide. This is illustrated in Fig. 147, where Z represents the observer's position when the bearing or azimuth of the object, angle PZX, is 90°—*i.e.*, the object is situated on the observer's prime vertical. This is the best time at which to measure the altitude of a body for ascertaining the longitude from one observation.

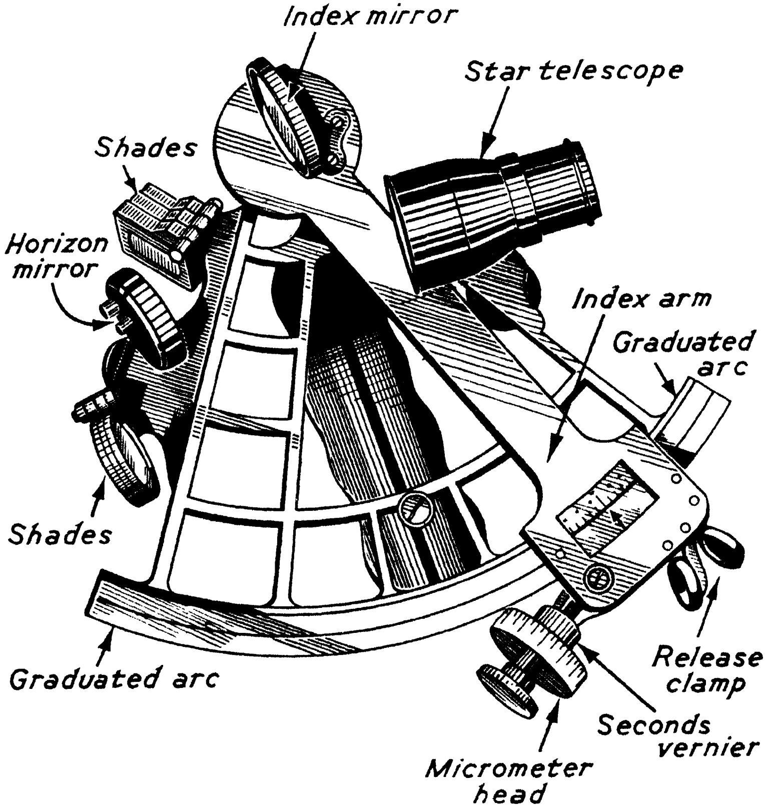


Fig. 148. Marine Pattern Sextant

The Sextant. The sextant is the modern instrument for measuring the altitudes of celestial bodies. It evolved out of the quadrant, and its essential features are two mirrors, one fixed and one movable, and a scale or other device for measuring the angle between them. There are two types in use to-day, the marine sextant (Fig. 148), which requires the use of the visible horizon above which to measure altitudes, and the bubble sextant (Fig. 150), incorporating an artificial horizon.

Marine Pattern. Fig. 148 illustrates a modern type of marine sextant with its fixed horizon mirror and movable index mirror fitted at the pivot of the index arm which, rotating like a radius about the centre of the arc, turns the mirror. Both mirrors should be perpendicular to the plane of the instrument, but whereas the index mirror is fully silvered the horizon mirror is half clear to allow a direct view of the horizon to be obtained. The angle through which the mirror moves can be measured by graduations on the arc, which is fitted with a vernier for accurate measurement, and a microscope for reading the small divisions, unless a micrometer screw-gauge is attached for reading the minutes of arc. The angle between the two mirrors, however, is only half the angle measured, because when a ray of light is reflected twice, as at I and H (Fig. 149), the angle between the first ray and last reflection is equal to twice the angle between the mirrors. The extent of the arc need then be only half the measure of the maximum angle required. This is proved as follows. Because MN is perpendicular to the mirror I, angle XIM = angle MIH, the angle of incidence being equal to the angle of reflection. Also, because NK is perpendicular to the mirror H, angle IHN = angle NHA.

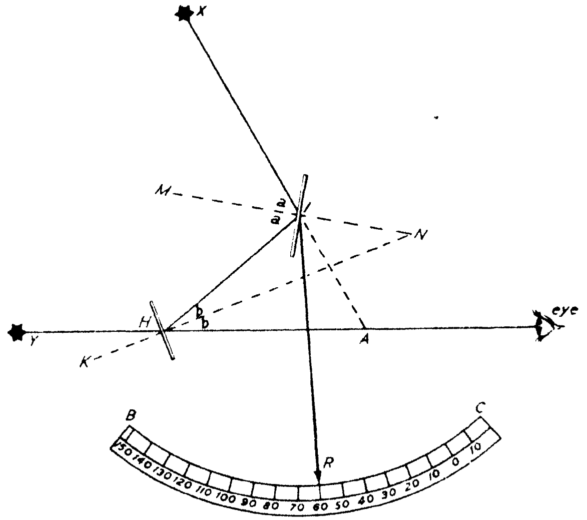


FIG. 149. PRINCIPLE OF THE SEXTANT

Now, in the triangle INH, $a = b + \mathrm{INH}$

$\therefore \quad 2a = 2b + 2\,(\mathrm{INH})$

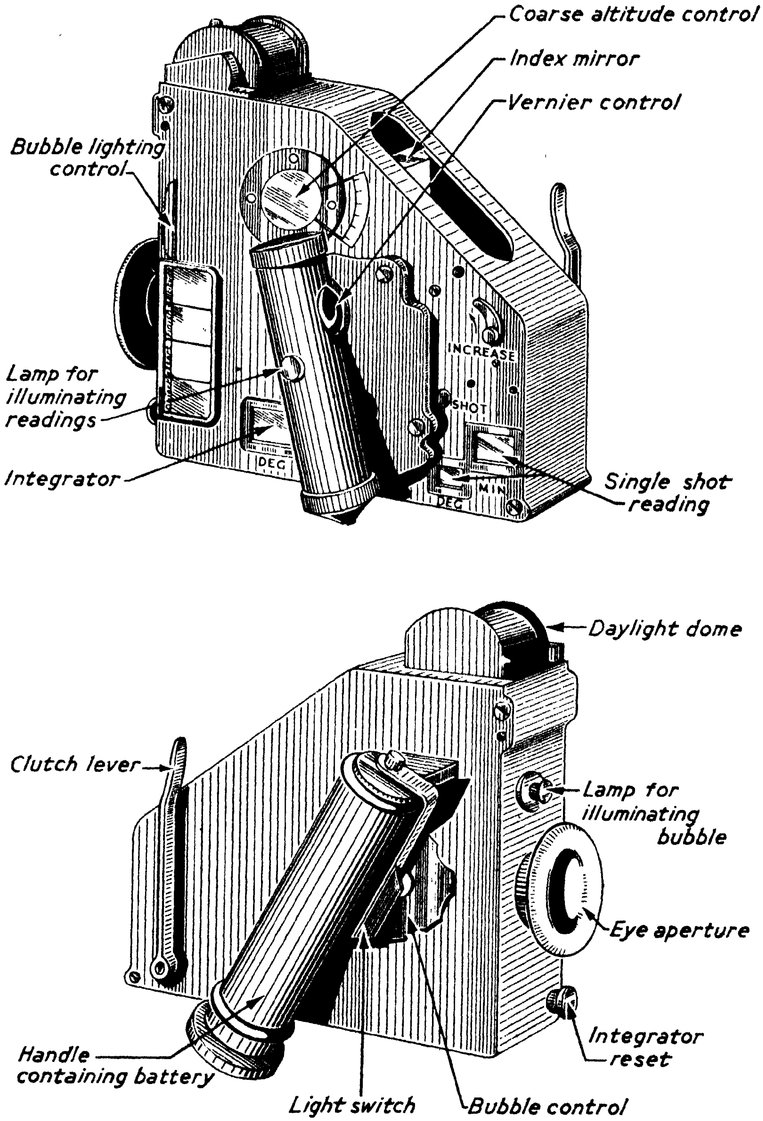


Fig. 150. THE BUBBLE SEXTANT

In the triangle IAH, $2a = 2b + IAH$

$\therefore$ angle IAH = 2 angle INH—*i.e.*, the angle between the first ray and the second reflection is equal to twice the angle between the mirrors.

Bubble Sextant. The bubble sextant is used principally in the air, where the natural horizon is often too distant to be distinct. Mirrors similar to those of the marine sextant provide the reflection of the celestial body, as shown in Fig. 151, but the mirror H is now entirely silvered on one side and allows no view of the horizon. The horizontal direction is provided by the reflection of a bubble B, which acts as in a spirit-level. Natural light passing through a dome above the bubble, or artificial light from an electric bulb, projects an image of the bubble through the mirror M on to the collimating lens and reflector assembled at L, which reflects back the image in parallel rays. These, on striking the mirror L at an angle of 45°, are partly reflected to the eye in a horizontal direction and so provide the artificial horizon necessary as a datum plane, above which altitudes of celestial bodies can be measured. The image of the body is thereby brought into contact with the bubble, and the angle between both directions is measured by the movement of the mirror I. This is controlled by a mechanism designed also to act as an adding machine, whereby in the Mark IX model a series of six readings is totalled and averaged, or, as in the Mark IXA, the average is given of continuous measurements taken over a period of two minutes. This is a considerable advance mechanically on the marine type sextant, from which Mark IX and Mark IXA sextants developed, but the marine type still scores over the bubble sextant in that it is capable of giving measurements of horizontal angles.

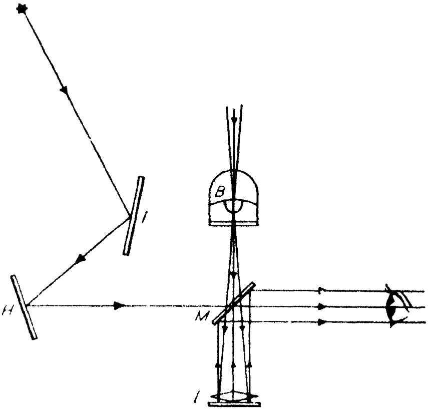


Fig. 151. THE ARTIFICIAL HORIZON

Correction of Observed Altitudes. Certain corrections are necessary to altitude readings in order to arrive at a body's true altitude. Some of these, applicable for the most part to the marine sextant, are illustrated by Fig. 152, in which C represents the centre of the earth and B the centre of a celestial body; O represents the observer; and OH his height; OV indicates the direction of the **visible horizon,** and the angle BOV is a measure of the observed altitude of the body's centre. Should the body be the sun or moon, either the lower or the upper edge, better known as the lower or upper limb, would be brought into contact with the visible horizon when measuring its altitude, necessitating the addition or subtraction of the angle subtended at the observer by the body's **semi-diameter** (S.D.), represented by the angle BOL, in order to obtain the altitude of the body's centre. As celestial measurements require to be referred to the one definite point of the earth—*i.e.*, the centre—altitudes must be deduced above a plane passing through the centre, and for each observer this means the plane parallel to his tangent plane. In Fig. 152 the tangent plane, known as the **sensible horizon,** is indicated by KOS, and the parallel plane, known as the celestial, or **rational horizon,** by MCR. To reduce BOV to BCR we must first find BOS by subtracting the **dip,** the angle of depression between the sensible and visible horizons, and then add the **parallax,** the angle at the body subtended by a line joining the observer's position to the earth's centre. The only other correction required to altitudes obtained by the marine sextant is **refraction,** the deflection of rays of light as they pass through atmosphere of differing density. This makes the object appear to be in the direction of F instead of B, so that refraction is always subtractive. In correcting observed altitudes of stars only dip and refraction need be applied, but for the sun, semi-diameter must also be considered, while for the moon parallax is an additional correction. Parallax for the sun and planets is so small that it may be disregarded in practice. For the moon it may be as much as 61·5′ when that body is on the horizon, but decreases as the cosine of its

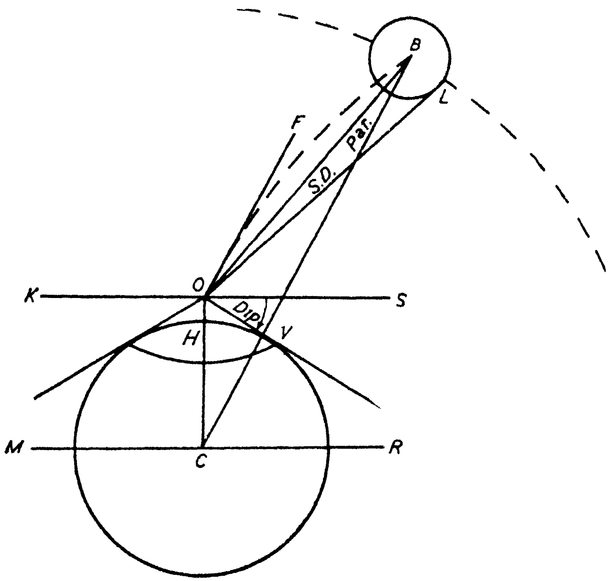


Fig. 152. Altitude Corrections

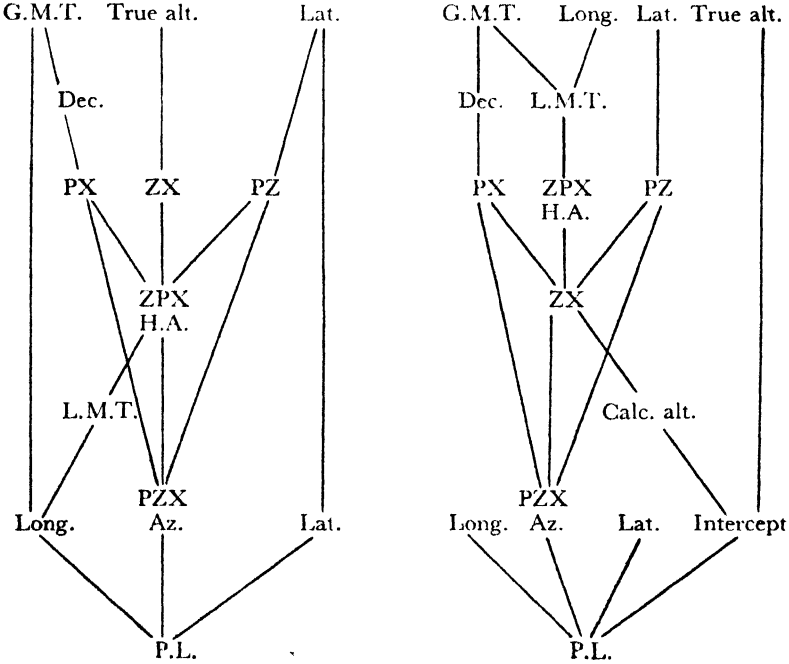
altitude, becoming nil at altitude 90°. Observations taken with the bubble sextant require no correction for dip, and at high altitudes in the air refraction becomes quite small, if not negligible. Further, the semi-diameter correction is not necessary, owing to the object being brought into actual contact with the bubble. A correction peculiar to the bubble sextant, however, is made necessary by displacement of the bubble itself, due to the aircraft's velocity. This is known as the 'Z' correction, and reaches its maximum when the observation is taken at right angles to the longitudinal axis of the aircraft.

Looking Ahead. During recent years, progress in the construction of navigational instruments has been very considerable, and the budding navigator may look forward with confidence to much simplification in his work by means of mechanical assistance hitherto undreamed of. For example, the astrograph projects position lines from a film direct on to a chart, and the astrocompass supplies true bearings by solving the astronomical triangle mechanically. Whatever aids are devised, however, the principle of position-line navigation is likely to remain fundamental.

SUMMARY

Determination of Position Lines

Longitude Method *Marcq St Hilaire Method*



General Exercises

1. Explain fully and illustrate: (*a*) a position line; (*b*) a circle of position.

2. What do you understand by the term geographical position in connexion with celestial bodies?

3. How may a position line be obtained from an observation of a celestial body?

4. What is the value of a single position line?

5. Under what conditions may you expect the longitude to be correct as the result of one observation?

6. In temperate latitudes what position line would result from an observation of Polaris?

7. How may latitude be obtained direct?

Draw figures on the plane of the observer's meridian and deduce his latitude from the following information:

	Cel. body	*Greenwich date*	*True Mer. altitude*	*Bearing*
(*a*)	Sun	January 5 10.00 hrs.	38° 40′	South
(*b*)	Canopus		57° 52′	South
(*c*)	Moon	January 27 10.36 hrs.	34° 46′	South
(*d*)	Venus	January 14 04.00 hrs.	48° 22′	South
(*e*)	Vega		66° 24′	North
(*f*)	Capella	lower transit	25° 36′	North
(*g*)	Achernar	lower transit	23° 12′	South
(*h*)	Sun	July 3 16.00 hrs.	44° 19′	North
(*i*)	Mars	January 21 20.00 hrs.	62° 14′	North
(*j*)	Moon	July 9 18.00 hrs.	53° 32′	North

8. Complete the following table:

	G.M.T.			*Local H.A.T.S.*			*E*			*Longitude*
	hrs.	*mins.*	*secs.*	*hrs.*	*mins.*	*secs.*	*hrs.*	*mins.*	*secs.*	
(*a*) May 14	18	12	40	03	24	16	12	03	47	—
(*b*) November 26	11	36	22	02	35	14	12	12	42	—
(*c*) January 5	05	30	00	04	27	10		—		—
(*d*) July 2	22	17	38	23	00	50		—		—
(*e*) January 8	16	41	11		—			—		153° 48′ E.
(*f*) July 7	02	12	20		—			—		45° 20′ W.

9. Complete the following table:

	G.M.T.			*Star*	*H.A.*			*R*			*Longitude*
	hrs.	*mins.*	*secs.*		*hrs.*	*mins.*	*secs.*	*hrs.*	*mins.*	*secs.*	
(*a*) June 4	12	17	00	Denebola	03	09	00	16	50	08	—
(*b*) October 21	23	42	16	Vega		—		02	00	05	65° 20′ W.
(*c*) March 15	15	11	27	Fomalhaut	18	46	15	11	31	19	—
(*d*) July 8	04	44	08	Achernar	01	02	33		—		—
(*e*) January 5	11	32	40	Rigel		—			—		105° 15′ E.
(*f*) July 4	07	24	36	Hamal		—			—		32° 45′ W.

10. From the following data find the longitude and azimuth in order to obtain the position line:

	G.M.T. *hrs. mins. secs.*	*D.R. lat.*	*Sun's True alt.*
(*a*) January 4	07 56 10	25° 16′ S.	40° 32′ east
(*b*) July 7	12 15 24	51° 42′ N.	36° 14′ east
(*c*) July 2	10 58 32	10° 12′ S.	32° 24′ west
(*d*) January 6	19 11 48	32° 24′ N.	21° 08′ west

11. From the following data find the longitude and azimuth in order to obtain the position line:

	G.M.T. *hrs. mins. secs.*	*D.R. lat.*	*Star's True alt.*
(*a*) January 1	05 13 35	53° 20′ N.	Capella 52° 20′ west
(*b*) July 8	20 51 10	22° 48′ S.	Achernar 45° 36′ east
(*c*) January 7	22 34 40	14° 06′ N.	Aldebaran 51° 08′ east
(*d*) July 3	17 16 25	35° 40′ S.	Antares 34° 48′ west

12. On January 4 at 20.45 hrs. G.M.T. in an assumed position 52° 00′ N. 4° 07′ E., the true altitude of Pollux was found to be 42° 17′. About five minutes later the latitude by Polaris was 52° 20′. Find the longitude at the time of taking the first observation if the track were 090° T.

13. Find the positions of the observer from the following data:

	D.R. lat.	*Observation of :*	*Bearing*	*Long.*
(*a*)	45° 12′ N.	Regulus	228°	gives 35° 30′ W.
	45° 12′ N.	Arcturus	123°	gives 35° 18′ W.
(*b*)	25° 06′ S.	Achernar	205°	gives 9° 45′ E.
	25° 06′ S.	Canopus	143°	gives 9° 58′ E.
(*c*)	12° 56′ N.	Vega	049°	gives 84° 09′ E.
	12° 56′ N.	Antares	160°	gives 83° 55′ E.
(*d*)	10° 04′ S.	Altair	285°	gives 34° 16′ W.
	10° 04′ S.	Fomalhaut	213°	gives 34° 02′ W.
(*e*)	54° 20′ N.	Deneb	103°	gives 15° 24′ W.
	54° 20′ N.	Dubhe	328°	gives 15° 10′ W.

14. Find position of observer at second observation from following data:

	D.R. lat.	*Observation of:*	*Bearing*	*Long.*	*Run between Observations*	
(*a*)	38° 42′ S.	Sun	050° gives	161° 48′ W.	240° T.	30 n.m.
		Sun	330° gives	162° 18′ W.		
(*b*)	27° 42′ N.	Sun	250° gives	62° 14′ W.	300° T.	48 n.m.
		Capella	334° gives	63° 12′ W.		
(*c*)	53° 25′ N.	Moon	202° gives	32° 08′ W.	080° T.	35 n.m.
		Sun	095° gives	31° 02′ W.		
(*d*)	15° 06′ S.	Canopus	218° gives	98° 20′ E.	140° T.	45 n.m.
		Sun	100° gives	98° 36′ E.		
(*e*)	68° 00′ N.	Sun	080° gives	14° 10′ E.	045° T.	40 n.m.
		Sun	142° gives	15° 40′ E.		
(*f*)	34° 12′ S.	Sun	055° gives	179° 13′ W.	260° T.	50 n.m.
		Sun	315° gives	179° 32′ E.		

15. Find the longitude and azimuth from the following observations and plot the position lines to obtain a fix:

(*a*)

G.M.T. February 23 16 hrs. 05 mins. 24 secs. (R = 10 hrs. 12 mins. 37 secs.)
D.R. lat. 35° 17′ N.
True alt. of Spica west of meridian 24° 43′.
True alt. of Altair east of meridian 38° 35′.

(*b*)

G.M.T. October 8 21 hrs. 05 mins. 12 secs. (R = 1 hr. 08 mins. 24 secs.)
D.R. lat. 14° 00′ S.
True alt. of Canopus east of meridian 35° 54′.
True alt. of Fomalhaut west of meridian 29° 06′.

16. What is an intercept?

17. Find the intercept in the following cases:

	Calculated zenith distance	*True zenith distance*
(*a*)	37° 14′	37° 05′
(*b*)	56° 52′	56° 59′
(*c*)	44° 03′	43° 51′
(*d*)	60° 55′	61° 06′
	Calculated altitude	*True altitude*
(*e*)	28° 25′	28° 17′
(*f*)	53° 06′	52° 57′
(*g*)	70° 34′	70° 40′
(*h*)	48° 59′	49° 12′

18. Complete the following tables:

(*a*)

		G.M.T.			*G.H.A. Sun*	*Longitude*	*L.H.A. Sun*
		hrs.	*mins.*	*secs.*			
(i)	August 12	22	37	15	158° 06′	105° 12′ W.	—
(ii)	February 25	02	20	15	211° 45′	99° 15′ E.	—
(iii)	July 28	18	45	15	99° 44′	—	37° 20′
(iv)	January 21	23	33	00	170° 22′	—	325° 00′
(v)	January 4	00	38	07	—	145° 44′ W.	—
(vi)	July 9	09	20	26	—	10° 58′ E.	—
(vii)	July 9	09	51	15	—	—	11° 16′
(viii)	January 4	21	12	36	—	—	21° 36′
(ix)	July 9	13	33	48	—	—	343° 53′
(x)	January 4	00	04	22	—	172° 14′ E.	—

(*b*)

		G.M.T.			*S.H.A.*✱	*G.H.A.*♈	*Long.*	*L.H.A.*✱
		hrs.	*mins.*	*secs.*				
(i)	March 30	03	18	15	194° 58′	350° 23′	47° 20′ W.	—
(ii)	September 9	00	57	15	259° 21′	20° 00′	60° 45′ E.	—
(iii)	January 29	07	51	03	Procyon	220° 32′	—	35° 20′
(iv)	July 17	22	05	20	Aldebaran	266° 40′	—	306° 48′
(v)	July 9	13	36	55	Altair	—	145° 50′ W.	—
(vi)	January 4	15	29	37	Achernar	—	134° 32′ E.	—
(vii)	July 9	02	15	40	Betelgeuse	—	—	295° 51′
(viii)	January 4	10	02	46	Regulus	—	—	27° 24′

19. Find the observer's actual position from the following data:

	Assumed position *Lat.*	*Long.*	*Body*	*Calc. alt.*	*True alt.*	*Bearing*
(*a*)	40° 15′ N.	35° 52′ W.	Denebola	51° 38′	51° 30′	238° T.
			Antares	15° 09′	15° 15′	146° T.
(*b*)	37° 25′ S.	131° 17′ E.	Canopus	55° 05′	55° 15′	229° T.
			Betelgeuse	23° 07′	23° 01′	300° T.
				Calc. Z.D.	*True Z.D.*	
(*c*)	46° 11′ S.	80° 36′ W.	Venus	60° 48′	60° 40′	263° T.
			Fomalhaut	22° 22½′	22° 12′	310° T.
(*d*)	10° 50′ N.	59° 24′ E.	Capella	42° 10½′	42° 18′	028° T.
			Sirius	56° 35′	56° 44′	118° T.
(*e*)	32° 20′ N.	64° 45′ W.	Moon	56° 40′	56° 48′	239° T.
			Mars	63° 05′	62° 59′	148° T.

20. Find the observer's position at the time of the second observation.

	Assumed position *Lat.*	*Long.*	*Body*	*Calc. Alt.*	*True Alt.*	*Bearing*	*Run*
(*a*)	19° 40′ S.	53° 58′ E.	Sun	30° 28½′	30° 17′	311° T.	235° T. 46 n.m
	20° 14′ S.	53° 27′ E.	Alphacca	33° 43½′	33° 52′	034° T.	
(*b*)	39° 32′ S.	179° 40′ W.	Antares	45° 39′	45° 50′	271° T.	240° T. 56 n.m.
	40° 00′ S.	179° 16′ E.	Sun	23° 58′	23° 53′	049° T.	
(*c*)	20° 19′ S.	00° 12′ E.	Sun	38° 40′	38° 47′	037° T.	320° T. 59 n.m.
	19° 34′ S.	00° 28′ W.	Sun	40° 37′	40° 29′	325° T.	
				Calc. Z.D.	*True Z.D.*		
(*d*)	35° 07′ N.	19° 52′ E.	Vega	48° 57½′	49° 07′	067° T.	095° T. 51 n.m.
	35° 02½′ N.	20° 54′ E.	Sun	62° 14′	62° 07′	150° T.	

21. Using the *Air Almanac*, find by means of intercepts the position at the time of observations in the following problems:

	G.M.T.	*hrs.*	*mins.*	*secs.*	*D.R. position* *Lat.*	*Long.*	*Body*	*True alt.*
(*a*)	July 9	23	24	00	54° 20′ N.	00° 50′ E.	Altair	42° 07′
							Arcturus	29° 48′
(*b*)	January 4	19	02	16	54° 10′ N.	4° 35′ E.	Betelgeuse	26° 25′
							Dubhe	32° 36′
(*c*)	January 4	04	10	22	51° 20′ N.	46° 35′ W.	Procyon	43° 44′
							Capella	62° 40′
(*d*)	January 4	12	40	36	52° 25′ N.	146° 28′ W.	Regulus	49° 35′
							Aldebaran	19° 30′
(*e*)	July 9	08	21	40	50° 05′ S.	79° 40′ W.	Fomalhaut	68° 20′
							Achernar	59° 30′
(*f*)	July 9	01	22	19	51° 30′ S.	65° 10′ W.	Antares	64° 40′
							Peacock	53° 50′

22. On January 3 local time, G.M.T. 4 days 00 hrs. 55 mins. 20 secs., in a position by D.R. 30° 16′ S. 49° 40′ W., the true altitude of Jupiter was 36° 40′. At G.M.T. 01 hrs. 14 mins. 45 secs., after covering

5 n.m. on a track 210° T., the true altitude of Achernar was 47° 15′. Find by plotting on plain paper the position of the observer at the second observation.

23. Using the *Nautical Almanac*, find by means of intercepts the position of the observer in each of the following cases:

Local date	*G.M.T.*				*D.R. position*		*Body*	*True alt.*
	days	*hrs.*	*mins.*	*secs.*	*Lat.*	*Long.*		
(*a*) January 6	6	20	39	16	54° 50′ N.	16° 05′ W.	Capella	64° 30′
							Hamal	57° 34′
(*b*) January 2	3	00	15	48	54° 20′ N.	48° 35′ W.	Aldebaran	51° 22′
							Alpheratz	42° 15′
(*c*) January 7	8	04	10	24	53° 20′ S.	77° 16′ W.	Rigel	43° 42′
							Achernar	52° 40′
(*d*) January 1	1	12	16	54	49° 24′ N.	139° 42′ W.	Arcturus	28° 50′
							Regulus	52° 24′
(*e*) January 5	4	17	42	10	51° 10′ N.	169° 45′ E.	Spica	25° 24′
							Pollux	37° 34′

24. On November 1 G.M.T. was 1 day 02 hrs. 51 mins. 48 secs. (R = 02 hrs. 40 mins. 01 secs.) when the true altitude of Mars (R.A. 00 hrs. 45 mins. 43 secs.) was 47° 31′, and the true altitude of Jupiter (R.A. 05 hrs. 19 mins. 32 secs.) was 54° 16′ to an observer in a D.R. position 35° 08′ N. 41° 38′ W.

Find by plotting on plain paper the observer's actual position.

PART III

RADIO NAVIGATION

CHAPTER XVI

RADIO NAVIGATION

Electro-magnetic waves—The directional loop—Difference of phase—'Sense'—Quadrantal error—Convergency—Conversion angle—Position line from radio bearing taken by D.F. station—Fix by radio bearings from D.F. station—Position line from radio bearings taken by ship or aircraft—Fix by radio bearings taken by ship or aircraft—Large-scale plotting of position lines—Running fix—Reliability of radio position lines—Radar—The hyperbolic method of obtaining position.

Electro-magnetic Waves. In Chapter V we discussed some of the properties of magnetism, with the emphasis on the earth as a magnet. We showed how the influence of such a magnet (Fig. 37) extended over the surface of the whole earth and for some miles above it, and that ferrous metal exposed in this 'field' acquired transient polarity by means of induction.

When an electric current travels along a conductor a similar magnetic field is set up in the immediate surroundings. If the conductor is exposed horizontally in the open air it constitutes an aerial, and when the current is allowed to flow intermittently through it a series of strains, or pulsations, is produced between it and the earth. These pulsations radiate in all directions along the lines of force and become waves travelling at a speed of 186,000 miles per second, the speed of light waves. If you imagine yourself blowing soap bubbles from the end of a tube, you will get an idea of what is meant by "all directions" as you watch the bubbles expand. You may regard each concentric bubble to be the crest of a wave and their distance apart the wavelength, which, in the case of electro-magnetic waves, depends upon the rate of pulsation, or frequency. In navigation we are concerned primarily with direction along the earth's surface and need to consider the waves' direction of progress horizontally. An impression of how the waves travel horizontally may be obtained by visualizing the progress of ripples on a pond when a stone is thrown into the middle of it. The ripples are seen to form concentric circles about their point of origin, and their height, like the amplitude of electro-magnetic waves, will depend upon the initial impulse and distance from the point of origin. The number of waves which pass a fixed point in a given time is a measure of their frequency. Waves, however, must not be regarded as having any translatory movement, but merely a rise and fall.

The Directional Loop. If a stick of wood were set up vertically in a pool where waves were in motion, the height of water along the stick would not be the same at two consecutive instants but would show a continuous rise and fall. A vertical aerial would act similarly to the electro-magnetic waves issuing from a radio transmitter, as, whenever lines of magnetic force, along which the waves travel, move across a conductor, an electro-motive force is induced in it. This E.M.F. would vary in amount, and produce an alternating current in the aerial. In Fig. 153 AB represents a vertical aerial in the path of an approaching wave crest C. As the crest approaches the aerial an increase in E.M.F. will take place followed by a decrease when the crest has passed. When two vertical aerials are placed in line with the wave's point of origin the amount of E.M.F. in each will be different unless their distance apart happens to coincide with the wavelength, which is highly improbable as radio wavelengths are usually measured in terms of metres. Two such vertical aerials are represented by DK and EH (Fig. 153). If the end D be joined by a parallel conductor to the end E, and end K be similarly joined to end H, a circuit will be completed round which an electrical current will alternate as long as a difference of E.M.F. is caused to exist in the vertical members of the circuit. This current, when passed into a radio receiver, may be used to produce sound. The volume of this sound, or signal strength, will vary with the amount of current passing through the circuit, being at its maximum when the vertical portions are in line with the transmitting station and decreasing as the plane of the circuit, when rotated about the axis XX, becomes more inclined to the direction of the incoming waves. When the plane of this simple loop aerial, as DEHK has now become, faces the direction of the incoming waves no signal will be heard at all, as the vertical parts of the loop now have the same E.M.F. and no current will flow. There is then said to be no **difference of phase** between these parts.

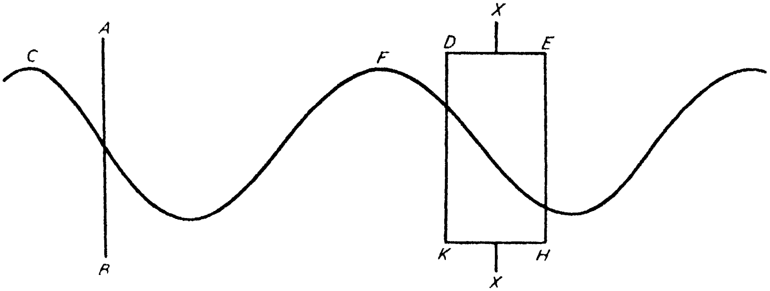


Fig. 153. The Directional Loop

According to signal strength in the receiver, therefore, we are able to judge whether the plane of the loop is in line with, or facing, the transmitter, or at some intermediate position between the two. Our loop aerial has thus become a direction finder.

Most loop direction finders fitted on ships and aircraft are now rotatable, so that they can be turned round until the signal received is at its minimum or maximum strength. It is easier to detect when the signal is absent than to detect when it is at its maximum. For this reason the direction in which the plane of the loop is facing is taken to indicate the direction of the transmitting station, and the angle between this direction and the true direction of the craft's head is measured on a bearing plate as a relative bearing—*i.e.*, from 0° at the craft's head clockwise through 360°. As explained in Chapter VIII, the relative bearing when added to the true course gives the true bearing and thus provides a position line along which the craft must be situated.

'Sense.' The plane of the loop faces in two directions, however, one being the reciprocal of the other, so that the direction of the transmitting station may be the one indicated or it may be 180° different. The operation performed to discover which of these two is correct is known as determining the 'sense.'

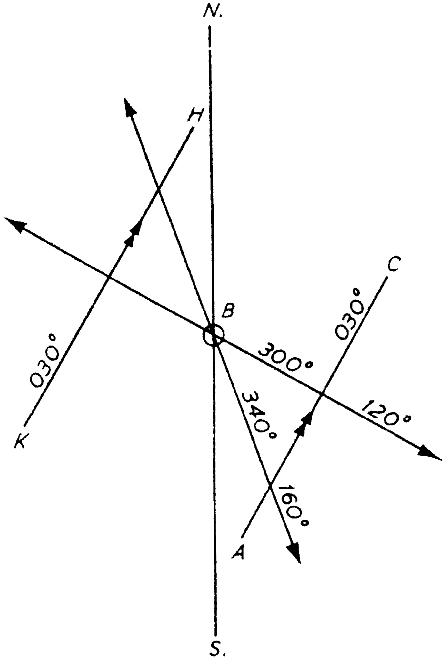


Fig. 154. 'SENSE' BY CHANGE OF BEARING

This may be done by taking two or more bearings in succession, with an interval of time between them sufficiently long to allow a change of bearing to take place. In Fig. 154 B represents the position of a radio transmitting station of which a craft on a track of 030° T.

finds the bearing by loop aerial to be 160° T., or 340° T. To determine the sense a second bearing is taken later and found to be 120° T., or 300° T. A consideration of the diagram will show that the correct bearings were 340° T. and 300° T., and that the craft was on a track AC to the eastward of the station and not on a track KH to the westward. Briefly, if the bearing decreases the station lies to port, if it increases the station lies to starboard. Should the bearing not change at all, the craft must be moving directly towards or away from the station; which of the two conditions is the correct one may be ascertained by altering course and taking further bearings. If the course is altered to port and the bearings decrease, the craft must be moving away from the station. If they increase, the craft must be moving towards the station.

Sense may also be determined mechanically by coupling the loop aerial to an open aerial whose signal strength is regulated to equal that of the loop's maximum. By reversing the current direction in the loop the two aerials may be made to combine their respective signal strengths and so produce double the previous maximum, or one may counteract the effect of the other and so produce no signal at all. Sense is then determined by reference to the original direction of current in the loop.

Quadrantal Error. When electro-magnetic waves impinge on the metallic parts of an aircraft they are reflected, and produce a local magnetic field which interferes with reception by the loop aerial. An error thus creeps into the loop reading to which a correction must be applied before adding to the true course. The amount of correction is found by swinging the aircraft over a prominent point and comparing the indicated direction with the known correct direction of a distant transmitter. The results are tabulated against each 10° interval of relative bearing, when it is found that the correction is at its maximum on the quadrantal points and nil, or 0°, 90°, 180°, and 270°—hence the name. The calibration of a loop aerial for Q.E. on one frequency is not strictly applicable to another frequency.

Convergency. Because electro-magnetic waves radiate simultaneously in all directions they travel direct from their point of origin to the receiver. They take the shortest possible route—*i.e.*, they follow the great circle passing through transmitter and receiver. True direction along a great circle is not constant, owing to the changing angle at which all great circles, except the equator, meet each meridian. The angle made by the intersection of the great circle and the transmitter's meridian will, therefore, be different from the angle made by the intersection of the great circle and the receiver's meridian unless, of course, the positions are both on the same meridian, or on the equator. This angular difference is due to the converging of meridians, and is known as **convergency.** Fig. 155 shows how the meridians through positions A and B converge towards the pole, P. The direction of the meridian through A is indicated by the line DAH, representing a very small portion of the meridian and, therefore, for practical purposes taken as straight. Similarly the direction of the meridian through B is indicated by the straight line EBF. The difference in direction between DAH and EBF is a measure of the convergency of the meridians at A and B respectively. If, then, we join A to B by a straight line representing the arc of the great circle passing through both places, the angle HAK made by the great circle at A will be found to exceed the angle FBA made by the great circle at B by an amount represented by the angle HAM, which is then a measure of the convergency. The amount of convergency depends upon (i) the mean latitude of the places concerned, being least near the equator but increasing with latitude to a maximum at the poles, and (ii) the difference, or change, of longitude between the two places. If the hypotenuse of a right-angled triangle be drawn to scale to represent the d. long. and one angle be constructed equal to the mean latitude, the side opposite this angle will represent the convergency in the same units as used for the d. long. Expressed mathematically, the convergency varies directly as the sine of the mean lat. and directly

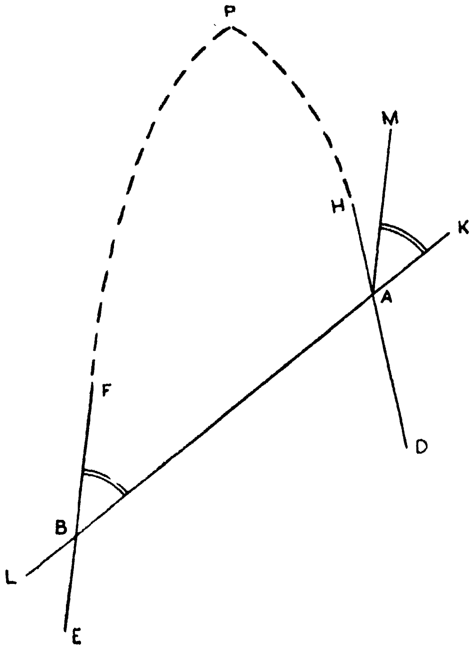


Fig. 155. CONVERGENCY

as the d. long. The product of these two variables gives the convergency—*i.e.*, convergency = sine mean lat. × d. long.

Example. What would be the convergency of the meridians between the Lizard lighthouse (50° 00′ N. 5° 12′ W.) and Bermuda (32° 15′ N. 64° 50′ W.).

Lizard lighthouse lat.	50° 00′ N.	long.	5° 12′ W.
Bermuda lat.	32° 15′ N.	long.	64° 50′ W.
Sum	82° 15′	d. long.	59° 38′
Mean lat.	41° 7½′	log. sin.	9·818030
d. long.	59·6°	log.	1·775246
Convergency	39·2°	log.	1·593276

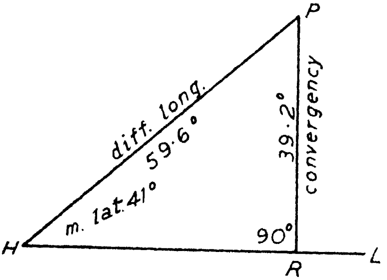


Fig. 156. How to Find the Convergency

Fig. 156 shows how the same result may be obtained graphically. An angle LHP equivalent to the mean lat.—*i.e.*, 41°—is made at one end H of the base-line LH. The line HP is then drawn to scale to represent the d. long. A perpendicular from P on to LH will then represent the convergency to the same scale. The convergency will thus always be less than the d. long., except at the poles, where they are equal.

Exercise 1

Find the convergency between the following pairs of places.

1. Ushant 48° 27′ N. 5° 05′ W.
   Fayal 38° 35′ N. 28° 45′ W.
2. C. St Vincent 37° 00′ N. 8° 58′ W.
   Barbados 13° 40′ N. 59° 50′ W.
3. C. Hatteras 35° 15′ N. 75° 30′ W.
   St Vincent 16° 50′ N. 25° 05′ W.
4. Mauritius 20° 10′ S. 57° 30′ E.
   Fremantle 32° 01′ S. 115° 45′ E.
5. Colombo 7° 00′ N. 79° 45′ E.
   Zanzibar 6° 10′ S. 39° 12′ E.
6. Halifax 44° 40′ N. 63° 40′ W.
   Trinidad 10° 30′ N. 61° 20′ W.
7. Auckland 36° 50′ S. 174° 42′ E.
   Sydney 33° 52′ S. 151° 15′ E.

Conversion Angle. Bearings obtained by loop aerial are used for finding the position of a ship or aircraft and must, like visual bearings, be plotted on a map or chart in order to provide position lines. For short distances bearings may be plotted as straight lines on either map or chart, as great circle and rhumb line are then almost coincident. Maps, however, are usually limited to comparatively small land areas, and, although great circles may be represented on them approximately by straight lines, they become inaccurate over long distances.

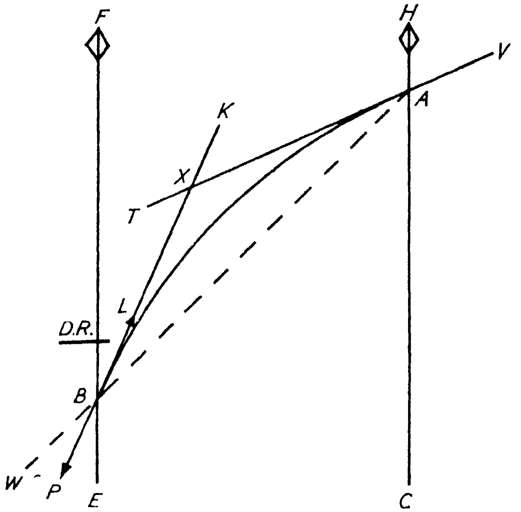


Fig. 157. Position Line when Radio Station takes the Bearing

As topographical maps are not designed for plotting the Mercator chart alone is used, but, because a great circle bearing can no longer be represented on it accurately by a straight line, a relationship must be established between great circle and rhumb line. In Fig. 157 HC represents the meridian of A on a Mercator chart, and FE the meridian of B. The straight pecked line will then represent the rhumb-line direction between A and B, and the curved line the great-circle arc joining the two places. BK is the tangent to the great circle at B, so that FBK represents the direction of the great circle at that point. AV is the tangent to the great circle at A, so that HAV represents the direction of the great circle at that point. Now, it was shown in Fig. 155 that the difference between the direction of the great circle at B and its direction at A was equivalent to the convergency of the meridians passing through the two places. In Fig. 157, therefore, HAV − FBK = the convergency. If VA be produced it will intersect

BK at a point X, so that AXK and BXT equal the convergency. Now, if the curve AB be regarded as the arc of a circle, then AX = BX and BAX = ABX. Also AXK, or BXT = BAX + ABX—*i.e.*, the convergency = 2 BAX, or 2 ABX. Therefore, the angle at a point between the great circle and rhumb line joining that point to another is equal to half the convergency, and is known as the **conversion angle.** Its value may be found in the same way as that explained for finding convergency, except that the half difference of longitude between the two places should be used instead of the full difference. A much quicker way, however, is provided on a plotting-sheet. It consists of three parallel straight lines, and is known as an **abac** (Latin, *abacus*, counting-machine). One of the outside lines is divided into ch. long. (d. long.) degrees of varying length and the other into a varying scale of mean-latitude degrees. The ch. long. and mean lat. concerned are noted on their respective lines and joined by a straight line. The conversion angle in degrees is then read off the central line at the point of intersection.

Position Line from Radio Bearing Taken by D.F. Station. If a ship not fitted with a loop aerial requires a W/T bearing, a request for it must be made to a land station. Craft which are fitted with a loop aerial sometimes find it more convenient to obtain a bearing from a land station than to take one themselves, especially when two or more widely spaced land stations in communication with each other combine their bearings to give the craft an actual position.

When a land station takes the bearing of a ship or aircraft transmitting a signal, the craft must lie somewhere along the great circle whose direction from the station is that indicated by the D.F. This is the direction submitted to the craft, but being a great-circle bearing it cannot be plotted direct on to a Mercator chart, it must be converted into a rhumb-line direction by the application of the conversion angle. To find this the craft's D.R. position must be used with the position of the shore station. A glance at Fig. 157 will indicate how the conversion angle should be applied. Because the great circle through two points always lies on the polar side of the rhumb line through the same points, **the conversion angle is always applied towards the equator** irrespective of hemisphere or whether the craft is situated to the east or to the west of the station. A rough figure will always determine the direction and avoids memorizing rules. In Fig. 157, if the bearing of B from A (angle HAT) were given as 240°, the conversion angle TAB would be applied towards the equator—*i.e.*, subtracted. If the bearing of A from B (angle FBK) were given as 030°, the conversion angle KBA would still be applied towards the equator, but this time added. A rhumb line may not be regarded as a position line, however, but it must be used to establish the locality through which the position line passes. The point at which the rhumb line intersects the D.R. meridian is regarded as a point on the position line, but, because the craft lies on the great circle, the direction of the position line must be that of the tangent at the point, and can be found by applying the conversion angle towards the equator again—as shown by angle WBP in Fig. 157, where PL is now seen to represent the craft's position line when the radio station takes the bearing.

Fix by Bearings from D.F. Station. Two position lines intersecting at a reasonable angle, exceeding 30°, will give a fix.

Example. A craft in a position by D.R. 50° 10′ N. 15° 45′ W. receives on request the following simultaneous radio bearings:

(*a*) from Valentia (51° 56′ N. 10° 19′ W.) 247°.
(*b*) from Ushant (48° 33′ N. 5° 10′ W.) 288°.

What is her position?

We must first find the conversion angle in each case and apply it to the respective radio bearing in order to obtain the rhumb-line directions. Triangles constructed on the principle of Fig. 156 will provide the values if the hypotenuse is measured to scale to represent the ½ d. long., or the same results may be found from the abac or by calculations as follows:

D.R. position	50° 10′ N.		15° 45′ W.
Valentia	51° 56′ N.		10° 19′ W.
	102° 06′	d. long.	5° 26′
		Half d. long.	2° 43′
Mean lat.	51° 03′	log. sin. =	9·890809
Half d. long	163′	log. =	2·212188
Conversion angle	127′	log. =	2·102997
Conversion angle	2° 07′		
Bearing from Valentia	247° 00′ T.		
Rhumb-line direction	244° 53′ T.		

If we now plot a line in this direction from Valentia, as shown on the small chart (Fig. 158), and note the latitude at which it crosses the D.R. meridian of 15° 45′ W., we shall obtain a position A through which to draw the position line. This is then drawn in a direction found by applying the conversion angle to the rhumb line—*i.e.*, 244° 53′ — 2° 07′ = 242° 46′—and also in the reciprocal direction 62° 46′.

We now proceed to find the second position line:

D.R. position	50° 10′ N.		15° 45′ W.
Ushant	48° 33′ N.		5° 10′ W.
	98° 43′	d. long.	10° 35′
		Half d. long.	5° 17·5′
Mean lat.	49° 21·5′	log. sin. =	9·880126
Half d. long.	317·5′	log. =	2·501744
Conversion angle	241′		
Conversion angle	4° 01′	log. =	2·381870
Bearing from Ushant	288° 00′ T.		
Rhumb-line direction	283° 59′ T.		
Conversion angle	4° 01′		
Position line	279° 58′ T.		
Position line	99° 58′ T.		

A rhumb line is therefore plotted from Ushant (Fig. 158) in a direction 284° T., and at the position B where it crosses the D.R.

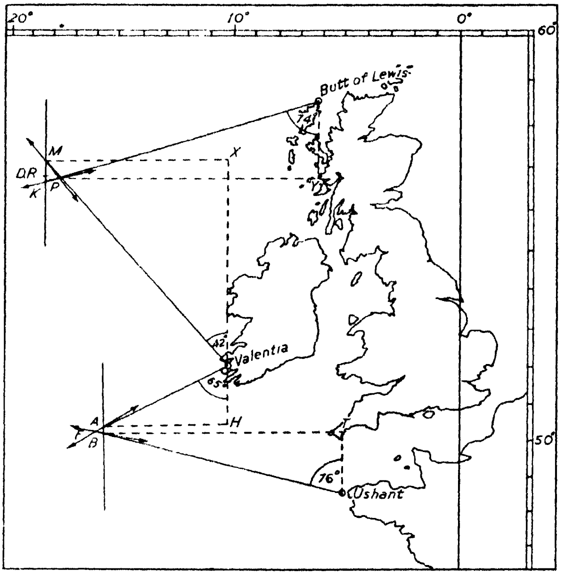


Fig. 158. Plotting Radio Position Lines

meridian the position line is drawn. This is found to intersect the first position line at a point marked F, which is the actual position of the craft at the time the bearings were taken. From the chart we obtain 50° 15′ N. 16° 00′ W. To ensure accuracy the position lines should cross at an angle of not less than 30°. Owing to the comparative nearness of the craft to the W/T D.F. stations the conversion angles are small and the position at which the rhumb lines intersect is little different from the actual position. It may, therefore, in such a case be regarded as the approximate position.

Position Line from Bearing Taken by Ship or Aircraft. When a ship or aircraft uses its own direction-finding equipment to obtain the bearing of a radio beacon the rhumb-line direction for laying off on the chart is obtained from it as before by applying the conversion angle to the initial great-circle direction. The rhumb line cannot, however, be laid off from an uncertain D.R. position in the certain knowledge that it will pass through the transmitting station, but must be reversed and laid back from the fixed position of the radio beacon. The position at which this rhumb line crosses the D.R. meridian is regarded as being a point on the position line. Suppose the true bearing, found by adding the relative bearing to the true course, were 070°, as illustrated in Fig. 159, where B represents the craft and A the beacon. The rhumb-line direction would then, in the northern hemisphere, lie to the right and, given a conversion angle of 5°, would be 075° T. The reciprocal of this would be 255° T., and a rhumb line is laid back from A in this direction intersecting the D.R. meridian at B.

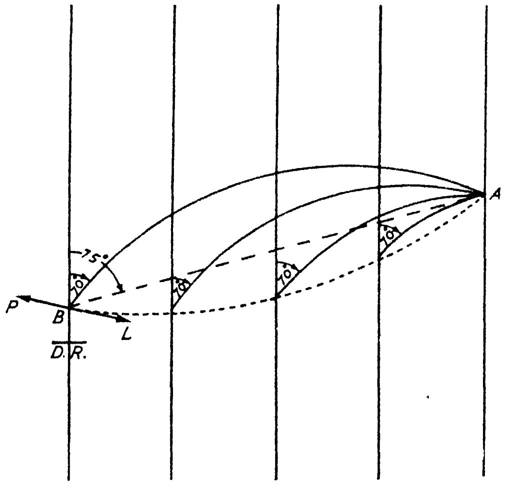


Fig. 159. Position Line when Ship or Aircraft takes the Bearing

The position line cannot lie along the rhumb line, as already explained, and, although a great circle can be drawn to pass through both A and B, there is only one point on it from which the bearing of

A from B is 070° T. Fig. 159 shows positions on different meridians from which the bearing of A is 070° T. A line joining all such points from which the bearing of an object is constant is known as a **curve of constant bearing.** The craft taking the bearing must lie along this locus. The direction of the position line passing through any point ascertained on it may be found by applying the convergency to the great-circle bearing. As B (Fig. 159) is regarded as being on the position line we apply 10°, twice the conversion angle, to the original bearing of 070°, making the direction of the position line 080° and 260°. The angles in Fig. 159 are exaggerated for the sake of clarity.

Fix by Radio Bearings Taken by Ship or Aircraft. Two position lines obtained almost simultaneously from two beacons suitably spaced would provide a fix at their point of intersection.

Example. A craft in a position by D.R. of 56° 50′ N. 18° 15′ W. steering 100° T. takes the following bearings almost simultaneously by loop aerial:

(*a*) Butt of Lewis (58° 31′ N. 6° 16′ W.) 329°.
(*b*) Valentia (51° 56′ N. 10° 19′ W.) 035°.

What is the craft's position?

The position lines are obtained respectively as follows:

	Butt of Lewis	*Valentia*
Craft's true course	100°	100°
Relative bearing	329°	035°
True bearing	429°	135°
True bearing	− 360°	
	069°	135°
Conversion angle	5° (nearest degree)	3°
Rhumb-line direction	074°	138°
Reciprocal	254°	318°
True bearing	069°	135°
Convergency	10° (nearest degree)	6°
Position line	079° 259°	141° 321°

The rhumb-line reciprocals are now laid off on the chart from their respective radio beacons (Fig. 158). The positions K and M, at which they intersect the D.R. meridian, are the points through which to plot the position lines. These are seen to cut at a point P, which we take to be the craft's position. From the chart it is found to be 56° 40′ N. 17° 40′ W.

If an appreciable period of time had elapsed between the taking of the two bearings, during which the craft had covered a number of miles, the movement would have to be allowed for as in the running fix, explained in Chapter VIII for visual bearings. It just means that the first position line would have to be carried forward—*i.e.*, transferred—in the direction and for the amount of movement in the interim. The point of intersection of the transferred position line with the second position line would then be the actual position at the time of taking the second observation. The small scale of the chart (Fig. 158) makes a clear illustration of this type of problem difficult. An enlargement of the area (Fig. 161), however, shows the necessary detail.

Large-scale Plotting of Position Lines. It will be appreciated that, owing to the general smallness of the chart scale required for such large distances as are involved in plotting position lines from radio bearings, an accurate position is not easily attainable on them. If, instead of plotting the rhumb line on a chart, we find by calculation, or by means of the traverse table, the latitude at which it crosses the D.R. meridian, we may plot the position line on a scale drawing and combine it with any other position line to find the actual position.

If you consider the first example, you will notice in Fig. 158 a right-angled triangle formed by the meridian of Valentia, the rhumb line VA through Valentia, and a parallel of latitude HA. Two parts of this triangle are known to us—*viz.*, the angle of 65° at Valentia, between the meridian and the rhumb line, and the difference of longitude 326′, between Valentia and the D.R. meridian. By using these two parts we can calculate by plane-sailing methods the difference of latitude represented by VH. This, when applied to the latitude of Valentia, will give the latitude through which the position line should be drawn. As d. lat. and d. long. are not measured in the same units, except at the equator, we must convert the d. long. into the same units as the d. lat.—*i.e.*, into departure—before being able to solve the triangle for VH. The method of conversion is explained in Chapter VII, and illustrated by Fig. 69. A figure drawn to scale will not give sufficiently accurate results, so we must proceed to find the departure by calculation:

departure = d. long. × cos. mid. lat.

As the distance does not exceed 600 miles, the mid. lat. may be regarded as equivalent to the mean lat.

Mean lat. = ½ (50° 10′ + 51° 56′)
Mean lat. = 51° 03′ log. cos. = 9·798403
d. long. = 326′ log. = 2·513218

∴ Departure = 205′ log. = 2·311621

To find the d. lat., we have from Chapter VII:

D. lat. = departure × cot. course
Departure = 205′ log. = 2·311621
Course = 65° log. cot. = 9·668672

D. lat. (VH) = 95·57′ log. cot. = 1·980293

Lat. Valentia = 51° 56′ N.
D. lat. (VH) = 1° 35·5′ S.

Lat. A (Fig. 158) = 50° 20·5′ N.

The point through which to draw the position line is therefore 50° 20·5′ N. 15° 45′ W. In Fig. 160 NS is drawn to represent the D.R. meridian of 15° 45′ W., and A is the position just established at which the rhumb line from Valentia meets it, and through which the position line must be drawn in the directions 243° T.—063° T.

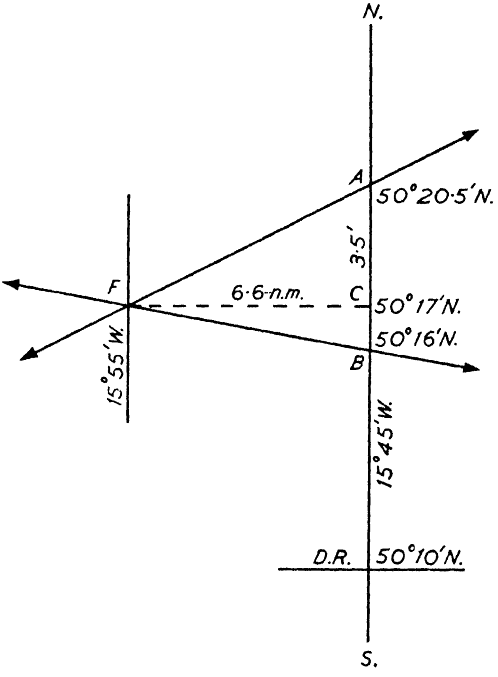


Fig. 160. Fix by Simultaneous Bearings from W/T D.F. Stations

The same procedure will serve to establish the point at which the rhumb line from Ushant meets the D.R. meridian. The calculation is as follows:

Mean lat.	= ½ (50° 10′ + 48° 33′)	
Mean lat.	= 49° 21·5′	log. cos. = 9·813799
D. long.	= 63·5′	log. = 2·802774
Departure	= 413·6′	log. = 2·616573
Course	= 76° 00·0′	log. cot. = 9·396771
D. lat. (UT)	= 103·1′	log. = 2·013344
Lat. Ushant	= 48° 33′ N.	
D. lat. (UT)	= 1° 43′ N.	
Lat. B (Fig. 158)	= 50° 16′ N.	

This point is now placed on the meridian in Fig. 160 and marked B, so that AB = 4·5 nautical miles to scale. The position line drawn through B in the direction 280° T.—100° T. intersects the position line through A in F, which is then taken to be the actual position. A

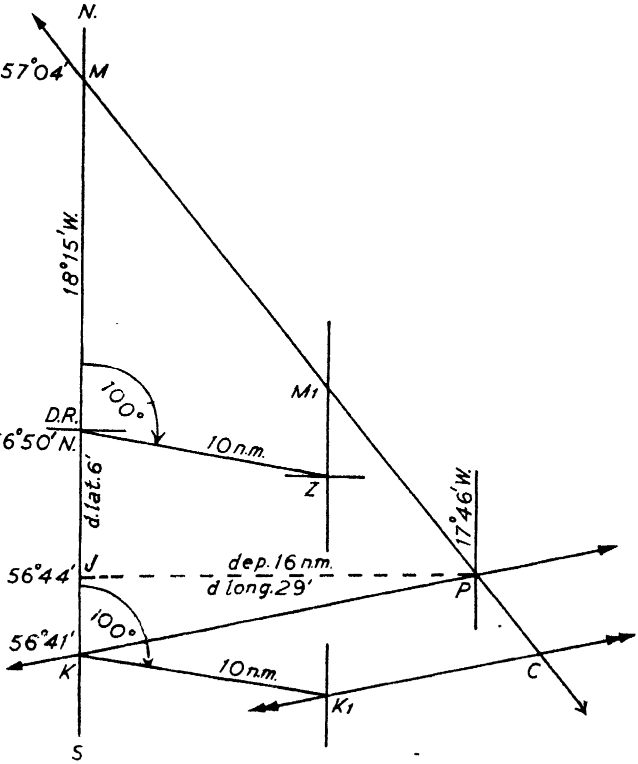


Fig. 161. Fix by Loop-aerial Bearings from Ship or Aircraft

perpendicular from F on to NS meets the meridian at C, whose latitude on measurement is found to be 50° 17′ N., which must also be the latitude of F. The longitude of F may be ascertained by converting the 6·6 nautical miles into d. long. and applying it to the D.R. longitude. The d. long. is found by scale drawing, use of tables, or calculation to be 10′, making the longitude of F 15° 55′ W. Final result (position of F) 50° 17′ N. 15° 55′ W.

Turning to the next example, and using the same technique in connexion with the bearing of Butt of Lewis, we have:

Mean lat. =	½ (56° 50′ + 58° 31′)		
Mean lat. =	57° 40·5′	log. cos. =	9·728127
D. long. =	719′	log. =	2·856729
Course =	74° 00′	log. cot. =	9·457496
D. lat. (BY) =	1° 50′ S.		
Lat. B of L =	58° 31′ N.	log. =	2·042352
Lat. Y =	56° 41′ N. = lat. K		

A point to represent this latitude is now marked on the D.R. meridian of 18° 15′ W. on Fig. 161, and a position line drawn through it in the direction 079° T.—259° T.

Similarly, for the bearing of Valentia we have:

Mean lat. =	½ (56° 50′ + 51° 56′)		
Mean lat. =	54° 23′	log. cos. =	9·765191
D. long. =	476′	log. =	2·637607
Course =	42° 00′	log. cot. =	10·045563
D. lat. (VX) =	5° 08′ N.		
Lat. V =	51° 56′ N.	log. =	2·488361
Lat. X =	57° 04′ = lat. M		

This point is now placed on the D.R. meridian (Fig. 161) and a position line drawn through it in the direction 141° T.—321° T. This is found to intersect the position line through K in P, which is taken, therefore, to be the craft's actual position. A perpendicular from P on to NS meets the meridian at J, whose latitude is then found by measurement from K or M to be 56° 44′, which is the latitude of P. The longitude is then found, as before, by converting PJ from nautical miles into minutes of d. long. and applying the result to the D.R. longitude. This becomes 18° 15′ − 0° 29′ = 17° 46′ W.

The final position is thus 56° 44′ N. 17° 46′ W.

Running Fix. Suppose that, instead of the bearings being taken almost simultaneously, an interval of time had elapsed between them, during which the craft had covered 10 n.m. in a direction of 100° T. The craft's D.R. position would now be at Z, and the rhumb line would meet the new D.R. meridian in M_1. Neither the rhumb line nor the convergency would have changed materially, so the second position line would pass through M_1 in the same direction as it previously passed through M. The first position line through K would have to be transferred, as shown, in a direction 100° T. for 10 n.m. It would then pass through K_1 and intersect the second position line in C, which would then be the position.

Reliability of Radio Position Lines. The degree of accuracy obtainable in finding a position by means of radio bearings depends firstly on reception. Generally speaking, errors of reception are least when near the transmitting station but increase with distance from it. The ground wave which, as we have seen, meets the loop aerial horizontally, provides its strongest signal near its point of origin, but as it progresses its energy becomes absorbed by the ground with a consequent proportionate loss in signal strength. This does not apply to waves which, after travelling obliquely through the atmosphere, are reflected downward by the Heaviside layer to meet the receiver at a distance from the point of origin when the ground wave has lost some of its impetus. The reflected wave, coming in to the aerial obliquely, produces an E.M.F. in the horizontal parts of the loop and disturbs the effect of the ground wave. This combination may produce an indistinct minimum signal or a removal of the zero-point several degrees either way, but whatever it is, the effect is undesirable and tends to impair the efficiency of the direction finder. At night time, due to a raising of the Heaviside layer, the effect is accentuated, as the reflected wave then becomes receivable over a greater distance than the ground wave and is solely responsible for the signal when the receiver is out of range of the ground wave. In this case the reflected wave may not be in the same vertical plane as the ground wave, due to the Heaviside layer not being perfectly horizontal. The result is that the minimum signal cannot give accurately the direction of the transmitting station. This is known as **night effect.** There may be a gap between the maximum range of the ground wave and the reception of the reflected wave. The extent of such a gap is called the **skip.**

When electro-magnetic waves pass obliquely over a coast their direction often undergoes a change, so that directions obtained from them must be treated with caution. This deflection is known as **coastal refraction.** Land D.F. stations are usually able to make allowance for it.

In taking radio bearings one can never overlook entirely the effect of the weather. Electrical charges in the atmosphere may distort, if not obliterate, signals which are normally receivable. Electricity in the air is capable of fantastic tricks.

The plotting of radio position lines assumes that both the great circle and the curve of constant bearing may be drawn on the Mercator chart as arcs of circles. This is an approximation which ceases to be valid over long distances. The larger the conversion angle, therefore, the greater is the chance of error due to this cause. One might ask why the gnomonic chart could not be used for plotting radio bearings. The reason is that, except at the tangent-point, bearings on the gnomonic chart cannot be measured accurately without the use of special protractors or compass roses of which too many would be required by a mobile craft.

Lastly there are the errors that creep into the calculation from personal causes. Unless the relative bearing is taken accurately and the craft is on its course at the time of the operation, the correct bearing cannot be obtained. Further, the bearing must be plotted accurately, as an error of ½° in plotting will displace the position line by 1 mile at a distance of 120 miles from the station. Two such displaced position lines would probably lead to an error of several miles in the position at which they intersect.

Radar. Like waves of sound, radio magnetic waves can be reflected back to their source after striking hard material—*i.e.*, they produce an echo. Such an echo, when reproduced visibly as a greenish light on the flat glass end of a cathode-ray tube, the screen, indicates to a navigator the presence of an object perhaps not otherwise visible. The interval of time occupied by a pulsation in its journey out and back, though extremely small, can be measured, and, when multiplied by half the speed of radio waves, gives to a navigator his distance from the reflecting object. This distance is represented to scale by measurement from the centre of the screen.

Instead of being transmitted in all directions the pulsations are confined within the limits of a beam extending horizontally for not more than 3° but considerably more vertically. The aerial producing this beam is caused to rotate and is synchronized with a corresponding radial line on the screen. Reflected objects appear as luminous marks along this line as the aerial passes their direction, which can then be read from the screen after a datum of reference such as north or the craft's heading has been fixed for it.

The navigator is thus provided with the direction as well as the distance of an object. This, however, is not all. The luminous reflections on the screen continue to glow until the beam comes round to them again, so that a stationary object appears continuously in the same position while a moving object is shown to advance. A continuous reproduction is thereby built up of all suitable objects within the minimum and maximum ranges of the instrument. The navigator then has before him on the screen, now become a Plan Position Indicator (P.P.I.), an all-round view of those objects susceptible to radio reflection. Those of vertical extent respond readily to an observer at sea, so that coastal features, especially cliffs, lighthouses, icebergs, and ships, are most easily discerned. Objects within fifty yards, however, do not appear owing to the shortness of the interval. From the air, ground objects of distinct horizontal shape usually stand out well. Distance above the ground is, of course, an aircraft's altitude, so that the radar principle can be applied to suit the requirements of an altimeter. Further, the P.P.I. may be adjusted in scale to fit a map or chart.

The value of Radar for pilotage purposes during periods of bad visibility either at sea or in the air is considerable. Besides saving time its introduction has increased the safety factor in both cases.

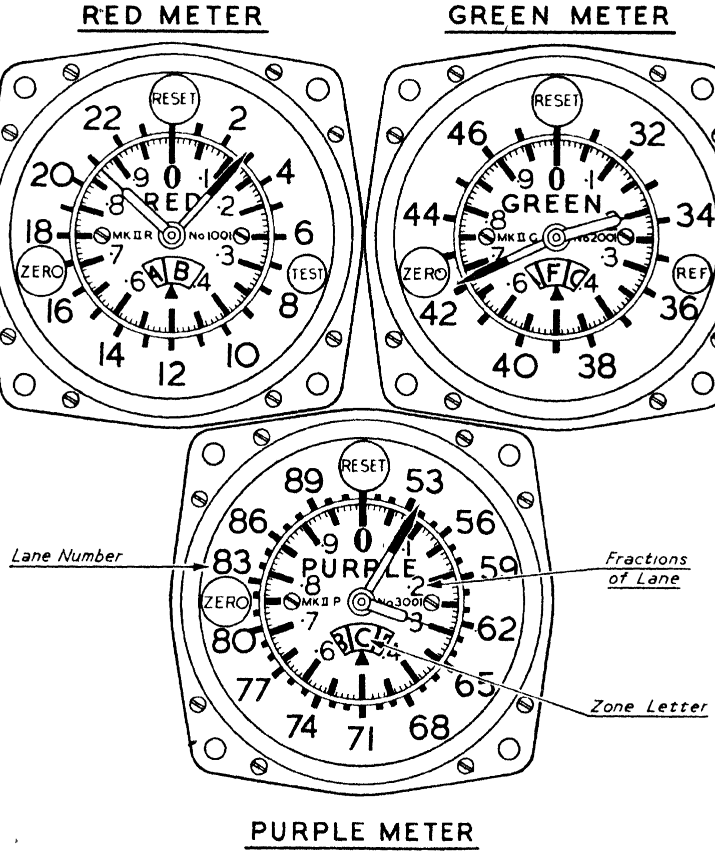
The Hyperbolic Method of Obtaining Position. Quite recently a new method of position finding by radio has been introduced into both marine and air navigation.

We all know that it is possible to draw a line between two points A and B so that any spot on it is equidistant from both A and B. Such a line is known as a locus line—which in this case bisects the base-line joining A and B at right angles. Many other locus lines could be drawn about A and B. If we mark a succession of points by making the distance from A always twice that from B, we should obtain a curved locus line known as a hyperbola. Similarly, if we make the distance from A always three times that from B, we should obtain another hyperbola. From this it can be seen that we obtain a fresh hyperbola every time we vary the ratio between distance from A and distance from B, and such curves will radiate out from the base-line.

Now, the distances travelled by radio waves can be measured, although it must be remembered that their speed is 186,000 miles per second, and for small distances the unit of time must be the microsecond (millionth part of a second). If we receive signals from two synchronized radio transmitters, we can tell which is the nearer by noting which wave arrives first. Suppose A and B to represent the two transmitters, then, as in the simpler case for distances, the ratio between the times taken by the waves to travel to us from A and B will determine which of the hyperbolic loci, or position lines, is ours.

Special radio receivers are used to detect and time the incoming waves. In the Gee and Loran systems the waves are 'marked' by radiating them in pulses, which are displayed at the receiver on a calibrated cathode-ray time base. In the Decca system (see Decca Lattice Chart in endpaper envelope) the time differences are measured indirectly as phase differences in the waves received from fixed transmitters, which radiate pure unmodulated C.W. The space between any two in-phase hyperbolæ is called a lane, and the number of lanes depends on the wavelength employed.

As explained earlier, one line is not sufficient to determine a position, so a second pair of synchronized transmitters is required to provide another set of hyperbolæ to cross the first. A network of position lines is thus built up, and the position of a receiving ship or aircraft is indicated by a point of intersection. For identification each set of hyperbolæ has a colour allotted to it, and each individual curve is numbered.


RED METER

GREEN METER

PURPLE METER

Fig. 162

The signals received automatically operate two indicators on the receiver, one designated RED and one GREEN (Fig. 162). Each of these continuously indicates a number, the value of which depends upon the coloured position line of the craft, the indicated number changing, of course, with the craft's movement. Charts covering the service area of three transmitting stations—*i.e.*, master and two slaves

—are gridded with red and green hyperbolic position lines which are all numbered. At any time when the position of the craft is required it is merely necessary to read off the number indicated by the RED meter and the number indicated by the GREEN meter, then look for the correspondingly numbered RED and GREEN lines on the chart. The point of intersection of the two lines is the position of the ship or aircraft. Combinations of RED PURPLE and GREEN PURPLE are similarly used. The position is established instantaneously and to a very high degree of accuracy, and a continuous record of progress is provided.

The ground stations necessary for the Decca Navigator system consists of a master station and two or three slaves. The ideal lay-out is a master with three slaves about 70 miles distant, so arranged that the slaves are at the corners of an equilateral triangle and the master at the middle. The angle between the base-lines is then 120°, and good angles of cut are given in all directions.

General Exercises

1. The following radio bearings were given to a craft stated to be in lat. 54° 10′ N. long. 3° 20′ E. by D.R.:

070° from Mablethorpe (53° 20′ N. 0° 17′ E.)
108° from Cullercoats (55° 02′ N. 1° 25′ W.)

What was the actual position?

2. A craft in 54° 20′ N. 2° 55′ E. by D.R. received the following radio bearings:

054° from Mablethorpe
318° from Willemsoord (52° 57′ N. 4° 43′ E.)

What was her actual position?

3. A ship in a position by D.R. 54° 30′ N. 2° 10′ E., steering 170° T. at 10 knots, receives a radio bearing of 045° from Mablethorpe. Six hours later the radio bearing received from the same station was 080°. What was the actual position of the ship at the time of taking the second bearing if the tide set on the average 340° T. at 1·5 knots in the interval?

4. A ship in a position by D.R. 53° 50′ N. 2° 20′ E., steering 085° T. at 15 knots, was given a radio bearing of 306° from Willemsoord. Four hours later the same station gave the ship's bearing by radio as 350°. Supposing the tide to set on the average 180° T. at 2 knots in the interval, what was the actual position of the vessel when the second bearing was given?

5. An aircraft, in a position by D.R. 54° 15′ N. 3° 00′ E., steering 200° T. at a T.A.S. 150 knots, receives a radio bearing of 104° from Cullercoats. Five minutes later a radio bearing of 060° is received from Mablethorpe. What is the actual position of the aircraft at the time of receiving the second bearing, supposing the W/V to be 270° 30 knots?

6. A craft in a position by D.R. 53° 20′ N. 3° 45′ E., steering 210° T., finds the relative bearings by loop aerial of North Foreland W/T station (51° 20′ N. 1° 25′ E.) and Mablethorpe W/T station to be 010° and 060° respectively. Find the craft's actual position.

7. An aircraft in 54° 30′ N. 5° 40′ E. by D.R., steering 240° T. at T.A.S. 180 knots, finds the relative bearing by loop aerial of Cullercoats W/T station to be 040°. Four minutes later the relative bearing by loop aerial of North Foreland is found to be 340°. If the W/V at the time were 300°/40 knots, what is the aircraft's actual position when the second bearing is taken?

8. An aircraft in 54° 25′ N. 2° 20′ E. by D.R., steering 330° T. at T.A.S. 160 knots, finds the relative bearing by loop aerial of Cromer (52° 55 N. 1° 15′ E.) to be 230°. Six minutes later Cullercoats bore 315° by loop aerial. What would be the actual position of the aircraft if the W/V at the time were 045°/30 knots?

9. A craft in a position by D.R. 50° 10′ N. 16° 45′ W. received the following radio bearings:

250° from Valentia W/T D.F. station (51° 56′ N. 10° 19′ W.)
290° from Ushant W/T D.F. station (48° 33′ N. 5° 10′ W.)

Find the conversion angle, rhumb-line bearing, and direction of position line in each case. Give the position obtained by combining both position lines.

10. A craft, whose position by D.R. was calculated to be 49° 10′ N. 13° 40′ W., received the following simultaneous radio bearings:

220° from Valentia W/T D.F. station.
280° from Ushant W/T D.F. station.

Find the conversion angle, rhumb-line bearing, and direction of position line in each case. Give the position obtained by combining both position lines.

11. The following radio bearings were obtained simultaneously and given to the craft whose position was stated to be 54° 05′ N. 16° 15′ W.:

260° from Malin Head W/T D.F. station (55° 22′ N. 7° 20′ W.)
300° from Valentia W/T D.F. station.

Find and apply each conversion angle to its respective bearing in order to obtain the rhumb-line bearings. Deduce the direction of each position line, and combine both to obtain a fix.

12. A craft giving her position as 37° 30′ N. 73° 05′ W. by D.R. received the following simultaneous radio bearings:

080° from Virginia Beach W/T D.F. station (36° 52′ N. 75° 59′ W.)
140° from Cape May W/T D.F. station (38° 56′ N. 75° 55′ W.)

Find the craft's actual position.

13. A craft in a position by D.R. 48° 30′ N. 13° 20′ W., steering 045° T., took the following relative bearings by loop aerial:

Ushant bearing 045°.
Valentia bearing 340°.

Find each conversion angle and rhumb-line bearing. Deduce the direction of each position line, and combine both to obtain a fix.

14. A craft in a position by D.R. 52° 50′ N. 16° 20′ W., steering 120° T., took the following relative bearings by loop aerial:

Valentia bearing 345°.
Malin Head bearing 305°.

Find each conversion angle and rhumb-line bearing. Deduce the direction of each position line, and combine both to obtain a fix.

15. Doubtful of her correct position, owing to bad weather, a craft in a position by D.R. 55° 50′ N. 2° 05′ E., and steering 320° T., obtained the following relative bearings by loop aerial:

Wick W/T station (58° 26′ N. 3° 06′ W.) bearing 340°.
FEI W/T station (58° 50′ N. 5° 32′ E.) bearing 070°.

Find the correct position.

16. In order to establish her position accurately, a craft in a position by D.R. 51° 45′ N. 23° 00′ W., and steering 100° T., obtained the following relative bearings by loop aerial:

Malin Head W/T station (55° 23′ N. 7° 25′ W.) bearing 323°.
Ferrol W/T station (43° 29′ N. 8° 15′ W.) bearing 024°.

Establish the correct position.

MISCELLANEOUS EXERCISES

MISCELLANEOUS D.R. AND TACTICAL PLOTTING QUESTIONS

1. Prepare a plan for a flight from SOUTHEND via WHITBY and CROMER back to SOUTHEND, giving the tracks, distances, true courses, G.S. and total time involved. T.A.S. 180 knots. W/V 270°/30 knots.

2. Complete preparations for a round flight beginning at CHELMSFORD and proceeding via CROMER, X (54° 30′ N. 0° 10′ W.), LINCOLN, and back to CHELMSFORD, giving the tracks, distances, true courses, G.S. and total time involved. T.A.S. 160 knots. W/V 340°/35 knots.

3. Leaving KING'S LYNN at 10.10 hrs. you set course for ROTTERDAM with the intention of continuing to ANTWERP and then returning to KING'S LYNN via CAMBRIDGE. What would be the tracks, distances, G.S., and true courses en route, and the E.T.A. at each point of alteration? T.A.S. 150 knots. W/V 250/30 knots.

4. At 09.00 hrs. you set course from LUTON (51° 51′ N. 0° 23′ W.) on a round flight via NORWICH (52° 38′ N. 1° 15′ E.), A (54° 20′ N. 1° 30′ E.), B (53° 50′ N. 0° 40′ W.) back to LUTON. T.A.S. 150 knots. W/V 030°/40 knots. Give tracks and distances, course to NORWICH, and G.S.

At 09.30 hrs. you pinpoint at X (52° 30′ N. 1° 00′ E.). Find W/V experienced, and readjust course to NORWICH. Use the new wind in calculating the remaining courses and G.S., and give the E.T.A. at each turning-point.

5. Passing over WHITBY at 14.00 hrs., S/C to CAMBRIDGE via a buoy in 54° 20′ N. 1° 40′ E. and return to WHITBY. T.A.S. 150 knots. W/V 300°/24 knots.

Give: tracks, distances, G.S., courses (T., M., and C.) using Form 316*b*,[1] and E.T.A.

When the buoy is not sighted as expected, course is altered as calculated for CAMBRIDGE, and at 15.02 hrs. you pinpoint at SKEGNESS. Give the W/V experienced since leaving WHITBY and readjust course to CAMBRIDGE accordingly. What would be the new course to WHITBY and the E.T.A.?

6. Draw up a plan for the round flight HULL—MARGATE—LUTON—HULL, giving tracks, distances, courses (T., M., and C.), G.S., and E.T.A. for a T.A.S. 150 knots and W/V 120°/30 knots (use Form 316*b* for deviations).

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[1] See p. 111.

08.40 hrs. HULL. S/C MARGATE.
09.11 hrs. KING'S LYNN. Give W/V experienced and new course to MARGATE.
09.53 hrs. MARGATE. S/C LUTON using new W/V.
10.18 hrs. LUTON. S/C HULL using new W/V.
10.38 hrs. A (52° 50′ N. 0° 10′ W.) bears 040° C.
PETERBOROUGH (52° 35′ N. 0° 13′ W.) bears 140° C.
Give: position, T.M.G., and G.S., and readjust course to HULL giving new E.T.A.

7. Find the track and distance from GRAVESEND to SCARBOROUGH, and give the required details on the following flight:

10.20 hrs. GRAVESEND (51° 28′ N. 0° 24′ E.). S/C SCARBOROUGH T.A.S. 140 knots. W/V 200°/26 knots.
Give: course (T., M., and C.), G.S., and E.T.A. (Use Form 316*b* for deviations.)
10.39 hrs. CAMBRIDGE.
Give: T.M.G., T.E. (track error), G.S., W/V, and new course to SCARBOROUGH.
10.46½ hrs. KING'S LYNN bears 055° C.
PETERBOROUGH bears 310° C. Give the position.
10.55 hrs. Give the D.R. position.
11.03 hrs. LINCOLN bears 305° C. Drift 5° stbd.
11.07 hrs. LINCOLN bears 250° C. Give the position.
Calculate the new course to destination and find the E.T.A.

8. Plot the following flight and give the required details:

13.15 hrs. CANTERBURY. S/C LOWESTOFT.
T.A.S. 150 knots. W/V 250°/28 knots.
Give: track, distance, course T., G.S., and E.T.A.
13.40 hrs. LOWESTOFT. Give: W/V experienced.
A/C 350° T. Drift 10° stbd. Give: track.
13.50 hrs. CROMER bears 270° T. Give: G.S.
14.10 hrs. Give: D.R. position.
A/C for FLAMBOROUGH HEAD lighthouse using W/V 230°/40 knots.
Give: track, distance, course, G.S., drift, and E.T.A.
14.45 hrs. FLAMBOROUGH HEAD A/C for CAMBRIDGE steering 190° T.
15.22 hrs. P.P. (pinpoint) 52° 52′ N. 0° 03′ W.
Give: T.M.G., G.S., drift, track error and W/V.
A/C for CAMBRIDGE using W/V 230°/36 knots.
Give: course T. and E.T.A.

9. From an aerodrome in 52° 25′ N. 0° 20′ W. you are to proceed to BRUSSELS at a T.A.S. 182 knots. W/V 250°/35 knots.

22.17 hrs. S/C BRUSSELS (50° 51′ N. 4° 22′ E.).
Give: track, course T., M., and C., G.S. (use Form 316*b* for deviations).
On crossing the Essex coast the back-bearing of a prominent object was 312° C.
Give: track, T.E., drift.
22.56 hrs. Crossing Belgian coast. Give: G.S.

23.03 hrs. Give: D.R. position and A/C by T.E. (track error) method to BRUSSELS.
23.13 hrs. BRUSSELS.
23.18 hrs. S/C for aerodrome in 51° 10′ N. 0° 20′ E. at T.A.S. 200 knots. W/V 260°/30 knots.
Give: track, distance, course T., M., and C., G.S., and E.T.A.
23.33 hrs. P.P 50° 56′ N. 3° 17′ E. Give: W/V.
23.48 hrs. Crossing French coast in 51° 02′ N 2° 12′ E.
23.50 hrs. B.B. (back bearing) of prominent object 106° C., wind-lanes 065°–245°.
Give: track, W/V (allowing 10° wind veer for altitude), and E.T.A.

10. From 52° 30′ N. 0° 10′ W. you are to proceed to an aerodrome in 52° 55′ N. 6° 45′ E.

14.42 hrs. S/C for destination at T.A.S. 185 knots. W/V 200°/40 M.P.H.
Give: track, distance, course T., M., and C., G.S., and E.T.A. (use Form 316*b* for deviations).
14.48 hrs. Crossing R. Ouse. Check G.S.
15.05½ hrs. NORWICH. Give: drift and latitude on crossing coast.
15.13 hrs. NEWARP light-vessel bears 000° C.
YARMOUTH bears 240° C. Give: position.
15.28 hrs. A/C 5° stbd.
15.37½ hrs. Give: D.R. position.
15.43½ hrs. B.B. of object on Dutch coast 277° C. Give: track.
16.08 hrs. Arrived destination.
16.20 hrs. S/C for CAMBRIDGE at T.A.S. 190 knots. W/V 200°/45 M.P.H.
Give: track, distance, course T., M., and C., G.S., and E.T.A.
16.50 hrs. P.P. 52° 37′ N. 4° 37′ E. Give: T.M.G., G.S., and T.E.
16.54 hrs. Surface wind direction observed to be 030°–210°.
Give: W/V (10° wind veer for altitude) and G.S.
17.31 hrs. ORFORDNESS. Give: T.M.G., G.S., and W/V experienced. Readjust course to CAMBRIDGE using new wind, and give E.T.A.

11. You are to proceed from 53° 25′ N. 0° 10′ W. to an aerodrome in 52° 20′ N. 4° 50′ E. T.A.S. 150 knots (use Form 316*b* for deviations).

14.24 hrs. S/C 126° C. Give: track, distance, and true course.
14.49 hrs. CROMER. Give: T.M.G., G.S., and drift.
14.56 hrs. YARMOUTH bears 190° C.
15.02 hrs. YARMOUTH bears 248° C.
Give position at 15.02 hrs.
15.39 hrs. P.P. 52° 15′ N. 4° 25′ E.
Give: T.M.G., G.S. (from CROMER), and W/V.
15.41 hrs. B.B. 300° C. Give: track and drift.
15.46 hrs. Arrived destination.
16.00 hrs. S/C for 52° 10′ N. 0° 10′ E., using W/V 180°/15 knots.
16.05 hrs. B.B. 094° C. Give: track.
16.08 hrs. Surface wind direction from wind-lanes 000°–180°.
Give: W/V (allowing 10° for wind veer), G.S., and E.T.A.

12. From an aerodrome in 52° 30′ N. 0° 10′ W. you are to fly with freight to 51° 10′ N. 6° 40′ E. W/V 220°/30 knots.

Give: track and distance.
09.42 hrs. S/C. What true air speed would be required so as to arrive over

your destination at 11.30 hrs.? Give the course T., M.. and C., using Form 316*b*.

10.10 hrs. Crossing the English coast in lat. 52° 10′ N. Give: G.S.

10.12 hrs. B.B. 292° C. Give: track.

10.25 hrs. Report the position of a derelict directly below you.

10.44 hrs. P.P. 51° 48′ N. 3° 53′ E.

Give: T.M.G., G.S., W/V, and alteration of course necessary to reach destination. What would be the E.T.A.?

11.45 hrs. Left 51° 10′ N. 6° 40′ E. for 51° 00′ N. 0° 10′ E. T.A.S. 150 knots. W/V 220°/40 knots.

Give: track, distance and course, T., M., and C. (dev. 3° W.).

13.12½ hrs. P.P 51° 00′ N. 2° 05′ E. Give: T.M.G. and G.S.

13.14 hrs. B.B. 100° C. Surface direction of wind by wind-lanes 020°–200°. Give: track, W/V (allowing for 10° veer), and G.S.

13. You are the navigator of a passenger aircraft detailed to proceed from 52° 30′ N. 0° 35′ E. to 52° 20′ N. 4° 50′ E. Use Form 316*b*.

10.24 hrs. S/C T.A.S. 190 knots. W/V 245°/30 M.P.H.

Give: track, distance, course T., M., and C., G.S., and E.T.A.

10.36 hrs. Crossing coast. NORWICH bears 319° C.

Give: T.M.G., T.E., drift, G.S., and W/V experienced.

10.39 hrs. Reset course to destination using W/V 270°/25 knots.

Give: new E.T.A.

11.01 hrs. SCHEVENINGEN bears 174° C.

IJMUIDEN bears 071° C.

Give: position at 11.01 hrs., and new course to destination.

At what time must you S/C from 52° 20′ N. 4° 50′ E. to 51° 40′ N. 0° 20′ E. in order to arrive at 13.00 hrs.? T.A.S. 195 knots. W/V 270°/30 knots.

12.01½ hrs. P.P. marine light in 52° 15′ N. 4° 25′ E.

12.04 hrs. B.B. marine light 080° C.

Surface wind direction by wind-lanes 110°–290°.

Give: track, G.S., and W/V (allowing 10° veer).

12.22 hrs. Give: D.R. position and proceed to 51° 10′ N. 0° 10′ E. so as to arrive at 13.00 hrs.

14. At 09.00 hrs. you leave an aerodrome in 52° 45′ N. 1° 15′ E. to patrol on a track of 350° T. T.A.S. 120 knots, W/V 045°/30 knots.

10.00 hrs. A/C 084° C., dev. 6° E., ——

11.10 hrs. Passing over raft with survivors.

A/C to intercept rescue-launch which had crossed your track 20 minutes earlier steering 200° M. at 30 knots.

Give: time and position of interception, track back to raft, time of rescue by launch, and course back to your aerodrome (dev. 2° W.) together with E.T.A.

15. At 06.00 hrs. you pass over FLAMBOROUGH HEAD on a track to a position 53° 00′ N. 3° 00′ E. T.A.S. 120 knots. Wind disregarded.

06.30 hrs. P.P. OUTER DOWSING light-vessel (53° 34′ N. 1° 00′ E.)

Give: W/V experienced.

Reset course so as to resume original track in 15 minutes. Var. 10° W. Use Form 316*c*.[1]

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[1] See Appendix, p. 396.

06.45 hrs. Reset course for 53° 00′ N. 3° 00′ E. and give E.T.A.
On E.T.A. S/C to COLCHESTER so as to arrive at 08.00 hrs.

16. From an aerodrome in 53° 00′ N. 0° 20′ W. you are to patrol on a track 070° T. and return in 3 hours. T.A.S. 120 knots, W/V 150°/24 knots.

10.00 hrs. Airborne. S/C, var. 10° W. Use Form 316*c*.
12.10 hrs. SOS from a craft in 52° 50′ N. 3° 30′ E.
What would be your E.T.A. at the aerodrome if you A/C to verify position of craft for rescue purposes?
Give: course and G.S. out and back, R. of A (radius of action), T.T.T. (time to turn), D.R. position at turn, time of sighting craft in distress, and course back.

17. You set course at 11.00 hrs. from an aerodrome in 53° 30′ N. 0° 40′ W. to intercept a ship reported to have been in a position 000° WHITBY 15 n.m. at 09.30 hrs. and steering for OUTER DOWSING light-vessel at 12 knots. You fly at a T.A.S. 180 knots disregarding wind.

11.15 hrs. FLAMBOROUGH HEAD lighthouse.
Find: W/V. Readjust course accordingly.
On intercepting the ship you proceed to patrol on a relative bearing from her of 310° to return in 1½ hours.
12.14 hrs. Stationary object sighted on a relative bearing 030° from the aircraft at an estimated distance of 10 n.m. After spending 5 minutes over the object you return to the ship.
Give: all true courses, time of altering course, and time of returning to the ship.

18. In response to an SOS sent out by an aircraft in distress, you leave your base in 53° 20′ N. 0° 10′ W. at 11.15 hrs. and proceed at a T.A.S. of 150 knots to the position given—*i.e.*, 54° 10′ N. 2° 50′ E. W/V 160°/30 knots. Give: track, distance, G.S., and E.T.A.

On arrival at the reported position no sign of aircraft or survivors is to be seen, so a fixed square search is begun first up-wind and then right-handedly, the visibility being 5 n.m. Give compass courses and G.S. involved, using Form 316 *c*.

At 13.04 hrs. a dinghy with survivors is sighted on a relative bearing of 330° at maximum visibility. What would you give as its position relative to FLAMBOROUGH HEAD?

MISCELLANEOUS NAUTICAL CHART QUESTIONS

1. Give the true bearing of:

(*a*) Flamborough Head lighthouse (54° 07′ N. 0° 05′ W.) from Humber light-vessel (53° 34′ N. 0° 22′ E.).

(*b*) Outer Dowsing light-vessel (53° 34′ N. 1° 00′ E.) from Cromer lighthouse (52° 55′ N. 1° 19′ E.).

(*c*) Spurn Head lighthouse (53° 35′ N. 0° 07′ E.) from 53° 30′ N. 2° 45′ E.

2. Give the true course and distance from:

(*a*) Cross Sand light-vessel (52° 38′ N. 1° 58′ E.) to Maas light-vessel (52° 01′ N. 3° 54′ E.).

(*b*) Newarp light-vessel (52° 47′ N. 1° 55′ E.) to Haaks light-vessel (52° 58′ N. 4° 19′ E.).

(*c*) Shipwash light-vessel (52° 02′ N. 1° 42′ E.) to Ostend (51° 12′ N. 2° 57′ E.).

3. Find the true course and distance from:

(*a*) 090° Whitby lighthouse (54° 32′ N. 0° 35′ W.) 10 n.m. to 070° Humber light-vessel (53° 34′ N. 0° 22′ E.) 5 n.m.

(*b*) Outer Dowsing light-vessel (53° 34′ N. 1° 00′ E.) bearing 180° T. distant 8 n.m. to 3 n.m. true north of Borkum Riff light-vessel (53° 46′ N. 6° 04′ E.).

(*c*) Flamborough Head lighthouse bearing 240° T. distant 8 n.m. to 4 n.m. true east of Outer Dowsing light-vessel.

4. (*a*) Using deviation 3° E. and variation 11° W., find the compass course steered and distance steamed from:

Haaks light-vessel bearing 300° C. distant 3 n.m. to 280° C. West Hinder light-buoy (51° 33′ N. 2° 37′ E.) 2 n.m.

(*b*) Using deviation 3° W. and variation 8° W., find the compass course steered and distance steamed from: Heligoland lighthouse (54° 11′ N. 7° 53′ E.) bearing 330° C. distant 8 n.m. to Borkum Riff light-vessel (53° 46′ N. 6° 04′ E.) bearing 150° C. distant 5 n.m.

5. (*a*) From a position 3 n.m. south magnetic of Humber light-vessel find the courses to steer by compass to Sunk light-vessel (51° 48′ N. 1° 35′ E.), altering course when Dudgeon Shoal buoy (53° 20′ N. 1° 00′ E.) bears 020° M. distant 2 n.m., so as to pass 3 n.m. 090° M. of Cross Sand light-vessel and Shipwash light-vessel. Variation 10° W., deviation card 2.[1]

(*b*) From a position 4 n.m. east magnetic of Flamborough Head lighthouse at 00.00 hrs. find the compass courses to steer to reach Ijmuiden (52° 27′ N. 4° 35′ E.), passing 3 n.m. 100° T. of Outer Dowsing light-vessel and altering course again when Winterton Ridge buoy (52° 47′ N. 2° 02′ E.) bears west true, distant 3 n.m., counteracting a tide for the first 7 hours setting S.S.E. magnetic at 2 knots, then N.E. magnetic at 2 knots for the remaining time. Speed for the first 12 hours 9 knots, then reduced to 7 knots owing to bad weather. Give the E.T.A. at Ijmuiden. Variation 10° W., deviation card 2.

6. Steering 130° C., Newarp light-vessel bore dead astern when Cross Sand light bore abeam to starboard. Give the ship's position, and state what course you would make good supposing the tide to set south true at 2 knots, ship steaming 8 knots. Where would you make the Dutch coast? Variation 10° W., deviation card 2.

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[1] See Appendix, p. 396.

7. You leave a position 3 n.m. north magnetic of Cross Sand light-vessel at 18.00 hrs. for Haaks light-vessel. What courses would you steer to counteract a tide setting SSW true at 2 knots for the first 3 hours, then NE true at 2 knots for the remaining time? When would you arrive if your steaming speed were 8 knots? Variation 10° W., deviation card 2.

8. From a position 3 n.m. abeam of Cromer lighthouse (52° 55′ N. 1° 20′ E.), set course by compass to Haaks light-vessel, altering course 3 n.m. south magnetic of Newarp light-vessel so as to counteract tides setting NNW magnetic at 3 knots for 6 hours then SSW magnetic at 2 knots. Steaming speed 10 knots. Give the total time taken. Variation 10° W., deviation card 2.

9. From a position at midnight, with Whitby lighthouse bearing 304° M. and Flamborough Head lighthouse bearing 184° M., find the compass course to steer to Ijmuiden, and the time it would take at 10 knots. At 05.00 hrs. Outer Dowsing light-vessel bore 170° C. and at 06.00 hrs. it bore 220° C. Find the ship's position on the second bearing supposing the tide to be slack. From this position find the course to your destination, counteracting a tide setting south true at 3 knots. Give the time at which you would expect to arrive. Variation 10° W., deviation 4° E. on both courses.

10. Find the courses to steer by compass from Outer Gabbard light-vessel (52° 00′ N. 2° 05′ E.) to East Dudgeon light-vessel (53° 20′ N. 1° 00′ E.), altering course when Cross Sand light-vessel (52° 38′ N. 1° 58′ E.) bears west magnetic, distant 4 n.m. and again when N. Haisbro' Lt. V. bears 050 M., distant 3 n.m. Give the distance along each course. While on the first course Lowestoft lighthouse (52° 28′ N. 1° 46′ E.) bore by compass 260° and Cross Sand light-vessel bore 337° by the same compass. Find the ship's position and the set and drift experienced, the ship having steamed for 2½ hours on this course at 12 knots. While on the second course Haisbro' lighthouse (52° 49′ N. 1° 32′ E.) bore by compass 290° and 30 minutes later it bore 240° by the same compass. Find the ship's position, supposing a tide to have set 090° T. at 2 knots in the interim. Ship's speed 12 knots. Variation 10° W., deviation card 2.

11. Stationed at Cromer you receive instructions to intercept a ship reported in a position 080° Hartlepool (54° 42′ N. 1° 11′ W.) 60 n.m. at 10.30 hrs. making good a course of 160° T. at 15 knots. Variation 10° W., deviation card 2.

11.20 hrs. Set course. Speed 30 knots. Tide setting 000° T. at 2 knots.
12.00 hrs. Ship reported altering course to 120° T.
Give: position of interception relative to Flamborough Head, E.T.I. and course back to Cromer, assuming 15 mins. spent with vessel in motion. Tide now setting 140° T. at 3 knots.
14.45 hrs. Loop bearing of Mablethorpe (53° 20′ N. 0° 17′ E.) 040°.
15.45 hrs. Loop bearing of Mablethorpe 085°.
Find position at 15.45 hrs. and readjust course to base, giving E.T.A.

12. At 08.00 hrs. you set course from Scarborough (54° 18′ N. 0° 24′ W.) at a speed of 25 knots to intercept a ship reported at 07.40 hrs. to be in a position 100° Whitby 100 n.m., and steering 030° T. at 10 knots. Estimated set of tide 150° T. at 3 knots. Var. 9° W. Deviation card 2.

Give the assumed position of interception and the E.T.I.

At 10.10 hrs. the ship reports that she is stopping owing to engine breakdown. You proceed to her position at that time. On arrival no ship is sighted, so you begin a fixed-square search with the tide, and then right-handedly, visibility 5 n.m. At 13.50 hrs. the ship is sighted on a relative bearing of 300° at maximum visibility.

Give the ship's position relative to Scarborough.

MISCELLANEOUS ASTRONOMICAL AND RADIO PLOTTING QUESTIONS

1. On September 5 at 02.40 hrs. G.M.T. in 52° 45′ N. 4° 20′ E. by D.R., when G.H.A.♈ was 23° 49′, the true altitude of Capella was 57° 40′ and the true altitude of Alpheratz 58° 36½′.

Find the correct position of the aircraft.

2. On September 15 at 22.14 hrs. G.M.T. in 53° 40′ N. 1° 10′ E. by D.R., when G.H.A.♈ was 327° 58′, the true altitude of Alpheratz was 56° 02′ and the true altitude of Vega was 52° 55′.

Find the actual position of the aircraft at the time of observation.

3. On July 19 at 23.24 hrs. G.M.T. in 54° 20′ N. 0° 50′ E. by D.R., when G.H.A.♈ was 288° 21′, the true altitude of Altair was 44° 02′ and the true altitude of Arcturus was 24° 00′.

What was the position by observation at 23.24 hrs.?

4. On July 5 at 22.34 hrs. G.M.T. (G.H.A.♈ 262° 01′) in a position by D.R. 53° 20′ N. 1° 45′ E., and steering 253° C., the true altitude of Arcturus was 38° 50′.

At 22.46 hrs. G.M.T. (G.H.A.♈ 265° 02′) the true altitude of Altair was 38° 50′.

Find the position at 22.46 hrs. T.A.S. 150 knots. W/V 150°/30 knots. Deviation for the course 3° W.

5. On September 30 at 00.05 hrs. G.M.T. (G.H.A.♈ 009° 36′) in a position by D.R. 54° 10′ N. 1° 40′ E., and steering 275° C. (dev. 5° W.) at T.A.S. 120 knots, the true altitude of Aldebaran was 32° 10′.

At 00.16 hrs. G.M.T. (G.H.A.♈ 012° 21′) the true altitude of Altair was 15° 25′.

Give the position at 00.16 hrs. W/V 180°/30 knots.

6. On September 30 at 04.56 hrs. G.M.T. (G.H.A.♈ 82° 33′) in a D.R. position 52° 45′ N. 1° 40′ E., and steering 260° T., the true altitude of Regulus was 23° 20′.

At 05.07 hrs. G.M.T. (G.H.A.♈ 85° 19′) the true altitude of Sirius was 19° 00′.

Give the position at 05.07 hrs. T.A.S. 180 knots, W/V 210°/36 knots.

7. On September 29 at 03.39 hrs. G.M.T. (G.H.A.♈ 62° 16′) D.R. position 53° 45′ N. 1° 42′ E., and steering 212° C. (dev. 1° W.), the true altitude of Capella was 78° 05′.

At 03.49 hrs. G.M.T. (G.H.A.♈ 64° 46′) the true altitude of Pollux was 46° 40′.

What was the position at 03.49 hrs.? T.A.S. 180 knots, W/V 270°/30 knots.

Give: true course back to base (52° 20′ N. 0° 20′ W.) and E.T.A.

8. On July 8 at 23.17 hrs. G.M.T. (G.H.A.♈ 275° 46′) in a position by D.R. 54° 50′ N. 2° 10′ E., the true altitude of Arcturus was 30° 17′, and the true altitude of Altair was 41° 25′.

Course 250° C. (dev. 3° W.), T.A.S. 183 knots, W/V 300°/24 knots.

At 23.37 hrs. G.M.T. (G.H.A.♈ 280° 47′) the true altitude of Arcturus was 28° 11′, and the true altitude of Altair was 43° 01′.

Find the position at each observation, T.M.G. between them, wind experienced in the interval, and true course to base (53° 50′ N. 0° 50′ W.), with E.T.A.

9. On January 1 at 00.19 hrs. G.M.T. (G.H.A.♈ 105° 01′) in a position by D.R. 49° 40′ N. 15° 50′ W., the true altitude of Regulus was 27° 25′, and the relative bearing by loop aerial of VALA (51° 45′ N. 10° 21′ W.) was 338°. Course 090° C. (dev. 7° E.), W/V 230°/36 knots, T.A.S. 158 knots, var. 17° W.

At 00.51 hrs. G.M.T. (G.H.A.♈ 113° 02′) the true altitude of Regulus was 33° 44′, and the relative bearing of LAN (50° 07′ N. 5° 40′ W.) was 009°.

Find the position of the aircraft at 00°51 hrs., T.M.G., wind experienced during the interval, and true course to base. (51° 50′ N. 5° 00′ W.), with E.T.A.

10. On August 2 at 08.42 hrs. G.M.T. in a position by D.R. 53° 45′ N. 15° 25′ W. the true altitude of the sun (G.H.A. 308° 58′, dec. 17° 50′ N.) was 28° 08′, and the true altitude of Jupiter (G.H.A. 007° 42′, dec. 21° 53′ N.) was 57° 27′. Course 155° C. (dev. 2° E.), T.A.S. 135 knots, W/V 240°/35 M.P.H., var. 18° W.

Find the position.

At 11.06 hrs. G.M.T. the true altitude of the sun (G.H.A. 344° 58′) was 52° 25′, and the true altitude of Jupiter (G.H.A. 043° 47′) was 50° 36′. (Sun's dec. 17° 49′ N., Jupiter's dec. 21° 53′ N.).

Find the position and give T.M.G., G.S., and W/V experienced since 08.42 hrs.

11. On January 5 at 08.30 hrs. G.M.T. you leave an aerodrome in 50° 55′ N. 4° 20′ W. steering 279° C. (dev. 5° W., var. 14° W.), T.A.S. 183 knots.

08.45 hrs. Drift observed by smoke-float to be 8° to port. Wind-lanes from wave movement 120°–300° (no veer).

Give: track, G.S., and W/V.
10.12 hrs. A/C 347° C. (dev. 0°, var. 17° W.).
10.57 hrs. True altitude of sun (G.H.A. 342° 54′) was 11° 50′ (dec. 22° 38′ S.). Drift 10° stbd., wind 270° T.
11.30 hrs. Relative bearing of VALA (51° 45′ N. 10° 21′ W.) was 139°.
Give: position.
A/C 017° C. (dev. 1° E., var. 18° W.).
12.02 hrs. True altitude of sun (G.H.A. 359° 08′) was 11° 14′ (dec. 22° 37½′ S.). Relative bearing of VALA was 151°.
Give: position, T.M.G., and W/V experienced since 08.30 hrs.

12. On July 8 you leave an aerodrome in 50° 20′ N. 5° 05′ W. at 07.05 hrs. to intercept a ship giving her position as 49° 40′ N. 13° 50′ W., course 351° M. (var. 18° W.), and speed 25 knots. T.A.S. 180 knots, W/V 240°/30 knots. Give: E.T.I.

09.30 hrs. True altitude of sun (G.H.A. 321° 17′) was 39° 29′. True altitude of Jupiter (G.H.A. 000° 19′) was 57° 40′.

Give: position, T.M.G., and W/V experienced since 07.05 hrs. (Sun's declination 22° 30′ N., Jupiter's declination 21° 14′ N.)

13. On July 30 at 14.10 hrs. G.M.T. you leave an aerodrome in 50° 55′ N. 3° 45′ W. for BREST (48° 23′ N. 4° 27′ W.), disregarding wind. T.A.S. 150 knots.

14.28 hrs. START POINT lighthouse (50° 13′ N. 3° 38′ W.). Find W/V and readjust course to destination, giving E.T.A.
14.55 hrs. You are notified of obstruction on aerodrome at destination.
14.57 hrs. A/C 270° C. (deviation 5° W., variation 12° W.), T.A.S. 145 knots.
15.08 hrs. A/C 310° C. (deviation 2° W., variation 12° W.).
15.21 hrs. A/C 030° C. (deviation 2° E., variation 12° W.).
15.28 hrs. A/C 100° C. (deviation 6° E., variation 12° W.).
1533 hrs. A/C 180° C. (deviation 0°, variation 12° W.).
15.40 hrs. A/C 240° C. (deviation 2° W., variation 12° W.).
15.48 hrs. Obstruction reported clear.
15.50 hrs. True altitude of sun $\left\{\begin{array}{l}\text{G.H.A. } 055^\circ\ 55'\\ \text{dec. } 18^\circ\ 31'\text{ N.}\end{array}\right\}$ was 38° 56′.
True altitude of moon $\left\{\begin{array}{l}\text{G.H.A. } 338^\circ\ 51'\\ \text{dec. } 7^\circ\ 54'\text{ S.}\end{array}\right\}$ was 28° 52′.
Give: position at 15.50 hrs.. and W/V experienced since 14.28 hrs. Readjust course to destination, giving E.T.A.
16.01 hrs. Arrived destination.
16.06 hrs. Left BREST for home. W/V 290°/38 knots, T.A.S. 145 knots.
16.30 hrs. You receive SOS from vessel giving position 200° BISHOP ROCK lighthouse (49° 52′ N. 6° 27′ W.) 30 n.m.
Give: true course to position, and E.T.A.
On arrival no trace of ship is found, so a fixed square search is begun up-wind and continued right-handedly (vis. 5 n.m.).
17.44 hrs. Directly over lifeboat. Give: position of lifeboat, true course to your aerodrome, and E.T.A.

14. On September 23 you leave 55° 08′ N. 6° 15′ W. at 04.40 hrs. G.M.T. to intercept a ship whose position at 04.00 hrs. was given by W/T bearings as 300° TI (55° 16′ N. 8° 15′ W.), 250° BUT (58° 31′ N.

6° 15′ W.) and which was steering 110° T. at 15 knots. W/V 220°/30 knots, T.A.S. 180 knots.

05.05 hrs. True altitude of Aldebaran was 50° 35′ (G.H.A. ♈ 77° 54′).
05.15 hrs. True altitude of Procyon was 28° 23′ (G.H.A. ♈ 80° 25′).
Give: position at 05.15 hrs.
05.50 hrs. No ship being sighted, a square search is begun using the ship's relative wind (visibility 7 n.m.).
06.40 hrs. Ship sighted, bearing 030° T. at maximum visibility.
Give: ship's position relative to INISHTRAHULL (55° 26′ N. 7° 14′ W.).

EXTRACTS FROM THE *NAUTICAL ALMANAC*

II JANUARY

Date		Day of Year	Sun's Semi-diameter	THE MOON'S						
				Semi-diameter	Horizontal Parallax	Meridian Passage				Age at 0^h
						Upper		Lower		
			′	′	′	h m	m	h m	m	d
Wed.	1	1	16·3	15·4	56·4	15 10		02 46		3·1
Thur.	2	2	16·3	15·2	55·7	15 55	45	03 33	47	4·1
Fri.	3	3	16·3	15·0	55·1	16 39	44	04 17	44	5·1
Sat.	4	4	16·3	14·9	54·7	17 22	43	05 01	44	6·1
Sun.	5	5	16·3	14·8	54·4	18 05	43	05 43	42	7·1
Mon.	6	6	16·3	14·8	54·2	18 48	43	06 26	43	8·1
Tues.	7	7	16·3	14·8	54·3	19 32	44	07 10	44	9·1
Wed.	8	8	16·3	14·9	54·5	20 18	46	07 55	45	10·1
Thur.	9	9	16·3	15·0	54·9	21 06	48	08 41	46	11·1
Fri.	10	10	16·3	15·1	55·4	21 56	50	09 30	49	12·1
Sat.	11	11	16·3	15·3	56·0	22 48	52	10 22	52	13·1
Sun.	12	12	16·3	15·4	56·6	23 41	53	11 14	52	14·1
Mon.	13	13	16·3	15·6	57·2	* *		12 08	54	15·1
Tues.	14	14	16·3	15·8	57·8	00 35	54	13 01	53	16·1
Wed.	15	15	16·3	15·9	58·3	01 28	53	13 54	53	17·1
Thur.	16	16	16·3	16·0	58·7	02 21	53	14 47	53	18·1
Fri.	17	17	16·3	16·1	59·0	03 13	52	15 38	51	19·1
Sat.	18	18	16·3	16·1	59·2	04 04	51	16 30	52	20·1
Sun.	19	19	16·3	16·1	59·2	04 56	52	17 22	52	21·1
Mon.	20	20	16·3	16·1	59·2	05 48	52	18 15	53	22·1
Tues.	21	21	16·3	16·1	59·1	06 42	54	19 09	54	23·1
Wed.	22	22	16·3	16·1	59·0	07 36	54	20 04	55	24·1
Thur.	23	23	16·3	16·0	58·8	08 32	56	21 00	56	25·1
Fri.	24	24	16·3	15·9	58·5	09 28	56	21 57	57	26·1
Sat.	25	25	16·3	15·8	58·1	10 24	56	22 52	55	27·1
Sun.	26	26	16·3	15·7	57·6	11 19	55	23 45	53	28·1
Mon.	27	27	16·3	15·6	57·1	12 11	52	* *		29·1
Tues.	28	28	16·3	15·4	56·5	13 00	49	00 36	51	0·5
Wed.	29	29	16·3	15·3	56·0	13 48	48	01 24	48	1·5
Thur.	30	30	16·3	15·1	55·4	14 33	45	02 10	46	2·5
Fri.	31	31	16·3	15·0	54·9	15 16	43	02 55	45	3·5
Sat.	32	32	16·3	14·9	54·6	15 59	43	03 38	43	4·5

THE SUN

G.M.T.	R	Dec.	E	G.M.T.	R	Dec.	E
		Wednesday 1				**Sunday 5**	
h	h m s	° ′	h m s	h	h m s	° ′	h m s
00	06 41 00·8	S. 23 03·1	11 56 38·6	00	06 56 47·0	S. 22 40·8	11 54 47·0
02	06 41 20·5	23 02·7	11 56 36·2	02	06 57 06·7	22 40·3	11 54 44·8
04	06 41 40·2	23 02·3	11 56 33·8	04	06 57 26·4	22 39·7	11 54 42·5
06	06 41 59·9	23 01·9	11 56 31·5	06	06 57 46·1	22 39·2	11 54 40·3
08	06 42 19·7	23 01·5	11 56 29·1	08	06 58 05·9	22 38·6	11 54 38·1
10	06 42 39·4	23 01·1	11 56 26·7	10	06 58 25·6	22 38·1	11 54 35·8
12	06 42 59·1	23 00·7	11 56 24·3	12	06 58 45·3	22 37·5	11 54 33·6
14	06 43 18·8	23 00·3	11 56 21·9	14	06 59 05·0	22 36·9	11 54 31·4
16	06 43 38·5	22 59·9	11 56 19·6	16	06 59 24·7	22 36·3	11 54 29·1
18	06 43 58·3	22 59·4	11 56 17·2	18	06 59 44·5	22 35·8	11 54 26·9
20	06 44 18·0	22 59·0	11 56 14·9	20	07 00 04·2	22 35·2	11 54 24·7
22	06 44 37·7	22 58·6	11 56 12·5	22	07 00 23·9	22 34·6	11 54 22·4
		Thursday 2				**Monday 6**	
00	06 44 57·4	S. 22 58·2	11 56 10·2	00	07 00 43·6	S. 22 34·0	11 54 20·2
02	06 45 17·1	22 57·8	11 56 07·8	02	07 01 03·3	22 33·4	11 54 18·0
04	06 45 36·8	22 57·3	11 56 05·5	04	07 01 23·0	22 32·8	11 54 15·8
06	06 45 56·5	22 56·9	11 56 03·1	06	07 01 42·7	22 32·3	11 54 13·5
08	06 46 16·2	22 56·5	11 56 00·8	08	07 02 02·4	22 31·7	11 54 11·3
10	06 46 35·9	22 56·0	11 55 58·4	10	07 02 22·1	22 31·1	11 54 09·1
12	06 46 55·7	22 55·6	11 55 56·1	12	07 02 41·9	22 30·5	11 54 06·9
14	06 47 15·4	22 55·2	11 55 53·8	14	07 03 01·6	22 29·9	11 54 04·7
16	06 47 35·1	22 54·7	11 55 51·4	16	07 03 21·3	22 29·3	11 54 02·5
18	06 47 54·8	22 54·3	11 55 49·1	18	07 03 41·0	22 28·7	11 54 00·4
20	06 48 14·5	22 53·8	11 55 46·8	20	07 04 00·7	22 28·1	11 53 58·2
22	06 48 34·2	22 53·4	11 55 44·4	22	07 04 20·4	22 27·5	11 53 56·0
		Friday 3				**Tuesday 7**	
00	06 48 53·9	S. 22 52·9	11 55 42·1	00	07 04 40·1	S. 22 26·9	11 53 53·8
02	06 49 13·6	22 52·4	11 55 39·8	02	07 04 59·8	22 26·3	11 53 51·6
04	06 49 33·3	22 51·9	11 55 37·5	04	07 05 19·5	22 25·6	11 53 49·5
06	06 49 53·0	22 51·5	11 55 35·1	06	07 05 39·2	22 25·0	11 53 47·3
08	06 50 12·8	22 51·0	11 55 32·8	08	07 05 59·0	22 24·4	11 53 45·1
10	06 50 32·5	22 50·5	11 55 30·5	10	07 06 18·7	22 23·7	11 53 43·0
12	06 50 52·2	22 50·0	11 55 28·2	12	07 06 38·4	22 23·1	11 53 40·8
14	06 51 11·9	22 49·5	11 55 25·9	14	07 06 58·1	22 22·5	11 53 38·6
16	06 51 31·6	22 49·0	11 55 23·6	16	07 07 17·8	22 21·8	11 53 36·5
18	06 51 51·4	22 48·5	11 55 21·3	18	07 07 37·6	22 21·2	11 53 34·3
20	06 52 11·1	22 48·0	11 55 19·0	20	07 07 57·3	22 20·5	11 53 32·2
22	06 52 30·8	22 47·5	11 55 16·7	22	07 08 17·0	22 19·9	11 53 30·0
		Saturday 4				**Wednesday 8**	
00	06 52 50·5	S. 22 47·0	11 55 14·4	00	07 08 36·7	S. 22 19·2	11 53 27·9
02	06 53 10·2	22 46·5	11 55 12·1	02	07 08 56·4	22 18·6	11 53 25·8
04	06 53 29·9	22 46·0	11 55 09·8	04	07 09 16·1	22 17·9	11 53 23·7
06	06 53 49·6	22 45·5	11 55 07·5	06	07 09 35·8	22 17·3	11 53 21·5
08	06 54 09·3	22 45·0	11 55 05·2	08	07 09 55·5	22 16·6	11 53 19·4
10	06 54 29·0	22 44·5	11 55 02·9	10	07 10 15·2	22 16·0	11 53 17·3
12	06 54 48·8	22 44·0	11 55 00·6	12	07 10 35·0	22 15·3	11 53 15·2
14	06 55 08·5	22 43·5	11 54 58·3	14	07 10 54·7	22 14·6	11 53 13·1
16	06 55 28·2	22 42·9	11 54 56·1	16	07 11 14·4	22 13·9	11 53 11·0
18	06 55 47·9	22 42·4	11 54 53·8	18	07 11 34·1	22 13·3	11 53 08·8
20	06 56 07·6	22 41·9	11 54 51·5	20	07 11 53·8	22 12·6	11 53 06·7
22	06 56 27·3	22 41·3	11 54 49·3	22	07 12 13·5	22 11·9	11 53 04·6
24	06 56 47·0	S. 22 40·8	11 54 47·0	24	07 12 33·2	S. 22 11·2	11 53 02·5

JANUARY

MOON'S RIGHT ASCENSION AND DECLINATION

G.M.T.	R.A.		Dec.	
h	h m s	s	° ′	′
	Saturday 25			
00	18 16 48	288	S. 18 18·7	2·2
02	18 21 36	288	18 16·5	2·6
04	18 26 24	287	18 13·9	3·1
06	18 31 11	286	18 10·8	3·5
08	18 35 57	286	18 07·3	4·0
10	18 40 43	285	18 03·3	4·4
12	18 45 28	285	17 58·9	4·8
14	18 50 13	284	17 54·1	5·2
16	18 54 57	284	17 48·9	5·6
18	18 59 41	283	17 43·3	6·1
20	19 04 24	282	17 37·2	6·5
22	19 09 06	281	S. 17 30·7	6·8
	Sunday 26			
00	19 13 47	281	S. 17 23·9	7·3
02	19 18 28	280	17 16·6	7·7
04	19 23 08	279	17 08·9	8·0
06	19 27 47	279	17 00·9	8·5
08	19 32 26	277	16 52·4	8·8
10	19 37 03	277	16 43·6	9·2
12	19 41 40	276	16 34·4	9·5
14	19 46 16	275	16 24·9	9·9
16	19 50 51	274	16 15·0	10·3
18	19 55 25	274	16 04·7	10·6
20	19 59 59	272	15 54·1	10·9
22	20 04 31	272	S. 15 43·2	11·3
	Monday 27			
00	20 09 03	270	S. 15 31·9	11·6
02	20 13 33	270	15 20·3	11·9
04	20 18 03	268	15 08·4	12·3
06	20 22 31	268	14 56·1	12·5
08	20 26 59	267	14 43·6	12·8
10	20 31 26	266	14 30·8	13·1
12	20 35 52	265	14 17·7	13·4
14	20 40 17	264	14 04·3	13·7
16	20 44 41	263	13 50·6	14·0
18	20 49 04	262	13 36·6	14·2
20	20 53 26	261	13 22·4	14·5
22	20 57 47	260	S. 13 07·9	14·7
	Tuesday 28			
00	21 02 07	259	S. 12 53·2	14·9
02	21 06 26	258	12 38·3	15·2
04	21 10 44	257	12 23·1	15·4
06	21 15 01	257	12 07·7	15·6
08	21 19 18	255	11 52·1	15·9
10	21 23 33	254	11 36·2	16·0
12	21 27 47	254	11 20·2	16·2
14	21 32 01	253	11 04·0	16·4
16	21 36 14	251	10 47·6	16·6
18	21 40 25	251	10 31·0	16·8
20	21 44 36	250	10 14·2	17·0
22	21 48 46	249	9 57·2	17·1
24	21 52 55		S. 9 40·1	

G.M.T.	R.A.		Dec.	
h	h m s	s	° ′	′
	Wednesday 29			
00	21 52 55	248	S. 9 40·1	17·2
02	21 57 03	247	9 22·9	17·4
04	22 01 10	247	9 05·5	17·6
06	22 05 17	245	8 47·9	17·7
08	22 09 22	245	8 30·2	17·8
10	22 13 27	244	8 12·4	17·9
12	22 17 31	243	7 54·5	18·0
14	22 21 34	243	7 36·5	18·2
16	22 25 37	241	7 18·3	18·2
18	22 29 38	241	7 00·1	18·4
20	22 33 39	241	6 41·7	18·4
22	22 37 40	239	S. 6 23·3	18·5
	Thursday 30			
00	22 41 39	239	S. 6 04·8	18·6
02	22 45 38	238	5 46·2	18·7
04	22 49 36	238	5 27·5	18·7
06	22 53 34	237	5 08·8	18·8
08	22 57 31	236	4 50·0	18·8
10	23 01 27	236	4 31·2	18·9
12	23 05 23	235	4 12·3	19·0
14	23 09 18	234	3 53·3	18·9
16	23 13 12	234	3 34·4	19·0
18	23 17 06	234	3 15·4	19·0
20	23 21 00	233	2 56·4	19·1
22	23 24 53	232	S. 2 37·3	19·0
	Friday 31			
00	23 28 45	233	S. 2 18·3	19·1
02	23 32 38	231	1 59·2	19·0
04	23 36 29	231	1 40·2	19·1
06	23 40 20	231	1 21·1	19·1
08	23 44 11	231	1 02·0	19·0
10	23 48 02	230	0 43·0	19·1
12	23 51 52	230	0 23·9	19·0
14	23 55 42	229	S. 0 04·9	19·0
16	23 59 31	230	N. 0 14·1	18·9
18	00 03 21	229	0 33·0	19·0
20	00 07 10	228	0 52·0	18·8
22	00 10 58	229	1 10·8	18·9
24	00 14 47		N. 1 29·7	

PHASES OF THE MOON

		h	m
Jan. 5	☽ *First Quarter* ...	13	40
13	○ Full Moon ...	11	04
20	☾ *Last Quarter* ...	10	01
27	● New Moon ...	11	03

		h
Jan. 6	☾ Apogee	05
19	☾ Perigee	08

JANUARY XI

Date		VENUS					MARS				
		R.A.		Dec.		Meridian Passage	R.A.		Dec.		Meridian Passage
		h m s	s	° ′	′	h m	h m s	s	° ′	′	h m
Wed.	1	16 50 46	318	S. 21 25·0	10·9	10 10	15 40 59	166	S. 19 12·9	9·6	09 00
Thur.	2	16 56 04	319	21 35·9	10·3	10 12	15 43 45	166	19 22·5	9·5	08 58
Fri.	3	17 01 23	319	21 46·2	9·6	10 13	15 46 31	167	19 32·0	9·3	08 57
Sat.	4	17 06 42	321	21 55·8	9·0	10 15	15 49 18	167	19 41·3	9·2	08 56
Sun.	5	17 12 03	321	22 04·8	8·3	10 16	15 52 05	167	19 50·5	9·0	08 55
Mon.	6	17 17 24	322	22 13·1	7·7	10 17	15 54 52	168	19 59·5	8·9	08 54
Tues.	7	17 22 46	323	22 20·8	7·0	10 19	15 57 40	168	20 08·4	8·7	08 53
Wed.	8	17 28 09	323	22 27·8	6·4	10 20	16 00 28	169	20 17·1	8·6	08 51
Thur.	9	17 33 32	323	22 34·2	5·6	10 22	16 03 17	169	20 25·7	8·3	08 50
Fri.	10	17 38 55	324	22 39·8	5·0	10 23	16 06 06	169	20 34·0	8·3	08 49
Sat.	11	17 44 19	325	22 44·8	4·3	10 24	16 08 55	170	20 42·3	8·0	08 48
Sun.	12	17 49 44	325	22 49·1	3·7	10 26	16 11 45	170	20 50·3	7·9	08 47
Mon.	13	17 55 09	325	22 52·8	2·9	10 28	16 14 35	170	20 58·2	7·7	08 46
Tues.	14	18 00 34	325	22 55·7	2·2	10 29	16 17 25	171	21 05·9	7·6	08 45
Wed.	15	18 05 59	326	22 57·9	1·5	10 30	16 20 16	172	21 13·5	7·3	08 44
Thur.	16	18 11 25	326	22 59·4	0·9	10 32	16 23 08	171	21 20·8	7·2	08 43
Fri.	17	18 16 51	325	23 00·3	0·1	10 33	16 25 59	172	21 28·0	7·1	08 41
Sat.	18	18 22 16	326	23 00·4	0·6	10 35	16 28 51	172	21 35·1	6·8	08 40
Sun.	19	18 27 42	326	22 59·8	1·3	10 36	16 31 43	173	21 41·9	6·7	08 39
Mon.	20	18 33 08	326	22 58·5	1·9	10 38	16 34 36	173	21 48·6	6·5	08 38
Tues.	21	18 38 34	325	22 56·6	2·7	10 39	16 37 29	174	21 55·1	6·3	08 37
Wed.	22	18 43 59	326	22 53·9	3·4	10 41	16 40 23	173	22 01·4	6·1	08 36
Thur.	23	18 49 25	325	22 50·5	4·1	10 42	16 43 16	174	22 07·5	6·0	08 35
Fri.	24	18 54 50	324	22 46·4	4·8	10 44	16 46 10	175	22 13·5	5·7	08 34
Sat.	25	19 00 14	325	22 41·6	5·5	10 45	16 49 05	174	22 19·2	5·6	08 33
Sun.	26	19 05 39	324	22 36·1	6·1	10 47	16 51 59	175	22 24·8	5·4	08 32
Mon.	27	19 11 03	323	22 30·0	6·9	10 48	16 54 54	176	22 30·2	5·2	08 31
Tues.	28	19 16 26	323	22 23·1	7·5	10 50	16 57 50	175	22 35·4	5·0	08 30
Wed.	29	19 21 49	322	22 15·6	8·2	10 51	17 00 45	176	22 40·4	4·8	08 29
Thur.	30	19 27 11	321	22 07·4	8·9	10 52	17 03 41	176	22 45·2	4·6	08 28
Fri.	31	19 32 32	321	21 58·5	9·5	10 54	17 06 37	176	22 49·8	4·4	08 27
Sat.	32	19 37 53		S. 21 49·0		10 55	17 09 33		S. 22 54·2		08 26

Horizontal parallax 0′·1
Magnitude
1st, −3·4; 11th, −3·4; 21st, −3·4
Venus is a morning star, in the constellations Ophiuchus and Sagittarius.

Horizontal parallax 0′·1
Magnitude
1st, 1·8; 11th, 1·7; 21st, 1·6
Mars is a morning star, in the constellations Scorpio and Ophiuchus.

THE SUN

G.M.T.	R	Dec.	E	G.M.T.	R	Dec.	E
		Wednesday 2				**Sunday 6**	
h	h m s	°	h m s	h	h m s	° ′	h m s
00	18 38 33·7	N. 23 05·3	11 56 16·7	00	18 54 19·9	N. 22 45·4	11 55 33·1
02	18 38 53·4	23 05·0	11 56 15·7	02	18 54 39·6	22 44·9	11 55 32·2
04	18 39 13·1	23 04·6	11 56 14·8	04	18 54 59·3	22 44·4	11 55 31·4
06	18 39 32·8	23 04·3	11 56 13·8	06	18 55 19·0	22 44·0	11 55 30·5
08	18 39 52·5	23 03·9	11 56 12·8	08	18 55 38·8	22 43·5	11 55 29·7
10	18 40 12·2	23 03·6	11 56 11·9	10	18 55 58·5	22 43·0	11 55 28·8
12	18 40 32·0	23 03·2	11 56 10·9	12	18 56 18·2	22 42·5	11 55 28·0
14	18 40 51·7	23 02·8	11 56 10·0	14	18 56 37·9	22 42·0	11 55 27·2
16	18 41 11·4	23 02·4	11 56 09·0	16	18 56 57·6	22 41·5	11 55 26·4
18	18 41 31·1	23 02·1	11 56 08·1	18	18 57 17·4	22 40·9	11 55 25·5
20	18 41 50·8	23 01·7	11 56 07·2	20	18 57 37·1	22 40·4	11 55 24·7
22	18 42 10·5	23 01·3	11 56 06·2	22	18 57 56·8	22 39·9	11 55 23·9
		Thursday 3				**Monday 7**	
00	18 42 30·2	N. 23 00·9	11 56 05·3	00	18 58 16·5	N. 22 39·4	11 55 23·1
02	18 42 49·9	23 00·5	11 56 04·4	02	18 58 36·2	22 38·9	11 55 22·3
04	18 43 09·6	23 00·1	11 56 03·4	04	18 58 55·9	22 38·4	11 55 21·5
06	18 43 29·3	22 59·8	11 56 02·5	06	18 59 15·6	22 37·8	11 55 20·6
08	18 43 49·1	22 59·4	11 56 01·6	08	18 59 35·3	22 37·3	11 55 19·8
10	18 44 08·8	22 59·0	11 56 00·6	10	18 59 55·0	22 36·8	11 55 19·0
12	18 44 28·5	22 58·6	11 55 59·7	12	19 00 14·8	22 36·3	11 55 18·2
14	18 44 48·2	22 58·2	11 55 58·8	14	19 00 34·5	22 35·8	11 55 17·4
16	18 45 07·9	22 57·8	11 55 57·9	16	19 00 54·2	22 35·2	11 55 16·6
18	18 45 27·7	22 57·4	11 55 56·9	18	19 01 13·9	22 34·7	11 55 15·8
20	18 45 47·4	22 57·0	11 55 56·0	20	19 01 33·6	22 34·2	11 55 15·0
22	18 46 07·1	22 56·6	11 55 55·1	22	19 01 53·3	22 33·6	11 55 14·2
		Friday 4				**Tuesday 8**	
00	18 46 26·8	N. 22 56·2	11 55 54·2	00	19 02 13·0	N. 22 33·1	11 55 13·4
02	18 46 46·5	22 55·8	11 55 53·3	02	19 02 32·7	22 32·5	11 55 12·6
04	18 47 06·2	22 55·3	11 55 52·4	04	19 02 52·4	22 32·0	11 55 11·8
06	18 47 25·9	22 54·9	11 55 51·5	06	19 03 12·1	22 31·4	11 55 11·1
08	18 47 45·6	22 54·5	11 55 50·6	08	19 03 31·9	22 30·8	11 55 10·3
10	18 48 05·3	22 54·0	11 55 49·7	10	19 03 51·6	22 30·3	11 55 09·5
12	18 48 25·1	22 53·6	11 55 48·8	12	19 04 11·3	22 29·7	11 55 08·7
14	18 48 44·8	22 53·2	11 55 47·9	14	19 04 31·0	22 29·1	11 55 07·9
16	18 49 04·5	22 52·7	11 55 47·0	16	19 04 50·7	22 28·6	11 55 07·2
18	18 49 24·2	22 52·3	11 55 46·2	18	19 05 10·5	22 28·0	11 55 06·4
20	18 49 43·9	22 51·9	11 55 45·3	20	19 05 30·2	22 27·4	11 55 05·6
22	18 50 03·6	22 51·4	11 55 44·4	22	19 05 49·9	22 26·9	11 55 04·9
24	18 50 23·3	N. 22 51·0	11 55 43·5	24	19 06 09·6	N. 22 26·3	11 55 04·1
		Saturday 5				**Wednesday 9**	
00	18 50 23·3	N. 22 51·0	11 55 43·5	00	19 06 09·6	N. 22 26·3	11 55 04·1
02	18 50 43·0	22 50·5	11 55 42·6	02	19 06 29·3	22 25·7	11 55 03·4
04	18 51 02·7	22 50·1	11 55 41·7	04	19 06 49·0	22 25·1	11 55 02·6
06	18 51 22·4	22 49·6	11 55 40·9	06	19 07 08·7	22 24·6	11 55 01·9
08	18 51 42·2	22 49·1	11 55 40·0	08	19 07 28·5	22 24·0	11 55 01·1
10	18 52 01·9	22 48·7	11 55 39·1	10	19 07 48·2	22 23·4	11 55 00·4
12	18 52 21·6	22 48·2	11 55 38·2	12	19 08 07·9	22 22·8	11 54 59·6
14	18 52 41·3	22 47·7	11 55 37·3	14	19 08 27·6	22 22·2	11 54 58·9
16	18 53 01·0	22 47·3	11 55 36·5	16	19 08 47·3	22 21·6	11 54 58·1
18	18 53 20·8	22 46·8	11 55 35·6	18	19 09 07·1	22 21·0	11 54 57·4
20	18 53 40·5	22 46·3	11 55 34·8	20	19 09 26·8	22 20·4	11 54 56·7
22	18 54 00·2	22 45·9	11 55 33·9	22	19 09 46·5	22 19·8	11 54 55·9

JULY

Date		JUPITER					SATURN				
		R.A.		Dec.		Meridian Passage	R.A.		Dec.		Meridian Passage
		h m s	s	° ′	′	h m	h m s	s	° ′	′	h m
Tues.	1	04 25 50	55	N. 20 59·3	2·1	09 50	03 30 16	25	N. 16 52·5	1·4	08 54
Wed.	2	04 26 45	55	21 01·4	2·0	09 47	03 30 41	26	16 53·9	1·4	08 51
Thur.	3	04 27 40	55	21 03·4	2·0	09 44	03 31 07	24	16 55·3	1·4	08 47
Fri.	4	04 28 35	54	21 05·4	2·0	09 41	03 31 31	25	16 56·7	1·3	08 44
Sat.	5	04 29 29	54	21 07·4	2·0	09 38	03 31 56	25	16 58·0	1·3	08 40
Sun.	6	04 30 23	54	21 09·4	2·0	09 35	03 32 21	24	16 59·3	1·3	08 37
Mon.	7	04 31 17	54	21 11·4	1·9	09 32	03 32 45	24	17 00·6	1·3	08 33
Tues.	8	04 32 11	54	21 13·3	1·9	09 29	03 33 09	24	17 01·9	1·3	08 30
Wed.	9	04 33 05	53	21 15·2	1·8	09 26	03 33 33	23	17 03·2	1·2	08 26
Thur.	10	04 33 58	53	21 17·0	1·9	09 23	03 33 56	23	17 04·4	1·3	08 23
Fri.	11	04 34 51	53	21 18·9	1·8	09 20	03 34 19	23	17 05·7	1·2	08 19
Sat.	12	04 35 44	53	21 20·7	1·7	09 17	03 34 42	23	17 06·9	1·1	08 16
Sun.	13	04 36 37	52	21 22·4	1·8	09 13	03 35 05	22	17 08·0	1·2	08 12
Mon.	14	04 37 29	52	21 24·2	1·7	09 10	03 35 27	23	17 09·2	1·1	08 08
Tues.	15	04 38 21	52	21 25·9	1·7	09 07	03 35 50	22	17 10·3	1·2	08 05
Wed.	16	04 39 13	52	21 27·6	1·7	09 04	03 36 12	21	17 11·5	1·1	08 01
Thur	17	04 40 05	51	21 29·3	1·6	09 01	03 36 33	22	17 12·6	1·0	07 58
Fri.	18	04 40 56	51	21 30·9	1·6	08 58	03 36 55	21	17 13·6	1·1	07 54
Sat.	19	04 41 47	50	21 32·5	1·6	08 55	03 37 16	20	17 14·7	1·0	07 51
Sun.	20	04 42 37	51	21 34·1	1·6	08 52	03 37 36	21	17 15·7	1·0	07 47
Mon.	21	04 43 28	50	21 35·7	1·5	08 49	03 37 57	20	17 16·7	1·0	07 43
Tues.	22	04 44 18	49	21 37·2	1·5	08 46	03 38 17	20	17 17·7	1·0	07 40
Wed.	23	04 45 07	50	21 38·7	1·5	08 43	03 38 37	19	17 18·7	1·0	07 36
Thur.	24	04 45 57	49	21 40·2	1·4	08 40	03 38 56	20	17 19·7	0·9	07 33
Fri.	25	04 46 46	49	21 41·6	1·4	08 36	03 39 16	19	17 20·6	0·9	07 29
Sat.	26	04 47 35	48	21 43·0	1·4	08 33	03 39 35	18	17 21·5	0·9	07 25
Sun.	27	04 48 23	48	21 44·4	1·4	08 30	03 39 53	18	17 22·4	0·8	07 22
Mon.	28	04 49 11	47	21 45·8	1·3	08 27	03 40 11	18	17 23·2	0·9	07 18
Tues.	29	04 49 58	48	21 47·1	1·4	08 24	03 40 29	18	17 24·1	0·8	07 14
Wed.	30	04 50 46	47	21 48·5	1·3	08 21	03 40 47	17	17 24·9	0·8	07 11
Thur.	31	04 51 33	46	21 49·8	1·2	08 18	03 41 04	17	17 25·7	0·7	07 07
Fri.	32	04 52 19		N. 21 51·0		08 14	03 41 21		N. 17 26·4		07 03

Average magnitude −1·6

Jupiter is a morning star, in the constellation Taurus.

Average magnitude 0·5

Saturn is a morning star, in the constellation Taurus.

STAR ELEMENTS

No.	Name	Mag.	S.H.A.		Dec.		R.A.		
			°	′	°	′	hrs.	mins.	secs.
1	Achernar	0·6	336	07	*S*.57	32	01	35	32
2	Acrux	1·1	174	10	*S*.62	47	12	23	20
3	Aldebaran	1·1	291	52	*N*.16	23	04	32	32
4	Alpheratz	2·2	358	39	*N*.28	46	00	05	24
5	Altair	0·9	63	01	*N*. 8	43	19	47	56
6	Antares	1·2	113	33	*S*.26	18	16	25	48
7	Arcturus	0·2	146	45	*N*.19	29	14	13	00
8	Betelgeuse	[1]0·8	272	00	*N*. 7	24	05	52	00
9	Canopus	−0·9	264	20	*S*.52	40	06	22	40
10	Capella	0·2	281	55	*N*.45	56	05	12	20
11	Deneb	1·3	50	08	*N*.45	04	20	39	28
12	Dubhe	2·0	194	59	*N*.62	04	11	00	04
13	Fomalhaut	1·3	16	23	*S*.29	56	22	54	28
14	Peacock	2·1	54	44	*S*.56	55	20	21	04
15	Pollux	1·2	244	34	*N*.28	10	07	41	44
16	Procyon	0·5	245	57	*N*. 5	22	07	36	12
17	Regulus	1·3	208	41	*N*.12	15	10	05	16
18	Rigel	0·3	282	04	*S*. 8	16	05	11	44
19	Rigel Kent.	0·1	141	06	*S*.60	36	14	35	36
20	Sirius	−1·6	259	22	*S*.16	38	06	42	32
21	Spica	1·2	159	28	*S*.10	51	13	22	08
22	Vega	0·1	81	15	*N*.38	44	18	35	00
23	*Adara*	1·6	255	55	*S*.28	53	06	56	20
24	*Alhena*	1·9	261	25	*N*.16	27	06	34	20
25	*Alnitak*	1·9	275	33	*S*. 1	58	05	37	48
26	*Alphacca*	2·3	126	57	*N*.26	55	15	32	12
27	*Alphard*	2·2	218	49	*S*. 8	24	09	24	44
28	*Anilam*	1·8	276	41	*S*. 1	14	05	33	16
29	*Bellatrix*	1·7	279	30	*N*. 6	18	05	22	00
30	*Denebola*	2·2	183	29	*N*.14	54	11	46	04
31	*Diphda*	2·2	349	50	*S*.18	18	00	40	40
32	*Hamal*	2·2	329	02	*N*.23	11	02	03	52
33	*Nath*	1·8	279	21	*N*.28	33	05	22	36
34	*Ras Alhague*	2·1	96	57	*N*.12	36	17	32	12
35	*Wezen*	2·0	253	30	*S*.26	18	07	06	00
36	*Alioth*	1·7	167	09	*N*.56	17	12	51	24
37	γ Argus	1·9	238	04	*S*.47	10	08	07	44
38	ε Argus	1·7	234	41	*S*.59	19	08	21	16
39	*Benetnasch*	1·9	153	42	*N*.49	37	13	45	12
40	*Castor*	1·6	247	17	*N*.32	01	07	30	52
41	β Centauri	0·9	150	05	*S*.60	06	13	59	40
42	β Crucis	1·5	168	56	*S*.59	22	12	44	16
43	γ Crucis	1·6	173	01	*S*.56	47	12	27	56
44	*Kaus Aust.*	2·0	84	55	*S*.34	25	18	20	20
45	*Miaplacidus*	1·8	221	52	*S*.69	29	09	12	32
46	*Mirfak*	1·9	309	58	*N*.49	39	03	20	08
47	*Polaris*	2·1	333	54	*N*.88	59	01	44	24
48	*Schedar*	2·3	350	42	*N*.56	13	00	37	12
49	*Shaula*	1·7	97	35	*S*.37	04	17	29	40
50	α Tri. Aust.	1·9	109	23	*S*.68	55	16	42	28

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[1] Betelgeuse, variable magnitude 0·5–1·1.

EXTRACTS FROM THE *AIR ALMANAC*

STARS, SUN and PLANETS — JAN. 4, GREENWICH DATE

G. M. T.	G.H.A. of First Point of ARIES = G.H.A.♈ 00ᵐ	10ᵐ	20ᵐ	30ᵐ	40ᵐ	50ᵐ	G. M. T.	VENUS −3·4 G.H.A.	Dec.	MARS 1·8 G.H.A.	Dec.
h	° ′	° ′	° ′	° ′	° ′	° ′	h	° ′	° ′	° ′	° ′
00	103 13	105 43	108 13	110 44	113 14	115 45	00	206 32	*S.21 56*	225 53	*S.19 42*
01	118 15	120 45	123 16	125 46	128 17	130 47	01	221 31	*21 56*	240 54	*19 42*
02	133 18	135 48	138 18	140 49	143 19	145 50	02	236 30	*21 57*	255 55	*19 42*
03	148 20	150 50	153 21	155 51	158 22	160 52	03	251 29	*21 57*	270 55	*19 43*
04	163 22	165 53	168 23	170 54	173 24	175 55	04	266 29	*21 58*	285 56	*19 43*
05	178 25	180 55	183 26	185 56	188 27	190 57	05	281 28	*S.21 58*	300 57	*S.19 43*
06	193 27	195 58	198 28	200 59	203 29	205 59	06	296 27	*21 58*	315 57	*19 44*
07	208 30	211 00	213 31	216 01	218 32	221 02	07	311 26	*21 59*	330 58	*19 44*
08	223 32	226 03	228 33	231 04	233 34	236 04	08	326 25	*21 59*	345 59	*19 45*
09	238 35	241 05	243 36	246 06	248 36	251 07	09	341 24	*21 59*	1 00	*19 45*
10	253 37	256 08	258 38	261 08	263 39	266 09	10	356 23	*S.22 00*	16 00	*S.19 45*
11	268 40	271 10	273 41	276 11	278 41	281 12	11	11 22	*22 00*	31 01	*19 46*
12	283 42	286 13	288 43	291 13	293 44	296 14	12	26 22	*22 01*	46 02	*19 46*
13	298 45	301 15	303 45	306 16	308 46	311 17	13	41 21	*22 01*	61 03	*19 46*
14	313 47	316 18	318 48	321 18	323 49	326 19	14	56 20	*22 01*	76 03	*19 47*
15	328 50	331 20	333 50	336 21	338 51	341 22	15	71 19	*S.22 02*	91 04	*S.19 47*
16	343 52	346 22	348 53	351 23	353 54	356 24	16	86 18	*22 02*	106 05	*19 48*
17	358 55	1 25	3 55	6 26	8 56	11 27	17	101 17	*22 02*	121 05	*19 48*
18	13 57	16 27	18 58	21 28	23 59	26 29	18	116 16	*22 03*	136 06	*19 48*
19	28 59	31 30	34 00	36 31	39 01	41 31	19	131 15	*22 03*	151 07	*19 49*
20	44 02	46 32	49 03	51 33	54 04	56 34	20	146 15	*S.22 04*	166 08	*S.19 49*
21	59 04	61 35	64 05	66 36	69 06	71 36	21	161 14	*22 04*	181 08	*19 50*
22	74 07	76 37	79 08	81 38	84 08	86 39	22	176 13	*22 04*	196 09	*19 50*
23	89 09	91 40	94 10	96 41	99 11	101 41	23	191 12	*22 05*	211 10	*19 50*

G. M. T.	G.H.A. of SUN 00ᵐ	10ᵐ	20ᵐ	30ᵐ	40ᵐ	50ᵐ	SUN'S *Dec*	G. M. T.	JUPITER −2·2 G.H.A	*Dec.*	Planets
h	° ′	° ′	° ′	° ′	° ′	° ′	° ′	h	° ′	° ′	Mean
00	178 49	181 19	183 49	186 18	188 48	191 18	*S.22 47*	00	69 25	*N.12 20*	S.H.A
01	193 48	196 18	198 48	201 18	203 48	206 18	*22 47*	01	84 28	*12 20*	°
02	208 48	211 18	213 48	216 18	218 48	221 18	*22 46*	02	99 30	*12 20*	V. 103
03	223 48	226 18	228 48	231 18	233 48	236 18	*22 46*	03	114 32	*12 20*	M. 122
04	238 47	241 17	243 47	246 17	248 47	251 17	*22 46*	04	129 35	*12 20*	J. 326
05	253 47	256 17	258 47	261 17	263 47	266 17	*S.22 46*	05	144 37	*N.12 20*	
06	268 47	271 17	273 47	276 17	278 47	281 17	*22 45*	06	159 40	*12 20*	Mean
07	283 47	286 17	288 47	291 16	293 46	296 16	*22 45*	07	174 42	*12 20*	R.A
08	298 46	301 16	303 46	306 16	308 46	311 16	*22 45*	08	189 45	*12 20*	h m
09	313 46	316 16	318 46	321 16	323 46	326 16	*22 45*	09	204 47	*12 20*	V. 17 09 M. 15 51
10	328 46	331 16	333 46	336 16	338 46	341 16	*S.22 44*	10	219 49	*N.12 20*	J. 02 15
11	343 45	346 15	348 45	351 15	353 45	356 15	*22 44*	11	234 52	*12 20*	
12	358 45	1 15	3 45	6 15	8 45	11 15	*22 44*	12	249 54	*12 20*	Mer
13	13 45	16 15	18 45	21 15	23 45	26 15	*22 44*	13	264 57	*12 20*	Pass.
14	28 45	31 15	33 45	36 14	38 44	41 14	*22 43*	14	279 59	*12 20*	h m
15	43 44	46 14	48 44	51 14	53 44	56 14	*S.22 43*	15	295 02	*N.12 20*	V. 10 15
16	58 44	61 14	63 44	66 14	68 44	71 14	*22 43*	16	310 04	*12 20*	M. 08 56
17	73 44	76 14	78 44	81 14	83 44	86 14	*22 42*	17	325 06	*12 20*	J. 19 19
18	88 43	91 13	93 43	96 13	98 43	101 13	*22 42*	18	340 09	*12 20*	
19	103 43	106 13	108 43	111 13	113 43	116 13	*22 42*	19	355 11	*12 20*	
20	118 43	121 13	123 43	126 13	128 43	131 13	*S.22 42*	20	10 14	*N.12 20*	Sun's
21	133 43	136 13	138 43	141 12	143 42	146 12	*22 41*	21	25 16	*12 20*	S.D.
22	148 42	151 12	153 42	156 12	158 42	161 12	*22 41*	22	40 19	*12 20*	16′
23	163 42	166 12	168 42	171 12	173 42	176 12	*22 41*	23	55 21	*12 20*	

Venus is conspicuous and is visible low in the east shortly before sunrise. Mars is observable low in the east for some time before sunrise. Jupiter is easily observable in the west until midnight.

◐ MOON — JANUARY 4, GREENWICH DATE

G.M.T.	00m G.H.A.	00m Dec.	10m G.H.A.	10m Dec.	20m G.H.A.	20m Dec.	30m G.H.A.	30m Dec.	40m G.H.A.	40m Dec.	50m G.H.A.	50m Dec.
h	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′
00	107 00	S.0 44	109 25	S.0 42	111 51	S.0 41	114 17	S 0 39	116 42	S.0 38	119 08	S.0 36
01	121 34	0 34	123 59	0 33	126 25	0 31	128 51	0 30	131 16	0 28	133 42	0 26
02	136 08	0 25	138 33	0 23	140 59	0 22	143 25	0 20	145 50	0 19	148 16	0 17
03	150 42	0 15	153 07	0 14	155 33	0 12	157 59	0 11	160 24	S.0 09	162 50	S.0 08
04	165 16	S.0 06	167 41	S.0 04	170 07	S.0 03	172 33	S.0 01	174 58	0 00	177 24	N.0 02
05	179 50	N.0 03	182 15	N.0 05	184 41	N.0 07	187 07	N.0 08	189 32	N.0 10	191 58	N.0 11
06	194 24	0 13	196 49	0 14	199 15	0 16	201 41	0 18	204 06	0 19	206 32	0 21
07	208 58	0 22	211 23	0 24	213 49	0 26	216 15	0 27	218 41	0 29	221 06	0 30
08	223 32	0 32	225 58	0 33	228 23	0 35	230 49	0 37	233 15	0 38	235 40	0 40
09	238 06	0 41	240 32	0 43	242 57	0 44	245 23	0 46	247 49	0 48	250 14	0 49
10	252 40	N.0 51	255 06	N.0 52	257 31	N 0 54	259 57	N.0 55	262 23	N.0 57	264 48	N.0 59
11	267 14	1 00	269 40	1 02	272 06	1 03	274 31	1 05	276 57	1 06	279 23	1 08
12	281 48	1 10	284 14	1 11	286 40	1 13	289 05	1 14	291 31	1 16	293 57	1 17
13	296 22	1 19	298 48	1 21	301 14	1 22	303 40	1 24	306 05	1 25	308 31	1 27
14	310 57	1 28	313 22	1 30	315 48	1 32	318 14	1 33	320 39	1 35	323 05	1 36
15	325 31	N.1 38	327 56	N.1 39	330 22	N.1 41	332 48	N.1 42	335 13	N.1 44	337 39	N 1 46
16	340 05	1 47	342 31	1 49	344 56	1 50	347 22	1 52	349 48	1 53	352 13	1 55
17	354 39	1 57	357 05	1 58	359 30	2 00	1 56	2 01	4 22	2 03	6 48	2 04
18	9 13	2 06	11 39	2 07	14 05	2 09	16 30	2 11	18 56	2 12	21 22	2 14
19	23 47	2 15	26 13	2 17	28 39	2 18	31 04	2 20	33 30	2 21	35 56	2 23
20	38 22	N.2 25	40 47	N.2 26	43 13	N.2 28	45 39	N.2 29	48 04	N.2 31	50 30	N 2 32
21	52 56	2 34	55 21	2 35	57 47	2 37	60 13	2 39	62 39	2 40	65 04	2 42
22	67 30	2 43	69 56	2 45	72 21	2 46	74 47	2 48	77 13	2 49	79 38	2 51
23	82 04	2 52	84 30	2 54	86 56	2 56	89 21	2 57	91 47	2 59	94 13	3 00

P. in A.

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Lat.	Twilight	Sunrise	Moonrise	Diff.
°	h m	h m	h m	m
N.68	09 14	11 42	11 02	2
66	08 51	10 23	11 03	4
64	08 32	09 47	11 04	5
62	08 17	09 21	11 04	7
N.60	08 04	09 01	11 05	8
58	07 54	08 45	11 06	9
56	07 44	08 31	11 06	10
54	07 35	08 19	11 06	10
52	07 27	08 08	11 07	11
N.50	07 20	07 58	11 07	12
45	07 04	07 38	11 08	14
40	06 51	07 22	11 08	15
35	06 40	07 09	11 09	16
30	06 30	06 57	11 10	17
N.20	06 12	06 36	11 10	19
N.10	05 55	06 18	11 11	20
0	05 39	06 01	11 12	21
S.10	05 21	05 45	11 12	23
S.20	05 02	05 26	11 13	24
S.30	04 37	05 05	11 14	26
35	04 22	04 52	11 15	27
40	04 04	04 38	11 16	28
45	03 43	04 21	11 16	30
S.50	03 15	03 59	11 17	31

Lat.	Sunset	Twilight	Moonset	Diff.
°	h m	h m	h m	m
N.68	12 28	14 55	(22 39)	42
66	13 47	15 18	23 59	40
64	14 24	15 37	23 57	38
62	14 49	15 52	23 55	36
N.60	15 09	16 05	23 53	35
58	15 25	16 15	23 52	34
56	15 40	16 25	23 50	33
54	15 52	16 34	23 49	32
52	16 02	16 42	23 48	32
N.50	16 12	16 49	23 47	31
45	16 32	17 05	23 44	29
40	16 48	17 18	23 42	28
35	17 01	17 29	23 41	27
30	17 13	17 40	23 39	26
N.20	17 34	17 58	23 37	24
N.10	17 52	18 14	23 34	23
0	18 09	18 31	23 32	21
S.10	18 26	18 48	23 30	20
S.20	18 44	19 08	23 28	19
S.30	19 06	19 32	23 25	17
35	19 18	19 47	23 24	16
40	19 33	20 04	23 22	15
45	19 50	20 26	23 20	14
S.50	20 12	20 54	23 18	12

Phase of Moon—First Quarter S D 15′ Meridian Passage 17 22 21

Figures in ( ) refer to the previous day as there is no moonset from 00^h to 24^h on Greenwich meridian.

STARS, SUN and PLANETS — JULY 9, G.D. (WEDNESDAY)

G.M.T.	G.H.A. of First Point of ARIES = G.H.A.♈ 00m	10m	20m	30m	40m	50m	G.M.T.	VENUS −3·3 G.H.A.	Dec.	MARS −0·5 G.H.A.	Dec.
h	° ′	° ′	° ′	° ′	° ′	° ′	h	° ′	° ′	° ′	° ′
00	286 32	289 03	291 33	294 04	296 34	299 04	00	155 48	*N.19 47*	281 39	*S.1 37*
01	301 35	304 05	306 36	309 06	311 37	314 07	01	170 47	*19 47*	296 40	*1 36*
02	316 37	319 08	321 38	324 09	326 39	329 09	02	185 47	*19 46*	311 41	*1 36*
03	331 40	334 10	336 41	339 11	341 41	344 12	03	200 46	*19 45*	326 42	*1 35*
04	346 42	349 13	351 43	354 13	356 44	359 14	04	215 45	*19 44*	341 44	*1 35*
05	1 45	4 15	6 46	9 16	11 46	14 17	05	230 45	*N.19 44*	356 45	*S.1 34*
06	16 47	19 18	21 48	24 18	26 49	29 19	06	245 44	*19 43*	11 46	*1 34*
07	31 50	34 20	36 50	39 21	41 51	44 22	07	260 43	*19 42*	26 47	*1 34*
08	46 52	49 23	51 53	54 23	56 54	59 24	08	275 43	*19 41*	41 48	*1 33*
09	61 55	64 25	66 55	69 26	71 56	74 27	09	290 42	*19 40*	56 49	*1 33*
10	76 57	79 27	81 58	84 28	86 59	89 29	10	305 41	*N.19 40*	71 51	*S.1 32*
11	92 00	94 30	97 00	99 31	102 01	104 32	11	320 41	*19 39*	86 52	*1 32*
12	107 02	109 32	112 03	114 33	117 04	119 34	12	335 40	*19 38*	101 53	*1 31*
13	122 04	124 35	127 05	129 36	132 06	134 36	13	350 39	*19 37*	116 54	*1 31*
14	137 07	139 37	142 08	144 38	147 09	149 39	14	5 39	*19 36*	131 55	*1 30*
15	152 09	154 40	157 10	159 41	162 11	164 41	15	20 38	*N.19 36*	146 56	*S.1 30*
16	167 12	169 42	172 13	174 43	177 13	179 44	16	35 38	*19 35*	161 58	*1 29*
17	182 14	184 45	187 15	189 46	192 16	194 46	17	50 37	*19 34*	176 59	*1 29*
18	197 17	199 47	202 18	204 48	207 18	209 49	18	65 36	*19 33*	192 00	*1 28*
19	212 19	214 50	217 20	219 50	222 21	224 51	19	80 36	*19 33*	207 01	*1 28*
20	227 22	229 52	232 23	234 53	237 23	239 54	20	95 35	*N.19 32*	222 02	*S.1 27*
21	242 24	244 55	247 25	249 55	252 26	254 56	21	110 34	*19 31*	237 03	*1 27*
22	257 27	259 57	262 27	264 58	267 28	269 59	22	125 34	*19 30*	252 05	*1 26*
23	272 29	274 59	277 30	280 00	282 31	285 01	23	140 33	*19 29*	267 06	*1 26*

G.M.T.	G.H.A. of SUN 00m	10m	20m	30m	40m	50m	SUN'S Dec.	G.M.T.	JUPITER −1·6 G.H.A.	Dec.	Planets
h	° ′	° ′	° ′	° ′	° ′	° ′	° ′	h	° ′	° ′	
00	178 46	181 16	183 46	186 16	188 46	191 16	*N.22 26*	00	218 16	*N.21 15*	Mean S.H.A.
01	193 46	196 16	198 46	201 16	203 46	206 16	*22 26*	01	233 18	*21 15*	°
02	208 46	211 16	213 46	216 16	218 46	221 16	*22 26*	02	248 20	*21 15*	V. 229
03	223 46	226 16	228 46	231 16	233 46	236 16	*22 25*	03	263 22	*21 15*	M. 355
04	238 46	241.16	243 46	246 16	248 46	251 16	*22 25*	04	278 24	*21 16*	J. 292
05	253 46	256 16	258 46	261 16	263 46	266 15	*N.22 25*	05	293 26	*N.21 16*	
06	268 45	271 15	273 45	276 15	278 45	281 15	*22 24*	06	308 28	*21 16*	Mean R.A.
07	283 45	286 15	288 45	291 15	293 45	296 15	*22 24*	07	323 30	*21 16*	°
08	298 45	301 15	303 45	306 15	308 45	311 15	*22 24*	08	338 31	*21 16*	V. 131
09	313 45	316 15	318 45	321 15	323 45	326 15	*22 24*	09	353 33	*21 16*	M. 5
10	328 45	331 15	333 45	336 15	338 45	341 15	*N.22 23*	10	8 35	*N.21 16*	J. 68
11	343 45	346 15	348 45	351 15	353 45	356 15	*22 23*	11	23 37	*21 16*	
12	358 45	1 15	3 45	6 15	8 45	11 15	*22 23*	12	38 39	*21 16*	Mer. Pass.
13	13 45	16 15	18 45	21 15	23 45	26 15	*22 22*	13	53 41	*21.16*	
14	28 45	31 15	33 45	36 15	38 45	41 15	*22 22*	14	68 43	*21 16*	h m
15	43 45	46 15	48 45	51 15	53 45	56 15	*N.22 22*	15	83 45	*N.21 16*	V. 13 37
16	58 45	61 15	63 45	66 14	68 44	71 14	*22 21*	16	98 47	*21 16*	M. 05 13
17	73 44	76 14	78 44	81 14	83 44	86 14	*22 21*	17	113 49	*21 16*	J. 09 26
18	88 44	91 14	93 44	96 14	98 44	101 14	*22 21*	18	128 51	*21 17*	
19	103 44	106 14	108 44	111 14	113 44	116 14	*22 21*	19	143 52	*21 17*	
20	118 44	121 14	123 44	126 14	128 44	131 14	*N.22 20*	20	158 54	*N.21 17*	Sun's S.D.
21	133 44	136 14	138 44	141 14	143 44	146 14	*22 20*	21	173 56	*21 17*	
22	148 44	151 14	153 44	156 14	158 44	161 14	*22 20*	22	188 58	*21 17*	16′
23	163 44	166 14	168 44	171 14	173 44	176 14	*22 19*	23	204 00	*21 17*	

Venus is a conspicuous object observable low in the west at sunset. Mars is easily observable in the east after midnight. Jupiter is observable low in the east for some time before sunrise.

Full ○ MOON — JULY 9, G.D. (WEDNESDAY)

G. M. T.	00m G.H.A.	00m Dec.	10m G.H.A.	10m Dec.	20m G.H.A.	20m Dec.
h	° ′	° ′	° ′	° ′	° ′	° ′
00	357 13	S.17 35	359 37	S.17 34	2 01	S.17 33
01	11 38	17 31	14 02	17 30	16 27	17 30
02	26 04	17 27	28 28	17 27	30 52	17 26
03	40 29	17 24	42 53	17 23	45 18	17 22
04	54 55	17 20	57 19	17 19	59 43	17 18
05	69 20	S.17 16	71 44	S.17 15	74 09	S.17 14
06	83 46	17 12	86 10	17 11	88 34	17 10
07	98 11	17 07	100 36	17 07	103 00	17 06
08	112 37	17 03	115 01	17 02	117 26	17 02
09	127 03	16 59	129 27	16 58	131 52	16 57
10	141 29	S.16 54	143 53	S.16 53	146 17	S.16 53
11	155 55	16 50	158 19	16 49	160 43	16 48
12	170 21	16 45	172 45	16 44	175 09	16 43
13	184 47	16 40	187 11	16 39	189 35	16 38
14	199 13	16 35	201 37	16 34	204 02	16 33
15	213 39	S.16 30	216 03	S.16 29	218 28	S.16 28
16	228 05	16 25	230 30	16 24	232 54	16 23
17	242 32	16 20	244 56	16 19	247 20	16 18
18	256 58	16 14	259 22	16 14	261 47	16 13
19	271 24	16 09	273 49	16 08	276 13	16 07
20	285 51	S.16 04	288 15	S.16 03	290 40	S.16 02
21	300 18	15 58	302 42	15 57	305 06	15 56
22	314 44	15 52	317 09	15 51	319 33	15 50
23	329 11	15 46	331 35	15 45	334 00	15 45

G. M. T.	30m G.H.A.	30m Dec.	40m G.H.A.	40m Dec.	50m G.H.A.	50m Dec.
h	° ′	° ′	° ′	° ′	° ′	° ′
00	4 26	S.17 33	6 50	S.17 32	9 14	S.17 32
01	18 51	17 29	21 15	17 29	23 39	17 28
02	33 16	17 25	35 41	17 25	38 05	17 24
03	47 42	17 22	50 06	17 21	52 30	17 20
04	62 07	17 18	64 32	17 17	66 56	17 16
05	76 33	S.17 14	78 57	S.17 13	81 21	S.17 12
06	90 59	17 10	93 23	17 09	95 47	17 08
07	105 24	17 05	107 49	17 05	110 13	17 04
08	119 50	17 01	122 14	17 00	124 39	16 59
09	134 16	16 56	136 40	16 56	139 04	16 55
10	148 42	S.16 52	151 06	S.16 51	153 30	S.16 50
11	163 08	16 47	165 32	16 46	167 56	16 46
12	177 34	16 42	179 58	16 42	182 22	16 41
13	192 00	16 38	194 24	16 37	196 48	16 36
14	206 26	16 33	208 50	16 32	211 15	16 31
15	220 52	S.16 28	223 17	S.16 27	225 41	S.16 26
16	235 18	16 22	237 43	16 22	240 07	16 21
17	249 45	16 17	252 09	16 16	254 34	16 15
18	264 11	16 12	266 36	16 11	269 00	16 10
19	278 38	16 06	281 02	16 05	283 27	16 04
20	293 04	S.16 01	295 29	S.16 00	297 53	S.15 59
21	307 31	15 55	309 55	15 54	312 20	15 53
22	321 58	15 49	324 22	15 48	326 47	15 47
23	336 24	15 44	338 49	15 43	341 13	15 42

P. in A. Alt.	P. in A. Corr.
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Lat.	Twilight	Sunrise	Moonrise	Diff.
°	h m	h m	h m	m
N.68	S.A.H.	S.A.H.	22 02	10
66	T.A.N.	01 05	21 42	13
64	T.A.N.	01 56	21 25	15
62	T.A.N.	02 28	21 11	16
N.60	01 25	02 52	20 59	17
58	02 04	03 10	20 49	18
56	02 28	03 26	20 40	19
54	02 48	03 39	20 32	20
52	03 04	03 50	20 25	21
N.50	03 18	04 00	20 19	21
45	03 45	04 22	20 05	22
40	04 06	04 39	19 53	23
35	04 24	04 53	19 43	24
30	04 39	05 05	19 35	25
N.20	05 03	05 26	19 20	26
N.10	05 22	05 44	19 07	27
0	05 39	06 01	18 55	28
S.10	05 55	06 18	18 43	29
S.20	06 12	06 36	18 30	30
S.30	06 30	06 56	18 15	31
35	06 40	07 08	18 06	31
40	06 51	07 21	17 56	32
45	07 03	07 37	17 45	33
S.50	07 19	07 57	17 31	34

Lat.	Sunset	Twilight	Moonset	Diff.
°	h m	h m	h m	m
N.68	S.A.H.	S.A.H.	02 52	42
66	23 01	T.A.N.	03 18	40
64	22 10	T.A.N.	03 38	38
62	21 40	T.A.N.	03 55	37
N.60	21 18	22 46	04 08	36
58	20 59	22 09	04 20	35
56	20 44	21 42	04 30	35
54	20 30	21 23	04 39	34
52	20 19	21 06	04 47	34
N.50	20 09	20 52	04 54	33
45	19 48	20 25	05 09	33
40	19 31	20 03	05 22	32
35	19 16	19 46	05 33	31
30	19 04	19 31	05 42	31
N.20	18 43	19 07	05 58	30
N.10	18 25	18 48	06 12	29
0	18 08	18 31	06 25	28
S.10	17 53	18 15	06 38	28
S.20	17 35	17 59	06 52	27
S.30	17 15	17 41	07 08	26
35	17 03	17 31	07 17	26
40	16 49	17 20	07 27	25
45	16 34	17 07	07 39	24
S.50	16 14	16 52	07 54	24

Phase of Moon—Full Moon. S.D. 16′ Meridian Passage 00 12 29

S.A.H.—Sun above the horizon all day. T.A.N.—Twilight all night.

EXTRACTS FROM THE ADMIRALTY TIDE TABLES

ENGLAND, EAST COAST—IMMINGHAM

Lat. 53° 37′ N. Long. 0° 11′ W.

Month Day	Week Day	SEPTEMBER.			
		HIGH WATER		LOW WATER	
		Time	Ht.	Time	Ht.
		H. M.	F.	H. M.	F.
1	𝔖.	07 26	20·4	01 24	3·0
		19 51	19·5	13 45	2·2
2	M.	07 56	20·0	01 53	3·1
		20 19	19·0	14 14	2·8
3	Tu.	08 26	19·6	02 21	3·5
		20 46	18·8	14 41	3·2
4	W.	09 00	18·8	02 51	4·0
		21 20	18·0	15 15	4·1
5	Th.	09 40	17·9	03 29	4·8
		22 00	17·2	15 55	5·1
6	F.	10 31	16·8	04 14	5·8
		22 56	16·2	16 46	6·2
7	S.	11 50	15·9	05 19	6·5
		—	—	18 05	7·0
8	𝔖.	00 19	16·0	06 51	6·7
		13 34	15·9	19 40	6·8
9	M.	01 51	16·7	08 30	5·5
		14 55	17·1	20 56	5·4
10	Tu	03 06	18·1	09 36	3·7
		15 56	18·9	21 56	3·8
11	W	04 04	20·1	10 30	1·7
		16 45	20·4	22 46	2·1
12	Th	04 51	21·8	11 20	0·0
		17 30	21·7	23 35	1·0
13	F	05 39	23·0	—	—
		18 15	22·2	12 04	*0·9
14	S	06 23	23·8	00 19	0·4
		18 56	22·2	12 46	*1·1
15	𝔖.	07 05	23·8	01 01	0·1
		19 39	21·9	13 30	*0·9
16	M.	07 50	23·1	01 44	0·6
		20 20	21·2	14 10	0·2
17	Tu	08 34	22·0	02 25	1·3
		21 00	20·1	14 50	1·8
18	W.	09 20	20·3	03 08	2·6
		21 41	18·7	15 30	3·4
19	Th	10 10	18·3	03 53	4·1
		22 31	17·2	16 16	5·3
20	F.	11 16	16·9	04 50	5·7
		23 39	16·1	17 15	7·2
21	S	—	—	06 11	6·6
		12 45	15·8	18 41	8·1
22	𝔖.	01 09	15·8	07 50	6·2
		14 15	15·9	20 15	7·8
23	M	02 30	16·4	09 05	5·2
		15 20	16·9	21 20	6·8
24	Tu.	03 29	17·6	09 55	4·0
		16 05	18·0	22 05	5·4
25	W.	04 14	18·8	10 36	3·0
		16 45	19·0	22 44	4·4
26	Th.	04 50	19·8	11 11	2·4
		17 20	19·7	23 20	3·8
27	F.	05 25	20·2	11 46	2·1
		17 51	20·0	23 54	3·1
28	S.	05 59	20·6	—	—
		18 24	20·1	12 19	2·1
29	𝔖.	06 30	20·6	00 26	3·0
		18 51	20·2	12 46	2·2
30	M.	07 00	20·3	00 56	3·0
		19 20	20·0	13 16	2·6
31					

Month Day	Week Day	OCTOBER.			
		HIGH WATER		LOW WATER	
		Time	Ht.	Time	Ht.
		H. M.	F.	H. M.	F.
1	Tu	07 31	20·2	01 26	2·9
		19 46	19·9	13 45	2·9
2	W.	08 03	19·7	01 59	3·0
		20 16	19·3	14 16	3·3
3	Th.	08 39	18·9	02 30	3·2
		20 50	18·8	14 50	4·1
4	F.	09 20	18·0	03 09	4·1
		21 35	18·0	15 31	5·0
5	S.	10 16	16·9	03 54	4·8
		22 31	17·0	16 25	6·1
6	𝔖.	11 35	16·0	04 58	5·8
		23 50	16·7	17 40	6·9
7	M.	—	—	06 30	6·0
		13 14	16·2	19 14	6·8
8	Tu	01 24	17·1	08 06	4·9
		14 35	17·7	20 34	5·5
9	W	02 40	18·7	09 14	3·1
		15 35	19·1	21 34	3·8
10	Th	03 40	20·4	10 08	1·3
		16 24	20·8	22 25	2·1
11	F.	04 30	22·1	10 55	0·2
		17 09	21·8	23 12	1·2
12	S.	05 16	23·0	11 40	*0·6
		17 50	22·2	23 58	0·5
13	𝔖	06 01	23·3	—	—
		18 31	22·3	12 24	*0·3
14	M	06 45	23·2	00 40	0·5
		19 12	21·9	13 05	0·2
15	Tu	07 30	22·3	01 24	0·9
		19 51	21·2	13 45	1·2
16	W.	08 15	21·2	02 05	1·3
		20 31	20·1	14 25	2·6
17	Th	09 00	19·7	02 47	2·6
		21 12	18·9	15 05	4·2
18	F.	09 50	18·0	03 31	3·9
		21 59	17·9	15 48	6·0
19	S.	10 52	16·4	04 25	5·1
		22 56	16·6	16 40	7·2
20	𝔖.	—	—	05 35	6·2
		12 12	15·6	17 55	8·2
21	M.	00 16	15·9	07 10	6·1
		13 36	15·7	19 29	8·0
22	Tu.	01 42	16·1	08 25	5·6
		14 41	16·7	20 39	7·0
23	W.	02 48	17·1	09 19	4·7
		15 30	17·7	21 30	5·7
24	Th	03 36	18·2	10 00	3·8
		16 10	18·9	22 12	4·8
25	F.	04 18	19·2	10 38	3·0
		16 46	19·6	22 50	3·8
26	S.	04 55	19·9	11 13	2·6
		17 20	20·1	23 26	3·1
27	𝔖.	05 30	20·2	11 45	2·3
		17 50	20·3	—	—
28	M.	06 04	20·2	00 00	3·0
		18 20	20·3	12 19	2·6
29	Tu.	06 38	20·2	00 34	2·9
		18 50	20·2	12 50	2·9
30	W.	07 11	19·9	01 06	2·9
		19 21	20·1	13 21	3·1
31	Th.	07 46	19·5	01 40	2·9
		19 54	19·9	13 56	3·3

Month Day	Week Day	NOVEMBER.			
		HIGH WATER		LOW WATER	
		Time	Ht.	Time	Ht.
		H. M.	F.	H. M.	F.
1	F.	08 25	18·9	02 15	3·0
		20 32	19·3	14 34	4·0
2	S	09 11	18·1	02 56	3·3
		21 19	18·7	15 18	4·8
3	𝔖.	10 09	17·1	03 44	4·0
		22 16	18·0	16 10	5·8
4	M	11 22	16·7	04 45	4·8
		23 29	17·4	17 20	6·5
5	Tu.	—	—	06 10	4·9
		12 50	16·6	18 45	6·6
6	W	00 55	17·6	07 38	4·3
		14 10	17·6	20 05	5·6
7	Th	02 14	18·7	08 48	3·0
		15 10	19·0	21 10	4·1
8	F.	03 16	20·0	09 44	1·8
		16 00	20·1	22 05	2·8
9	S	04 10	21·2	10 33	0·9
		16 46	21·1	22 54	1·8
10	𝔖.	04 58	22·0	11 20	0·9
		17 30	21·8	23 40	1·1
11	M	05 45	22·2	—	—
		18 10	21·7	12 02	1·0
12	Tu.	06 30	22·1	00 25	1·1
		18 50	21·3	12 44	1·7
13	W	07 15	21·3	01 08	1·2
		19 30	20·8	13 25	2·3
14	Th	08 00	20·2	01 50	1·9
		20 10	20·2	14 04	3·2
15	F	08 45	19·1	02 31	2·4
		20 49	19·3	14 41	4·6
16	S	09 33	17·9	03 14	3·2
		21 31	18·2	15 21	5·8
17	𝔖.	10 25	16·9	04 00	4·2
		22 21	17·2	16 08	6·9
18	M.	11 28	15·8	04 55	5·1
		23 24	16·4	17 05	7·7
19	Tu.	—	—	06 06	5·9
		12 40	15·8	18 21	7·9
20	W.	00 40	16·0	07 23	5·9
		13 49	16·0	19 40	7·3
21	Th	01 51	16·8	08 26	5·3
		14 45	16·9	20 44	6·4
22	F.	02 51	17·2	09 16	4·7
		15 30	18·0	21 35	5·2
23	S.	03 40	18·1	09 59	3·9
		16 10	18·9	22 16	4·3
24	𝔖.	04 24	18·7	10 36	3·3
		16 46	19·6	22 56	3·9
25	M.	05 03	19·2	11 15	3·1
		17 20	20·1	23 35	3·2
26	Tu.	05 40	19·6	11 50	3·0
		17 55	20·2	—	—
27	W.	06 19	19·9	00 12	3·0
		18 29	20·2	12 28	3·0
28	Th.	06 57	19·8	00 50	2·2
		19 05	20·3	13 05	3·0
29	F.	07 38	19·4	01 28	2·1
		19 43	20·3	13 44	3·0
30	S.	08 20	19·1	02 07	2·0
		20 25	20·1	14 24	3·1
31					

Month Day	Week Day	DECEMBER.			
		HIGH WATER		LOW WATER	
		Time	Ht.	Time	Ht.
		H. M.	F.	H. M.	F.
1	𝔖.	09 06	18·8	02 50	2·0
		21 10	19·8	15 08	3·9
2	M.	10 00	17·9	03 37	2·8
		22 05	19·0	15 57	4·9
3	Tu	11 04	16·9	04 34	3·4
		23 08	18·3	16 58	5·7
4	W.	—	—	05 44	3·9
		12 19	16·8	18 13	5·9
5	Th.	00 24	18·0	07 05	4·0
		13 35	17·0	19 34	5·8
6	F.	01 44	18·2	08 19	3·8
		14 44	18·0	20 45	4·8
7	S.	02 54	19·1	09 20	2·9
		15 39	19·1	21 45	3·8
8	𝔖.	03 51	20·1	10 14	2·2
		16 27	20·1	22 39	2·9
9	M.	04 45	20·6	11 00	2·1
		17 11	20·5	23 28	2·0
10	Tu.	05 34	20·8	11 45	2·3
		17 54	20·9	—	—
11	W.	06 20	20·4	00 14	1·8
		18 35	21·1	12 28	2·8
12	Th.	07 05	20·1	00 56	1·3
		19 14	20·6	13 06	3·1
13	F.	07 47	19·3	01 38	1·6
		19 51	20·2	13 45	3·3
14	S.	08 30	18·9	02 16	2·0
		20 29	19·9	14 21	4·0
15	𝔖.	09 10	18·0	02 55	2·8
		21 06	19·1	14 59	4·9
16	M.	09 54	17·1	03 34	3·2
		21 48	18·4	15 37	5·3
17	Tu.	10 40	16·5	04 15	4·0
		22 35	17·4	16 20	6·0
18	W.	11 34	15·9	05 04	5·0
		23 32	16·8	17 15	6·9
19	Th	—	—	06 05	5·8
		12 36	15·8	18 25	7·0
20	F.	00 44	16·1	07 15	6·0
		13 43	16·1	19 41	7·0
21	S.	01 56	16·1	08 20	5·9
		14 41	16·9	20 48	6·1
22	𝔖.	02 59	17·0	09 15	5·0
		15 30	17·8	21 41	5·1
23	M	03 50	18·0	10 01	4·2
		16 15	18·6	22 29	4·2
24	Tu	04 38	18·4	10 46	3·7
		16 54	19·2	23 14	3·2
25	W.	05 21	19·1	11 29	3·2
		17 34	20·1	23 55	2·7
26	Th.	06 04	19·5	—	—
		18 13	20·5	12 10	2·7
27	F.	06 46	20·0	00 36	1·9
		18 52	21·0	12 50	2·1
28	S.	07 30	20·1	01 18	1·0
		19 34	21·2	13 31	2·0
29	𝔖.	08 14	19·9	02 00	0·8
		20 15	21·3	14 14	2·0
30	M.	08 57	19·7	02 41	0·8
		21 00	21·0	14 56	2·8
31	Tu.	09 45	18·9	03 26	1·0
		21 50	20·3	15 41	3·5

Time meridian Greenwich (Zone 0); 00^h is midnight, 12^h is noon. Heights are referred to the datum of the largest scale Admiralty chart of the place.

SCOTLAND, WEST COAST—GREENOCK

Lat. 55° 57′ N. Long. 4° 46′ W.

Month Day	Week Day	JANUARY.				Week Day	FEBRUARY.				Week Day	MARCH.				Week Day	APRIL.				Month Day
		HIGH WATER		LOW WATER			HIGH WATER		LOW WATER			HIGH WATER		LOW WATER			HIGH WATER		LOW WATER		
		Time H. M.	Ht. F.	Time H. M.	Ht. F.		Time H. M.	Ht. F.	Time H. M.	Ht. F.		Time H. M.	Ht. F.	Time H. M.	Ht. F.		Time H. M.	Ht. F.	Time H. M.	Ht. F.	
1	Tu.	08 29	8·9	01 49	3·2	F.	10 13	9·6	03 31	2·8	F.	08 21	8·9	01 47	3·4	M.	10 34	9·4	03 45	1·1	1
		20 49	9·1	14 34	2·8		22 47	9·3	16 05	1·8		21 17	8·7	14 42	1·9		23 08	8·8	16 11	*0·5	
2	W.	09 45	9·2	02 56	2·8	S.	11 20	10·3	04 25	2·0	S.	09 52	9·4	03 09	2·7	Tu	11 23	9·9	04 27	0·5	2
		22 04	9·2	15 38	2·1		23 46	9·7	16 54	0·6		22 33	8·9	15 48	1·1		23 54	9·4	16 50	*1·0	
3	Th.	10 41	10·0	03 54	2·3	𝕾.	—	—	05 13	1·3	𝕾.	10 57	9·8	04 10	1·9	W.	—	—	05 08	0·0	3
		23 06	9·3	16 25	1·3		12 08	10·7	17 43	0·2		23 30	9·2	16 32	*0·3		12 09	10·6	17 32	*1·0	
4	F.	11 35	10·4	04 42	1·8	M.	00 40	10·0	05 59	0·8	M.	11 46	10·3	04 53	0·9	Th	00 38	10·0	05 47	*0·3	4
		23 59	9·8	17 10	0·5		12 55	11·1	18 24	*0·6		—	—	17 18	*0·5		12 54	10·6	18 14	*0·8	
5	S.	—	—	05 26	1·0	Tu	01 19	10·1	06 39	0·4	Tu	00 20	9·8	05 37	0·4	F.	01 13	10·2	06 30	*0·3	5
		12 24	10·6	17 58	0·2		13 38	11·4	19 08	*0·6		12 35	10·8	17 59	*0·9		13 34	10·7	18 57	*0·2	
6	𝕾.	00 46	10·1	06 13	0·6	W.	02 02	10·3	07 20	0·3	W.	00 59	9·9	06 14	0·0	S.	01 54	10·8	07 15	*0·1	6
		13 08	11·1	18 42	*0·2		14 25	11·7	19 50	*0·4		13 18	10·8	18 39	*1·4		14 17	10·7	19 40	0·0	
7	M.	01 32	10·3	06 59	0·4	Th	02 49	10·9	08 03	0·4	Th	01 37	10·1	06 54	*0·5	𝕾.	02 35	10·9	07 57	0·1	7
		13 53	11·3	19 25	*0·4		15 11	11·5	20 37	0·2		14 01	11·0	19 23	*0·5		15 00	10·4	20 24	0·7	
8	Tu	02 22	10·1	07 43	0·5	F.	03 32	11·0	08 50	1·0	F.	02 24	10·6	07 40	*0·3	M	03 11	10·8	08 43	0·7	8
		14 35	11·6	20 17	*0·2		15 51	11·8	21 22	1·1		14 41	11·3	20 05	0·0		15 38	9·9	21 09	2·1	
9	W.	03 10	10·3	08 30	0·7	S.	04 12	11·2	09 34	1·7	S.	03 04	11·0	08 21	0·2	Tu	03 52	10·6	09 34	1·1	9
		15 21	11·6	21 02	0·3		16 27	11·7	22 08	1·7		15 20	11·1	20 48	1·1		16 23	9·3	22 11	2·3	
10	Th	04 00	10·1	09 20	1·1	𝕾	04 51	10·9	10 25	2·2	𝕾.	03 42	11·0	09 05	0·9	W.	04 34	9·8	10 34	1·9	10
		16 10	11·4	21 56	0·7		17 06	10·8	23 06	2·6		15 56	10·6	21 37	2·1		17 27	8·6	23 19	3·0	
11	F.	04 45	10·7	10 11	1·1	M	05 26	10·3	11 23	2·8	M	04 19	10·7	09 57	1·6	Th	05 19	8·9	11 57	2·2	11
		16 59	10·9	22 48	1·4		17 57	9·6	—	—		16 40	9·7	22 36	2·5		18 37	7·8	—	—	
12	S.	05 25	10·7	11 04	1·9	Tu	06 24	9·5	00 19	3·0	Tu	04 58	10·4	10 58	2·2	F.	06 29	8·5	00 51	3·0	12
		17 45	10·8	23 54	2·3		19 33	8·4	13 06	2·7		17 36	8·9	23 50	3·0		20 23	7·2	13 29	2·2	
13	𝕾.	06 13	10·1	—	—	W	07 58	8·8	01 51	3·3	W.	05 48	9·3	—	—	S.	08 09	8·2	02 16	2·7	13
		18 39	9·9	12 11	2·6		21 23	8·2	14 38	3·1		19 10	8·1	12 31	2·5		21 43	7·5	14 45	1·7	
14	M.	07 22	9·4	01 03	2·6	Th	09 31	8·8	03 16	3·3	Th	07 21	8·4	01 25	3·2	𝕾.	09 26	8·4	03 14	2·4	14
		20 01	9·0	13 36	3·0		22 41	8·8	15 49	2·6		21 00	7·7	14 09	2·9		22 31	8·2	15 29	1·1	
15	Tu.	08 42	9·1	02 18	2·8	F.	10 36	9·5	04 14	2·8	F.	09 02	8·5	02 56	3·3	M.	10 13	8·7	03 51	1·7	15
		21 47	8·9	15 07	3·1		23 27	9·1	16 36	2·4		22 19	8·1	15 27	2·3		23 05	8·4	16 04	1·0	
16	W	09 55	9·5	03 31	2·8	S.	11 21	10·0	04 59	2·6	S.	10 10	8·9	03 48	2·9	Tu	10 48	9·4	04 26	1·3	16
		22 50	9·3	16 06	2·6		—	—	17 14	1·8		23 08	8·4	16 13	1·8		23 34	8·9	16 38	0·8	
17	Th	10 50	9·9	04 26	2·6	𝕾	00 04	9·4	05 34	1·9	𝕾.	10 52	9·2	04 33	2·2	W.	11 21	9·9	04 56	1·0	17
		23 37	9·5	16 49	2·1		11 54	10·4	17 43	1·2		23 43	8·7	16 46	1·2		—	—	17 06	0·5	
18	F.	11 39	10·3	05 08	2·2	M.	00 36	9·6	06 03	1·9	M	11 27	9·7	05 06	1·7	Th	00 04	9·4	05 27	0·9	18
		—	—	17 29	1·6		12 25	10·7	18 09	0·9		—	—	17 15	0·5		11 55	10·3	17 37	0·4	
19	S.	00 16	9·8	05 48	1·8	Tu	01 06	9·7	06 28	1·4	Tu	00 10	9·2	05 33	1·1	F.	00 36	9·9	05 55	0·9	19
		12 17	10·6	18 04	1·3		12 59	11·0	18 38	0·5		11 56	10·1	17 40	0·3		12 28	10·4	18 07	0·9	
20	𝕾.	00 55	9·7	06 25	1·6	W.	01 34	9·8	06 56	1·2	W.	00 36	9·3	05 56	1·0	S.	01 06	10·1	06 28	0·8	20
		12 49	10·9	18 36	0·9		13 30	11·2	19 08	0·5		12 28	10·3	18 07	0·2		13 01	10·2	18 39	1·0	
21	M.	01 28	9·8	06 55	1·5	Th.	02 05	9·9	07 26	0·9	Th	01 05	9·6	06 25	0·5	𝕾.	01 38	10·4	07 04	0·8	21
		13 21	11·2	19 05	0·8		14 03	11·1	19 39	0·6		13 04	10·5	18 36	0·5		13 39	10·1	19 15	0·9	
22	Tu.	02 02	9·8	07 27	1·7	F.	02 37	10·1	07 58	0·9	F.	01 34	9·8	06 54	0·5	M.	02 11	10·3	07 41	0·5	22
		13 53	11·4	19 37	0·6		14 33	11·0	20 08	0·8		13 31	10·9	19 06	0·7		14 21	9·8	19 52	1·1	
23	W.	02 34	9·6	07 56	1·4	S.	03 10	9·8	08 30	1·1	S	02 09	10·1	07 30	0·6	Tu.	02 47	10·3	08 20	0·4	23
		14 25	11·6	20 07	0·7		15 01	11·0	20 40	1·3		14 00	11·0	19 39	0·7		15 04	9·8	20 33	1·4	
24	Th.	03 08	9·7	08 26	1·6	𝕾.	03 41	9·6	08 58	1·4	𝕾.	02 40	10·0	08 01	0·8	W.	03 30	9·9	09 07	0·5	24
		14 58	11·2	20 39	1·2		15 32	10·7	21 09	1·8		14 35	10·3	20 08	0·9		15 48	9·3	21 26	1·6	
25	F.	03 38	9·4	08 56	1·9	M.	04 04	9·4	09 34	1·9	M.	03 11	9·8	08 34	0·7	Th	04 15	9·4	10 00	0·9	25
		15 32	11·1	21 12	1·6		16 05	10·4	21 53	2·4		15 11	9·8	20 44	1·7		16 36	8·9	22 27	1·8	
26	S.	04 08	9·7	09 32	2·2	Tu.	04 38	9·1	10 23	2·3	Tu.	03 41	9·6	09 15	1·1	F.	05 01	9·1	11 23	1·1	26
		16 06	10·7	21 46	2·3		16 48	9·4	22 49	2·8		15 50	9·5	21 33	2·2		17 57	8·4	23 51	2·4	
27	𝕾.	04 37	9·6	10 10	3·0	W.	05 24	9·0	11 35	2·3	W.	04 22	9·3	10 05	1·5	S.	06 13	8·8	—	—	27
		16 36	10·5	22 28	2·8		17 55	8·9	—	—		16 40	9·0	22 33	2·5		19 33	8·2	12 55	0·9	
28	M.	05 13	9·2	11 03	2·8	Th	06 32	8·9	00 12	3·2	Th.	05 12	9·0	11 27	1·9	𝕾.	07 39	8·6	01 20	2·3	28
		17 19	9·9	23 29	3·2		19 44	8·6	13 20	2·3		17 53	8·9	23 58	2·8		20 44	8·4	13 58	0·4	
29	Tu.	06 02	9·0	—	—						F.	06 14	8·7	—	—	M.	09 00	8·9	02 29	1·9	29
		18 22	9·4	12 11	3·0							19 45	8·1	13 09	1·8		21 37	8·7	14 52	*0·1	
30	W.	07 22	8·8	00 44	3·4						S.	08 01	8·6	01 38	2·9	Tu.	10 02	9·5	03 17	1·2	30
		20 00	8·8	13 44	2·8							21 14	8·2	14 25	1·2		22 35	9·0	15 46	*0·4	
31	Th.	08 49	9·0	02 12	3·4						𝕾.	09 29	8·9	02 55	2·3						31
		21 43	8·9	15 04	2·3							22 12	8·6	15 21	0·3						

Time meridian Greenwich (Zone 0); 00^h is midnight, 12^h is noon. Heights are referred to the datum of the largest scale Admiralty chart of the place and should be *added* to depths, unless preceded by an asterisk (*) when they should be *subtracted*.

ENGLAND, WEST COAST—LIVERPOOL

Lat. 53° 25′ N. Long. 3° 00′ W.

Month Day	Week Day	MAY.				Week Day	JUNE.				Week Day	JULY.				Week Day	AUGUST.				Month Day
		HIGH WATER		LOW WATER			HIGH WATER		LOW WATER			HIGH WATER		LOW WATER			HIGH WATER		LOW WATER		
		Time	Ht.	Time	Ht.		Time	Ht.	Time	Ht.		Time	Ht.	Time	Ht.		Time	Ht.	Time	Ht.	
		H. M.	F.	H. M.	F.		H. M.	F.	H. M.	F.		H. M.	F.	H. M.	F.		H. M.	F.	H. M.	F.	
1	W.	09 46	28·7	04 16	2·6	S.	11 00	27·8	05 30	2·4	M.	11 33	26·6	06 00	2·9	Th.	00 07	27·7	06 57	2·5	1
		22 14	28·6	16 45	1·2		23 23	28·1	17 50	2·8		23 50	27·4	18 15	4·1		12 29	27·0	19 04	3·7	
2	Th.	10 33	29·5	05 07	1·5	𝔖	11 45	27·6	06 15	2·2	Tu	—	—	06 40	2·8	F.	00 39	27·8	07 28	2·8	2
		22 58	29·1	17 32	0·4		—	—	18 32	3·2		12 14	26·7	18 53	4·2		13 01	26·8	19 32	3·8	
3	F.	11 18	29·6	05 52	0·9	M.	00 05	28·0	06 53	2·2	W.	00 28	27·5	07 17	2·9	S.	01 10	27·4	07 56	3·2	3
		23 40	29·2	18 17	0·9		12 30	27·2	19 07	3·6		12 52	26·5	19 25	4·4		13 33	26·4	19 58	4·0	
4	S.	—	—	06 35	0·7	Tu.	00 44	27·6	07 30	2·7	Th	01 04	27·3	07 51	3·3	𝔖.	01 38	26·9	08 23	3·9	4
		12 00	29·2	18 53	1·6		13 09	26·4	19 42	4·7		13 29	26·1	19 57	4·8		14 02	25·6	20 28	4·5	
5	𝔖	00 21	29·0	07 11	1·2	W.	01 23	26·9	08 07	3·7	F	01 37	26·8	08 23	3·9	M	02 08	26·1	08 52	4·7	5
		12 43	28·5	19 28	2·5		13 48	25·5	20 17	5·4		14 03	25·4	20 29	5·2		14 32	24·9	21 02	5·3	
6	M	01 01	28·1	07 46	2·2	Th	02 01	26·0	08 45	4·6	S.	02 11	26·2	08 57	4·6	Tu	02 40	25·1	09 27	5·8	6
		13 22	27·1	20 01	3·8		14 32	24·5	20 55	6·3		14 40	24·7	21 06	5·7		15 09	23·8	21 38	6·4	
7	Tu	01 39	26·8	08 23	3·6	F.	02 43	25·2	09 28	5·6	𝔖.	02 47	25·4	09 35	5·4	W.	03 21	23·9	10 08	6·9	7
		14 05	25·5	20 38	5·5		15 17	23·5	21 37	7·0		15 19	23·9	21 44	6·3		15 56	22·8	22 26	7·4	
8	W	02 20	25·5	09 07	5·1	S	03 27	24·0	10 15	6·5	M.	03 28	24·3	10 15	6·2	Th	04 14	22·8	11 03	8·0	8
		14 51	23·9	21 21	7·1		16 06	22·7	22 26	7·6		16 02	23·0	22 27	7·1		16 55	22·2	23 27	8·4	
9	Th	03 08	23·9	09 55	6·5	𝔖.	04 18	23·2	11 07	7·0	Tu	04 14	23·3	11 03	6·9	F.	05 22	22·0	—	—	9
		15 48	22·5	22 11	8·2		17 01	22·2	23 19	7·9		16 53	22·4	23 17	7·6		18 10	22·1	12 08	8·7	
10	F.	04 05	22·4	10 54	7·6	M.	05 16	22·6	—	—	W.	05 09	22·5	11 56	7·5	S.	06 49	21·9	00 46	8·6	10
		16 53	21·5	23 11	8·8		18 00	22·0	12 00	7·3		17 52	22·1	—	—		19 31	23·1	13 37	8·3	
11	S.	05 12	21·9	—	—	Tu	06 16	22·4	00 15	7·9	Th	06 15	22·2	00 17	7·9	𝔖.	08 11	23·0	02 17	7·6	11
		18 01	21·3	12 00	8·0		19 00	22·5	12 57	7·1		18 59	22·5	12 59	7·7		20 42	24·7	14 58	6·9	
12	𝔖.	06 26	21·9	00 15	9·1	W	07 19	22·9	01 14	7·7	F	07 27	22·6	01 25	7·8	M	09 18	24·9	03 32	5·6	12
		19 11	21·9	13 09	7·6		19 57	23·3	13 57	6·6		20 06	23·6	14 11	7·2		21 39	26·6	16 01	5·0	
13	M.	07 32	22·6	01 25	8·3	Th	08 18	23·6	02 15	6·9	S.	08 35	23·7	02 40	7·0	Tu	10 10	26·7	04 33	3·4	13
		20 09	22·9	14 11	6·5		20 46	24·5	14 55	5·8		21 05	25·2	15 19	6·1		22 29	28·6	16 57	3·4	
14	Tu.	08 26	23·6	02 23	7·1	F	09 07	24·6	03 14	5·9	𝔖	09 33	25·1	03 47	5·5	W.	10 57	28·3	05 28	1·8	14
		20 51	24·1	15 02	5·5		21 33	25·7	15 46	5·0		21 58	26·6	16 19	4·8		23 14	30·3	17 50	2·0	
15	W.	09 07	24·5	03 12	5·8	S	09 53	25·6	04 08	4·9	M	10 26	26·4	04 49	4·0	Th	11 42	29·5	06 20	0·4	15
		21 29	25·4	15 44	4·5		22 20	26·8	16 37	4·3		22 47	28·1	17 14	3·8		23 59	31·4	18 40	1·1	
16	Th	09 45	25·6	03 56	4·7	𝔖.	10 40	26·5	05 01	3·8	Tu	11 14	27·5	05 42	2·4	F	—	—	07 06	*0·5	16
		22 07	26·5	16 25	3·7		23 03	27·7	17 27	3·0		23 34	29·1	18 06	2·7		12 26	30·2	19 26	0·5	
17	F.	10 22	26·5	04 39	3·8	M.	11 27	27·2	05 50	3·0	W	—	—	06 32	1·4	S.	00 44	31·8	07 50	*0·6	17
		22 43	27·2	17 06	3·2		23 47	28·4	18 15	3·2		12 00	28·4	18 55	2·0		13 09	30·1	20 06	0·5	
18	S.	10 59	26·9	05 23	3·2	Tu	—	—	06 40	2·3	Th	00 18	30·2	07 21	0·6	𝔖.	01 26	31·3	08 29	*0·2	18
		23 21	27·7	17 48	2·9		12 12	27·5	19 03	2·7		12 45	28·9	19 41	1·7		13 50	29·3	20 43	1·2	
19	𝔖.	11 40	27·1	06 06	2·7	W.	00 32	28·8	07 27	1·7	F.	01 03	30·5	08 06	0·3	M	02 09	30·2	09 05	1·1	19
		—	—	18 28	2·7		12 50	27·5	19 46	2·6		13 31	29·0	20 22	1·6		14 33	28·0	21 21	2·4	
20	M.	00 02	28·0	06 48	2·5	Th	01 16	29·0	08 11	1·5	S.	01 48	30·3	08 48	0·5	Tu.	02 51	28·2	09 43	3·0	20
		12 20	27·1	19 10	3·0		13 45	27·4	20 31	2·8		14 17	28·4	21 02	2·1		15 20	26·2	22 04	4·4	
21	Tu.	00 42	27·8	07 30	2·5	F.	02 04	28·7	08 56	1·8	𝔖	02 32	29·6	09 26	1·3	W.	03 42	26·1	10 30	5·3	21
		13 03	26·6	19 52	3·3		14 35	26·9	21 16	3·4		15 02	27·3	21 43	3·1		16 15	24·4	23 00	6·3	
22	W.	01 24	27·5	08 13	2·8	S	02 53	28·1	09 42	2·4	M.	03 21	28·2	10 10	2·6	Th	04 45	24·1	11 30	7·4	22
		13 49	26·0	20 34	4·1		15 27	26·3	22 03	4·1		15 52	26·1	22 30	4·4		17 24	23·0	—	—	
23	Th	02 11	27·0	09 01	3·4	𝔖.	03 46	27·3	10 34	3·2	Tu.	04 12	26·7	11 00	4·4	F.	06 06	22·6	00 12	7·8	23
		14 42	25·4	21 24	4·9		16 22	25·5	22 55	4·9		16 48	24·8	23 25	5·9		18 50	22·5	12 53	8·7	
24	F.	03 04	26·5	09 53	4·1	M.	04 42	26·5	11 28	4·2	W.	05 17	25·1	—	—	S.	07 38	22·5	01 48	8·0	24
		15 41	24·7	22 16	5·7		17 25	24·8	23 56	5·8		17 56	23·8	12 01	6·0		20 14	23·1	14 24	8·5	
25	S.	04 04	25·8	10 51	4·7	Tu.	05 48	25·6	—	—	Th.	06 29	23·9	00 36	6·9	𝔖.	08 51	23·5	03 06	6·8	25
		16 45	24·4	23 19	6·2		18 30	24·6	12 33	5·0		19 13	23·6	13 17	7·1		21 17	24·5	15 31	7·3	
26	𝔖.	05 09	25·5	11 57	4·8	W.	06 58	25·2	01 05	6·2	F.	07 50	23·8	02 00	7·0	M.	09 43	24·7	04 01	5·3	26
		17 56	24·2	—	—		19 39	24·7	13 44	5·5		20 27	24·1	14 35	7·0		22 02	25·8	16 19	6·0	
27	M.	06 21	25·6	00 27	6·3	Th.	08 06	25·4	02 19	5·9	S.	08 58	24·3	03 14	6·2	Tu.	10 24	25·8	04 46	3·9	27
		19 05	24·7	13 08	4·7		20 42	25·4	14 54	5·3		21 27	25·0	15 41	6·5		22 39	26·7	17 00	4·9	
28	Tu.	07 29	26·1	01 42	5·8	F.	09 09	25·6	03 25	5·2	𝔖.	09 53	25·2	04 13	5·0	W.	10 57	26·7	05 26	2·9	28
		20 08	25·6	14 18	4·2		21 36	26·0	15 54	5·0		22 16	26·0	16 34	5·5		23 10	27·6	17 36	4·2	
29	W.	08 28	26·6	02 48	4·8	S.	10 01	26·1	04 20	4·3	M.	10 39	25·8	05 02	3·9	Th.	11 29	27·3	06 01	2·6	29
		21 03	26·5	15 20	3·5		22 25	26·6	16 46	4·5		22 56	26·8	17 19	4·8		23 41	27·9	18 09	3·5	
30	Th.	09 23	27·2	03 48	3·9	𝔖.	10 50	26·4	05 12	3·5	Tu.	11 18	26·3	05 45	3·1	F.	—	—	06 32	2·4	30
		21 52	27·3	16 14	3·0		23 10	27·2	17 33	4·2		23 33	27·4	17 58	4·2		12 01	27·6	18 39	3·2	
31	F.	10 12	27·8	04 40	3·0						W	11 55	26·6	06 24	2·6	S	00 10	28·1	07 02	2·4	31
		22 38	27·8	17 05	2·9							—	—	18 34	3·9		12 29	27·5	19 06	3·1	

Time meridian Greenwich (Zone 0); 00^h is midnight, 12^h is noon. Heights are referred to the datum of the largest scale Admiralty chart of the place and should be *added* to depths, unless preceded by an asterisk (*) when they should be *subtracted.*

APPENDIX

THE HAVERSINE FORMULA

Development from the fundamental cosine formula for the astronomical triangle PZX,

where ZPX = hour angle
PZX = azimuth
ZX = zenith distance
PX = polar distance
PZ = co-latitude

the fundamental formula would read:

$$\cos. ZPX = \frac{\cos. ZX - \cos. PZ \times \cos. PX}{\sin. PZ \times \sin. PX}$$

$$1 - \cos. ZPX = 1 - \frac{\cos. ZX - \cos. PZ \times \cos. PX}{\sin. PZ \times \sin. PX}$$

$$\text{versine } ZPX = \frac{(\sin. PZ \sin. PX + \cos. PZ \cos. PX) - \cos. ZX}{\sin. PZ \sin. PX}$$

$$\sin. PZ \sin. PX \text{ vers. } ZPX$$

$$= \cos. (PZ \sim PX) - \cos. ZX$$

Add and subtract 1 and rearrange:

$$= 1 - \cos. ZX - [1 - \cos. (PZ \sim PX)]$$

$$= \text{vers. } ZX - \text{vers. } (PZ \sim PX)$$

Divide throughout by 2:

$$\sin. PZ \sin. PX \frac{\text{vers. } ZPX}{2} = \frac{\text{vers. } ZX}{2} - \frac{\text{vers. } (PZ \sim PX)}{2}$$

$$\text{Thus, haversine } ZPX = \frac{\text{hav. } ZX - \text{hav. } (PZ \sim PX)}{\sin. PZ \sin. PX}$$

DEVIATION CARD 2

Compass course	*Deviation*	*Compass course*	*Deviation*
000°	0°	190°	0°
010°	1° E.	200°	1° W.
020°	1° E.	210°	1° W.
030°	2° E.	220°	1° W.
040°	3° E.	230°	2° W.
050°	4° E.	240°	2° W.
060°	5° E.	250°	3° W.
070°	6° E.	260°	4° W.
080°	7° E.	270°	5° W.
090°	7° E.	280°	5° W.
100°	7° E.	290°	4° W.
110°	7° E.	300°	3° W.
120°	6° E.	310°	2° W.
130°	5° E.	320°	2° W.
140°	3° E.	330°	1° W.
150°	2° E.	340°	0°
160°	2° E.	350°	0°
170°	1° E.	360°	0°
180°	0°		

FORM 316*c*

For magnetic course	*Steer by compass*	*Deviation*
000°	000°	0°
045°	042°	3° E.
090°	083°	7° E.
135°	131°	4° E.
180°	180°	0°
225°	226°	1° W.
270°	275°	5° W.
315°	317°	2° W.

ANSWERS

CHAPTER I

Exercise 1

	D. lat. (*ch. lat.*)	*D. long.* (*ch. long.*)
1.	5° 22′ N.	39° 19′ E.
2.	34° 23′ N.	3° 27′ E.
3.	22° 57′ S.	33° 10′ W.
4.	52° 08′ N.	8° 38′ W.
5.	42° 25′ S.	31° 43′ E.
6.	86° 20′ N.	62° 09′ E.
7.	47° 41′ N.	167° 23′ W.
8.	26° 40′ S.	73° 59′ W.
9.	10° 44′ S.	18° 55′ E.

Exercise 2

	Latitude	*Longitude*	*Place*
1.	51° 37′ N.	3° 57′ W.	—
2.	37° 58′ N.	23° 43′ E.	—
3.	34° 57′ S.	138° 38′ E.	—
4.	38° 36′ N.	90° 18′ W.	—
5.	13° 04′ N.	80° 17′ E.	Madras
6.	18° 05′ S.	178° 28′ E.	Suva
7.	8° 59′ N.	79° 32′ W.	Panama
8.	23° 12′ N.	113° 15′ E.	Canton
9.	31° 11′ N.	29° 51′ E.	Alexandria

Exercise 3 (*a*)

Difference in time:

	hrs.	*mins.*	*secs.*
1.	2	25	24
2.	9	37	08
3.	5	42	48
4.	0	22	24
5.	11	48	08
6.	0	59	40
7.	1	53	16
8.	4	19	04

Exercise 3 (*b*)

D. long.:

1.	64°	27′	30″
2.	172°	19′	00″
3.	13°	06′	00″
4.	95°	10′	45″
5.	56°	24′	30″
6.	136°	44′	30″
7.	15°	08′	00″
8.	87°	03′	00″

EXERCISE 4

Local Time:

	days	*hrs.*	*mins.*	*secs.*
1.	6	14	21	04
2.	25	9	58	21
3.	18	2	53	10
4.	12	2	04	50
5.	30	10	10	34
6.	1	21	53	50
7.	11	17	04	24
8.	28	3	27	02
9.	3	23	54	08

GENERAL EXERCISES

1.

	D. lat. (*ch. lat.*)	*D. long.* (*ch. long.*)
(i)	20° N.	100° E.
(ii)	110° S.	20° W.
(iii)	60° N.	30° W.
(iv)	30° S.	50° E.

3.

	Ch. lat. (*d. lat.*)	*Ch. long.* (*d. long.*)
(*a*)	90° 11′ S.	144° 28′ E.
(*b*)	6° 20′ S.	102° 00′ W.
(*c*)	54° 55′ N.	168° 05′ W.
(*d*)	44° 59′ S.	70° 46′ E.
(*e*)	63° 59′ S.	12° 47′ W.
(*f*)	21° 02′ S.	81° 55′ W.
(*g*)	81° 26′ N.	86° 33′ E.
(*h*)	7° 35′ N.	26° 00′ E.
(*i*)	62° 40′ N.	27° 19′ E.

4. 15°

	hrs.	*mins.*	*secs.*
(*a*)	1	11	20
(*b*)	1	58	08
(*c*)	2	48	41
(*d*)	5	17	12
(*e*)	0	17	44
(*f*)	10	12	38
(*g*)	14	07	04
(*h*)	19	03	40
(*i*)	22	25	27

5. May

	days	*hrs.*	*mins.*	*secs.*
(*a*)	6	4	06	10
(*b*)	6	8	53	50
(*c*)	6	6	14	54
(*d*)	5	22	48	30
(*e*)	5	19	34	30
(*f*)	6	4	58	14
(*g*)	5	17	12	50
(*h*)	6	11	50	30

6. June

	days	*hrs.*	*mins.*	*secs.*
(*a*)	19	01	21	16
(*b*)	18	07	39	56
(*c*)	18	22	57	48
(*d*)	18	12	29	16

10.

(*a*)	202 st. m.	324 km.
(*b*)	464 n.m.	860 km.
(*c*)	448 n.m.	515 st. m.

CHAPTER II

Exercise 1

1.	Distance on ground	31·6 st. mls.
2.	Distance on ground	25·6 st. mls.
3.	Distance on ground	75·0 km.
4.	Distance on map	14·0 cm.
5.	Distance on map	6·6 ins.
6.	Distance on map	3·5 ins.
7.	Representative fraction	$\frac{1}{253,440}$

Exercise 2

1. 3·33 ins.

2. 1·17 ins.

3. 2·11 ins.
3·40 ins.
1·65 ins.

Exercise 3

	Name	*Graticule Reference*	*Direction and Distance*	*Grid Reference*
1.	———	51° 26′ N. 0° 56′ W.	013° 54 n.m.	Q 2290
2.	Bath	———	321° — 61 n.m.	U 3281
3.	———	50° 06′ N. 1° 34′ W.	———	Z 8352
4.	Guildford	51° 14′ N. 0° 32′ W.	035° — 49 n.m.	———
5.	———	50° 36′ N. 2° 27′ W.	272° 43 n.m.	U 2602
6.	Christchurch	———	299° — 19 n.m.	U 7116
7.	———	49° 54′ N. 0° 42′ W.	———	V 3933
8.	Swindon	51° 32′ N. 1° 45′ W.	344° — 60 n.m.	———
9.	———	50° 48′ N. 0° 06′ W.	070° 47 n.m.	Q 7628
10.	Chichester	———	050° — 25 n.m.	Q 3429
11.	———	50° 12′ N. 0° 12′ E.	———	V 9765
12.	Portland Bill	50° 30′ N. 2° 28′ W.	263 — 44 n.m.	———

Scale: (i) $\frac{1}{3,040,000}$ (ii) 48 st. mls.

Exercise 4

1. Gradient = 1 in 21
2. Gradient = 1 in 74
3. Gradient = 1 in 52
4. Gradient = 1 in 88

CHAPTER III

General Exercises

6.

	Ch. lat. (*d. lat.*)	*Ch. long.* (*d. long.*)
(*a*)	1° 49′ S.	4° 14′ E.
(*c*)	26′ N.	5′ W.
(*d*)	32′ N.	36′ W.
(*e*)	11′ N.	1° 51′ W.
(*f*)	5′ S.	2° 12′ W.
(*g*)	57′ S.	2° 37′ W.

7. Scale: $\frac{1}{28,000,000}$ approx.

	(*T.*)	(*n.m.*)
(*a*)	043°	3840
(*b*)	108°	2518
(*c*)	256°	1925
(*d*)	272°	1677
(*e*)	062°	1821
(*f*)	266°	2107
(*g*)	330°	1633

---

10. Vertex positions:

L. to C. 65° N. 55° W.
C. to M. 76° N. 30° W.
M. to L. 56° N. 32° E.

11.

	Initial Direction (*T.*)	*Final Direction* (*T.*)	*Vertex*
(*a*)	038°	100°	58° 03′ N. 2° 30′ E.
(*b*)	080°	116°	44° 50′ N. 47° 30′ W.
(*c*)	281°	247°	51° 10′ N. 20° 00′ W.
(*d*)	285°	255°	53° 55′ N. 34° 00′ W.
(*e*)	050°	085°	58° 35′ N. 5° 00′ W.
(*f*)	289°	249°	51° 40′ N. 22° 30′ W.
(*g*)	318°	294°	62° 27′ N. 71° 00′ W.

Note: These results do not entirely agree with values obtained by calculation. Directions on a gnomonic chart cannot usually be measured with absolute accuracy by ordinary means.

12.

	Rhumb-line Direction (*T.*)	*Distance* (*n.m.*)
(*a*)	144°	88
(*b*)	094°	106
(*c*)	306°	82
(*d*)	282°	174
(*e*)	337°	67
(*f*)	152°	116

13.

	Latitude	*Longitude*
(*a*)	52° 44′ N.	1° 16′ E.
(*b*)	53° 37′ N.	1° 56′ E.
(*c*)	52° 38′ N.	2° 48′ E.
(*d*)	51° 48′ N.	1° 56′ E.
(*e*)	53° 16′ N.	0° 13′ W.
(*f*)	51° 46′ N.	0° 51′ E.

CHAPTER IV

EXERCISE 1 (*a*)

	High water					
	Time		*Height*	*Duration*		*Range*
	hrs.	*mins.*	(*ft.*)	*hrs.*	*mins.*	(*ft.*)
1.	14	03	25·4	6	26	20·2
2.	05	22	22·0	6	46	13·3
3.	23	32	16·8	6	33	11·0
4.	05	16	23·0	6	24	23·6
5.	12	24	10·6	5	34	10·4
6.	11	46	10·3	5	32	10·8

Exercise 1 (b)

	Low water Time hrs. mins.	Low water Height (ft.)	Duration hrs. mins.	Range (ft.)
1.	08 11	2·6	6 22	7·4
2.	18 17	*1·0	7 06	11·0
3.	12 04	1·5	5 34	25·9
4.	12 02	0·9	5 23	28·3
5.	04 59	*0·9	6 11	23·1
6.	17 32	1·0	6 08	20·7

Exercise 2

	Duration hrs. mins.	Interval hrs. mins.	M.T.L. (ft.)	Half range (ft.)	Angle	Height (ft.)	Depth (ft.)
1.	6 35	2 10	10·8	5·6	59°	13·7	33·7
2.	6 20	2 21	11·4	9·6	67°	15·2	51·2
3.	6 07	3 50	15·4	7·1	113°	12·7	42·7
4.	6 24	4 05	6·2	3·4	115°	4·8	29·8
5.	6 16	1 44	5·8	3·0	50°	7·7	31·7
6.	5 33	1 47	15·8	11·9	58°	22·0	55·0
7.	5 19	2 59	14·8	15·4	101°	11·9	53·9
8.	6 23	2 19	11·5	11·8	65°	16·3	64·3

Exercise 3

	Duration hrs. mins.	M.T.L. (ft.)	Half range (ft.)	Angle	Interval hrs. mins.	Time hrs. mins.
1.	6 46	15·25	12·75	91°	3 25	03 27
2.	5 23	16·1	14·1	82°	2 27	21 51
3.	6 09	11·1	8·4	84°	2 52	03 12
4.	6 23	11·5	11·8	124°	1 24	10 25
5.	5 21	4·7	6·1	96°	2 51	16 09
6.	6 58	4·95	4·45	63°	2 26	21 38

CHAPTER V

Exercise 1

Variation, 1944

1. 15° 42′ W.
2. 21° 45′ E.
3. 1° 20′ E.
4. 10° 58′ W.
5. 14° 33′ W.
6. 1° 29′ W.

Exercise 2

Magnetic 156°; 258°; 006°; 355°.
True 095°; 166°; 357°; 014°.
Variation 15° E.; 6° W.; 16° E.; 9° W.

Exercise 3

	(a) Bearing: 140° (M.)		(b) 267° (M.)		(c) 002° (M.)		(d) S. 89° E. (M.)	
	Dev.	Course M.	Dev.	Course M.	Dev.	Course M.	Dev.	Course M.
1.	2° W.	358°	1° E.	001°	4° E.	004°	3° W.	N. 3° W.
2.	1° E.	046°	4° E.	049°	3° E.	048°	1° W.	N. 44° E.
3.	3° E.	093°	5° E.	095°	Nil	090°	2° E.	S. 88° E.
4.	4° E.	139°	3° E.	138°	3° W.	132°	5° E.	S. 40° E.
5.	2° E.	182°	2° W.	178°	5° W.	175°	4° E.	S. 4° W.
6.	1° W.	224°	5° W.	220°	4° W.	221°	1° E.	S. 46° W.
7.	4° W.	266°	4° W.	266°	1° E.	271°	3° W.	S. 87° W.
8.	3° W.	312°	2° W.	313°	4° E.	319°	5° W.	N. 50° W.

EXERCISE 4

	Course (C.)	*Dev.*	*Course (M.)*	*Var.*	*Course (T.)*
1.	—	—	304°	—	292°
2.	—	—	137°	—	154°
3.	046°	—	051°	—	—
4.	297°	—	299°	—	—
5.	182°	—	—	—	180°
6.	312°	—	—	—	299°
7.	—	—	045°	10° E.	—
8.	—	—	286°	12° W.	—
9.	—	2° E.	140°	—	—
10.	—	7° W.	353°	—	—
11.	—	4° E.	—	—	286°
12.	—	3° W.	—	—	078°
13.	—	2° E.	202°	—	—
14.	004°	—	007°	—	—
15.	—	—	357°	—	002°
16.	—	4° E.	—	16° W.	—
17.	—	—	003°	11° W.	—

GENERAL EXERCISES

17. 3° E.

19.

Mag. Bearing S. 45° *W.* (225°).

Comp. Course	*Dev.*
N.	1° W.
N. by E.	1½° W.
N.N.E.	2° W.
N.E. by N.	2½° W.
N.E.	3° W.
N.E. by E.	3½° W.
E.N.E.	4° W.
E. by N.	4½° W.
E.	5° W.
E. by S.	5½° W.
E.S.E.	5½° W.
S.E. by E.	5° W.
S.E.	4° W.
S.E. by S.	2½° W.
S.S.E.	1° W.
S. by E.	0°
S.	1° E.
S. by W.	1½° E.
S.S.W.	2° E.
S.W. by S.	2½° E.
S.W.	3° E.
S.W. by W.	3½° E.
W.S.W.	4° E.
W. by S.	4½° E.
W.	5° E.
W. by N.	5½° E.
W.N.W.	5½° E.
N.W. by W.	5° E.
N.W.	4° E.
N.W. by N.	3° E.
N.N.W.	1½° E.
N. by W.	0°
N.	1° W.

Mag. Course	*Dev.*
356°	1° W.
007°	2° W.
028°	3° W.
056°	4° W.
075°	5° W.
126°	4° W.
136°	3° W.
143°	2° W.
152°	1° W.
163°	0°
174°	1° E.
194°	2° E.
216°	3° E.
238°	4° E.
259°	5° E.
312°	4° E.
325°	3° E.
334°	2° E.
342°	1° E.
348°	0°
356°	1° W.

20.

Mag. Bearing S. 89° *E.* (091°).

Comp. Course	*Dev.*
N.	2° W.
N. by E.	1° W.
N.N.E.	½° W.
N.E. by E.	½° E.
N.E.	1° E.
N.E. by E.	1½° E.
E.N.E.	2° E.
E. by N.	2½° E.
E.	3° E.
E. by S.	3½° E.
E.S.E.	3½° E.
S.E. by E.	4° E.
S.E.	4° E.
S.E. by S.	4° E.
S.S.E.	3½° E.
S. by E.	3° E.
S.	2° E.
S. by W.	1° E.
S.S.W.	½° E.
S.W. by S.	0°
S.W.	1° W.
S.W. by W.	1½° W.
W.S.W.	2° W.
W. by S.	2½° W.
W.	3° W.
W. by N.	3½° W.
W.N.W.	3½° W.
N.W. by W.	4° W.
N.W.	4° W.
N.W. by N.	4° W.
N.N.W.	3½° W.
N. by W.	3° W.
N.	2° W.

Mag. Course	*Dev.*
352°	2° W.
006°	1° W.
020°	0°
036°	1° E.
056°	2° E.
080°	3° E.
110°	4° E.
160°	3° E.
176°	2° E.
190°	1° E.
204°	0°
217°	1° W.
232°	2° W.
252°	3° W.
280°	4° W.
330°	3° W.
352°	2° W.

22. (*a*) 207° T.; (*b*) 7° E.

23.

			Course			Bearing		
	Var.	*Dev.*	*True*	*Mag.*	*Comp.*	*True*	*Mag.*	*Comp.*
(*a*)	—	—	—	064°	059°	281°	295°	—
(*b*)	—	—	—	160°	163°	051°	041°	—
(*c*)	—	—	—	002°	356°	354°	011°	—
(*d*)	—	4° E.		224°	—	—	354°	350°
(*e*)	13° W.	—	283°	296°	—	—	238°	—
(*f*)	13° E.	—	213°	—	198°	—	062°	—
(*g*)	11° E.	4° W.		—	088°	175°		—
(*h*)	18° W.	2° E.	—	014°	—	—		036°
(*i*)	—	4° E.	—	020°	020°	019°	—	—
(*j*)	12° E.	—	198°	—	181°	—	—	060°
(*k*)	14° E.	3° W.	—	—	—	—	196°	199°
(*l*)	16° W.	4° E.	358°	014°	—	—	—	—
(*m*)	—	—	162°	—	151°	239°	—	228°
(*n*)	—	—	040°	055°	—	074°	089°	—
(*o*)	—	5° E.	091°	—	—	—	323°	318°
(*p*)	6° W.	—		—	132°	027°	—	043°
(*q*)	—	—	010°	010°	—	—	344°	340°
(*r*)	—	3° E.		015°		214°	239°	—
(*s*)	4° W.	4° E.	—	—	230°	082°	—	—
(*t*)	14° E.	6° W.	114°		—	—	—	246°
(*u*)	—	4° W.	032°		—	—	350°	354°
(*v*)	11° E.	—	—	055°	—	174°	163°	—

24.

Variation, 1945

(*a*)	14° 00′ W.
(*b*)	17° 20′ E.
(*c*)	10° 08′ W.
(*d*)	3° 18′ W.
(*e*)	11° 10′ E.
(*f*)	16° 56′ W.

25.

			Course			Bearing		
	Var.	*Dev.*	*True*	*Mag.*	*Comp.*	*True*	*Mag.*	*Comp.*
(*a*)	17° W.	4° E.	—	177°	173°	287°	304°	—
(*b*)	18° E.	2° W.	—	024°	026°	010°	—	354°
(*c*)	7° E.	1° E.	216°	209°	—	—	358°	357°
(*d*)	8° W.	5° E.	164°	—	167°	051°	059°	—
(*e*)	14° E.	3° W.	004°	—	353°	026°	—	015°
(*f*)	26° W.	Nil	—	046°	046°	—	006°	006°
(*g*)	25° W.	6° E.	131°	156°	—	049°	—	068°
(*h*)	23° E.	3° E.	118°	—	092°	236°	213°	—

26. Deviation 4° W.
27. Deviation 4° E.
28. Deviation 2° W.; course 280° T.; bearing 210° C.
29. Bearing 144° T.
30. Deviation 2° E.; course 170° T.; bearing 142° M.
31. Bearing 013° C.
32. Deviation on steering (P4) compass 3° E.; deviation on bearing compass 2° W.
33. Deviation 1° E.; bearing 316° T.
34. Track 068° T.; deviation 2° W.

CHAPTER VI

EXERCISE 1

	Track	*G.S.* (*knots*)
1.	230°	115
2.	032°	182
3.	316°	165
4.	154°	177
5.	290°	174
6.	354°	140
7.	000°	180
8.	006°	177

For ship's speed over the ground, divide the G.S. by 10.

EXERCISE 2

	True course	*T.A.S.* (*knots*)
1.	025°	174
2.	248°	135
3.	164°	151
4.	322°	134
5.	066°	124
6.	007°	170
7.	259°	195
8.	137°	125

For ship's steaming speeds divide the T.A.S. by 10.

EXERCISE 3

	Course (*T.*)	*G.S.* (*knots*)	
1.	096°	122	(12·2)
2.	085°	205	(20·5)
3.	349°	172	(17·2)
4.	258°	209	(20·9)
5.	156°	185	(18·5)
6.	289°	219	(21·9)
7.	025°	204	(20·4)
8.	303°	176	(17·6)
9.	014°	116	(11·6)
10.	246°	270	(27·0)
11.	348°	131	(13·1)

EXERCISE 4

	W/V (*knots*)
1.	086°/38
2.	223°/30
3.	098°/25
4.	040°/41
5.	350°/28
6.	110°/40
7.	250°/36
8.	180°/32
9.	135°/45
10.	045°/33
11.	280°/36

EXERCISE 5

	W/V (*knots*)
1.	269°/32
2.	347°/35
3.	237°/25
4.	225°/31
5.	070°/35
6.	170°/44
7.	080°/34

Exercise 6

	W/V (knots)	*G.S. (knots)*
1.	180°/18	142
2.	300°/28	210
3.	230°/20	114
4.	350°/33	193
5.	240°/23	215
6.	270°/25	166
7.	010°/39	229

Exercise 7

	W.V. (knots)
1.	190°/17
2.	160°/45
3.	210°/40
4.	263°/30
5.	019°/34
6.	282°/38

Exercise 8

	W/V (knots)
1.	100°/21½
2.	300°/14
3.	080°/20
4.	270°/25
5.	225°/20

Exercise 9

	Course (T.)	*Track*	*G.S. (knots)*	*Position*
1.	072°	080°	130	54° 08′ N. 3° 08′ E.
2.	094°	105°	135	52° 40′ N. 3° 00′ E.
3.	024°	017°	155	54° 40′ N. 1° 21′ E.
4.	116°	105°	188	51° 57′ N. 5° 18′ E.
5.	258°	266°	126	51° 55′ N. 1° 48′ E.
6.	286°	276°	160	51° 28′ N. 0° 17′ E.

Exercise 10

	True course	*Course made good*	*Effective speed (knots)*	*Position*
1.	336°	328°	10·4	52° 16′ N. 2° 42′ E.
2.	080°	088°	11·8	52° 38′ N. 3° 23′ E.
3.	135°	136°	22·5	53° 08′ N. 1° 38′ E.
4.	045°	041°	15·5	52° 21′ N. 2° 47′ E.
5.	240°	245°	15·9	52° 23′ N. 2° 45′ E.
6.	005°	359°	15·9	54° 15′ N. 1° 18′ E.

Exercise 11

	Track	*Distance (n.m.)*	*G.S. (knots)*	*Course (T.)*	*Course (C.)*	*T.A.S. (knots)*
1.	151°	137	137	161°	166°	135
2.	096°	143	143	083°	089°	133
3.	248°	234	117	257°	270°	135
4.	325°	80	160	319°	334°	173
5.	358°	90	135	004°	016°	141
6.	116°	171	114	101°	106°	115

Exercise 12

	Course to make good (T.)	*Distance (n.m.)*	*Effective speed (knots)*	*True course*	*Compass course*	*Steaming speed (knots)*
1.	170°	36	12·0	167°	172°	13·9
2.	124°	70	14·0	117°	121°	13·7
3.	010°	56	14·0	005°	016°	12·5
4.	218°	100	16·7	223°	233°	14·9
5.	316°	76	13·8	309°	323°	13·2
6.	070°	57	15·5	072°	078°	17·2

EXERCISE 13

	Track	*Course (T.)*	*Course (C.)*	*G.S. (knots)*
1.	111°	124°	127°	106
2.	152°	160°	165°	160
3.	274°	278°	291°	156
4.	014°	004°	016°	90
5.	099°	088°	094°	164
6.	280°	271°	285°	144

EXERCISE 14

	Course to make good (T.)	*True course*	*Compass course*	*Effective speed (knots)*
1.	146°	141°	145°	16·6
2.	050°	055°	063°	21·5
3.	259°	265°	278°	12·9
4.	076°	083°	090°	13·0
5.	303°	306°	321°	14·8
6.	126°	130°	133°	13·3

EXERCISE 15

	Course (T.)	*T.M.G.*	*G.S. (knots)*	*W/V (knots)*
1.	100°	105°	162	327°/17
2.	179°	169°	107	231°/24
3.	040°	047°	122	307°/15
4.	300°	293°	134	022°/17
5.	130°	123°	158	272°/30
6.	320°	307°	144	018°/38

EXERCISE 16

	True course	*Course made good (T.)*	*Effective speed (knots)*	*Current set drift (n.m.)*
1.	083°	089°	10·2	168°/6·0
2.	260°	265°	12·5	326°/8·0
3.	230°	234°	19·0	350°/7·0
4.	334°	329°	16·1	283°/8·0
5.	155°	163°	16·8	205°/11·7
6.	010°	012°	14·2	182°/9·0

EXERCISE 17

1. Tracks: 095°; 215°; 274°; 022°.
Distances (n.m.): 226; 119; 206; 111.
First course: 089° T.; 098° M.; 095° C.
G.S. 176 knots, E.T.A. 11.32 hrs.
11.12 hrs. T.M.G. 090°, G.S. 181 knots, W/V 276°/21 knots.
New track 112°, Course 114° T., 121° M., 118° C.
Third course: 222° T.; 229° M.; 230° C.
G.S. 147 knots. Time 0 hrs. 48½ mins.
Fourth course: 274° T. 283° M. 287° C.
G.S. 137 knots. Time 1 hr. 30 mins.
Fifth course: 014° T. 024° M. 022° C.
G.S. 165 knots. Time 0 hrs. 40 mins.

2. Track: 119°, distance 179 n.m.
Course: 127° T. 136° M. 138° C., G.S. 172 knots.
E.T.A.: 11.22 hrs.
T.M.G.: 118°, G.S. 173 knots, W/V 242°/31 knots.
Adjusted course: 129° T., 137° M., 139° C.
G.S. 168 knots, E.T.A. 11.24 hrs.
3. Air position: 52° 29′ N. 1° 23′ E.
W/V: 252°/40 knots.
Track: 080°.
Course: 084° T.
E.T.A.: 20.12 hrs.
W/V: 237°/24 knots.
4. Course: 082° M. 072° T.
Air position: 52° 35′ N. 2° 02′ E.
W/V: 159°/25 knots.
Track: 063°.
Course: 072° T. 081° M. 079° C.
G.S.: 150 knots.
E.T.A.: 11.07 hrs.
Expected position: 52° 28′ N. 2° 42′ E.
5. Course: 062° T.
T.M.G.: 076°.
G.S.: 186 knots.
W/V: 298°/53 knots.
Crossing coast: 12.07½ hrs. 52° 08′ N. 1° 37′ E.
Course from Lowestoft: 062° T. 070° M. 068° C.
G.S.: 166 knots.
Expected position: 52° 46′ N. 3° 33′ E.
E.T.A.: 13.17 hrs.
6. Course 062° T.
T.M.G. 068°; drift 6° stbd.
G.S. 149 knots, W/V 310°/17 knots.
Track 094°; distance 107 n.m.
Course 094° T., 103° M., 103° C.
G.S. 160 knots, E.T.A. 11.50 hrs.
Course 247° M., 239° T.
Track 234°; G.S. 123 knots.
Position 51° 56′ N. 3° 29′ E.
Track 277°; distance 126 n.m.
Course 276° T., 285° M., 285° C.
G.S. 121 knots, E.T.A. 13.17½ hrs.
7. First course 082° M., 073° T.
T.M.G. 069°.
G.S. 159 knots.
W/V 150°/12 knots.
D.R. position 53° 01′ N. 2° 51′ E.
Second course 060° T., 068° M., 066° C.
T.A.S. 126 knots.
Third course 255° T., 264° M., 262° C.
Distance 183 n.m.
G.S. 169 knots.
E.T.A. 11.45 hrs.
8. W/V 107°/15 knots.
Track and course 324° T.
T.M.G. 316° T.
G.S. 169 knots.
Drift 8° P.
W/V 092°/29 knots.
New track 334°.
Course 344° T.
Distance 65 n.m.
G.S. 155 knots.

E.T.A. 17.10 hrs.

9. Track 109°.
Distance 187 n.m.
Course 101° T., 110° M., 111° C.
G.S. 115 knots.
E.T.A. 08.52 hrs.
Norwich bears 000° T.
Southwold bears 090° T.
Position 52° 20′ N. 1° 15′ E.
T.M.G. 104°.
  G.S. 126 knots.
  W/V 064°/21 knots.
Crossing coast 07.41½ hrs.
Adjusted course 108° T., 116° M., 117° C.

General Exercises

1.

	Track	*G.S. (knots)*	*Drift*
(*a*)	303°	210	—
(*b*)	358°	176	—
(*c*)	037°	237	—
(*d*)	341°	288	9° P.
(*e*)	022°	240	8° P.
(*f*)	008°	137	12° S.

2.

	Course (T.)	*G.S. (knots)*	*Drift*
(*a*)	205°	244	—
(*b*)	233°	224	—
(*c*)	305°	154	7° S.
(*d*)	106°	113	10° S.
(*e*)	003°	276	7° S.
(*f*)	052	140	7° P.

3.

	W/V (knots)
(*a*)	317°/43
(*b*)	255°/28
(*c*)	203°/38
(*d*)	260°/26
(*e*)	090°/27
(*f*)	120°/22

4.

	G.S. (knots)	*Wind Speed (knots)*
(*a*)	186	30
(*b*)	142	20
(*c*)	228	40

5.

	Course made good (T.)	*Effective Speed (knots)*
(*a*)	068°	8·5
(*b*)	350°	14·5
(*c*)	248°	18·4
(*d*)	178°	12·1

6.

	Course to steer (T.)	*Effective Speed (knots)*
(*a*)	048°	11·5
(*b*)	329°	12·3
(*c*)	116°	18·8
(*d*)	295°	7·4
(*e*)	008°	9·3
(*f*)	015°	15·0

7.

	Set of Current (*T.*)	*Rate of Drift* (*knots*)
(*a*)	330°	1·0
(*b*)	175°	0·8
(*c*)	082°	1·0
(*d*)	264°	1·6

8.

	Course to steer (*T.*)	*Steaming Speed* (*knots*)
(*a*)	310°	9·5
(*b*)	051°	12·0

9.

	W/V (*knots*)
(*a*)	240°/50
(*b*)	082°/24
(*c*)	220°/40

CHAPTER VII

EXERCISE 1

1.	Air position	51° 43′ N. 3° 08′ E.	D.R. position	52° 00′ N. 4° 28′ E.
	Track	256° T.	Distance	192 n.m.
	Course	260° T.	G.S.	100 knots
	Time	1 hr. 55 mins.	E.T.A.	16.25 hrs.

2.	Air position	52° 11′ N. 2° 06′ E.	D.R. position	53° 10′ N. 2° 45′ E.
	Track	233° T.	Distance	66 n.m.
	Course	230° T.	G.S.	80 knots
	Time	0 hr. 49½ mins.	E.T.A.	09.25½ hrs.

3.	Air position	53° 29′ N. 4° 18′ E.	W/V	264°/28 knots
	Track	264° T.	Distance	261 n.m.
	Course	264° T.	G.S.	92 knots
	Time	2 hrs. 50 mins.	E.T.A.	19.30 hrs.

4.	Air position	52° 43′ N. 2° 03′ E.	D.R. position	53° 13′ N. 3° 32′ E.	W/V 242°/32 knots
	Track	272° T.	Distance	154 n.m.	
	Course	264° T.	G.S.	92 knots	
	Time	1 hr. 40 mins.	E.T.A.	11.00 hrs.	

EXERCISE 2

D. long.	*Dep.*	*Lat.*
1. 11° 48·4′	**4.** 284·1 n.m.	**7.** 60° 00′
2. 8° 19·4′	**5.** 321·0 n.m.	**8.** 45° 55′
3. 23° 24·4′	**6.** 846·8 n.m.	**9.** 46° 34′

EXERCISE 3

	Departure	*Difference*
1.	274·0 n.m.	0·3
2.	376·0 n.m.	3·0
3.	369·1 n.m.	8·3
4.	389·6 n.m.	3·8

EXERCISE 4

	Course	*Distance* (*n.m.*)
1.	S. 29° 15′ W.	215·5
2.	S. 67° 26½′ E.	211·1
3.	N. 35° 04′ W.	189·4
4.	N. 41° 49′ E.	305·9
5.	S. 31° 26′ W.	263·7
6.	S. 66° 16′ E.	288·2
7.	N. 24° 37′ W.	298·1
8.	N. 60° 12′ W.	215·3

EXERCISE 5

1.	D.R. position	48° 57·9′ N.	6° 50·6′ W.
	Estimated position	49° 26·2′ N.	6° 08·0′ W.
	Course	S. 30° 43′ W.	Distance 16·05 n.m.
2.	D.R. position	49° 08·2′ N.	11° 22·5′ W.
	Estimated position	49° 08·2′ N.	10° 05·0′ W.
	Course	S. 9° W.	Distance 123·0 n.m.
3.	D.R. position	58° 04·5′ N.	1° 45′ W.
	Estimated position	57° 56·0′ N.	2° 16′ W.
	Course	N. 19° 51½′ W.	Distance 243·5 n.m.
4.	Position at start	49° 54·9′ N.	5° 02·2′ W.
	D.R. position	48° 48·5′ N.	7° 12·2′ W.
	Estimated position	48° 58·1′ N.	6° 31·7′ W.
	Course	S. 45° 39′ W.	Distance 81·2 n.m.
	Bearing	N. 40° 51′ E.	Distance 79·2 n.m.

EXERCISE 6

	Course	*Distance* (*n.m.*)
1.	S. 76° 51′ W.	2637
2.	N. 45° 50½′ E.	2584
3.	N. 59° 18½′ W.	3017
4.	S. 51° 45′ E.	4416
5.	N. 69° 22′ E.	3760
6.	S. 23° 12½′ W.	3786
7.	N. 68° 33½′ W.	3365

EXERCISE 7

	Distance (*n.m.*)	*Initial Course*
1.	4456	108° 39′
2.	5248	240° 13′
3.	1186	086° 28′
4.	3783	276° 50′
5.	4071	044° 01′
6.	5035	223° 50′
7.	3583	113° 07′

General Exercises

1.

	Mean lat.	*D. long.* (*ch. long.*)	*Departure* (*n.m.*)
(*a*)	—	—	350·0
(*b*)	47° 00′	—	—
(*c*)	—	14° 40′	—
(*d*)	—	—	356·3
(*e*)	26° 46′	—	—
(*f*)	—	13° 48′	—
(*g*)	—	1° 40′	—

2.
(*a*) 54° 22·5′ N. 00° 56·0′ E.
(*b*) 54° 41·3′ N. 00° 44·4′ E.
(*c*) 039·5° T. 40·5 n.m.

3.
(*a*) Ship's D.R. position } 56° 00′ N. 9° 05′ W.
(*b*) Aircraft's air position } 56° 00′ N. 9° 05′ W.
(*a*) Average set and drift 290° 30 n.m.
(*b*) W/V 110°/15 knots.

4. Latitude 48° 11′.
5. 503·9 knots.
6. Longitude 19° 02′ W.
7. D. lat. 6° 33′ S. Track 220° 54·5′
D.M.P. 589·7 Longitude 18° 07′ W.

8.

	Course (*T.*)	*Distance* (*n.m.*)
(*a*)	068° 38′	312·9
(*b*)	198° 45′	455·2
(*c*)	068° 09′	274·0
(*d*)	122° 39′	430·0
(*e*)	138° 23′	585·9
(*f*)	196° 25′	608·7
(*g*)	175° 40′	263·8
(*h*)	290° 13′	926·5

9. 100·8 n.m.
10. Rhumb-line distance 2085 n.m.
G.C. distance 2037 n.m.
Bearing from St John's 070° 39·5′ (T.).

11.

	Initial Course (*T.*)	*G.C. Distance* (*n.m.*)	*Mercator Course* (*T.*)	*R.L. Distance* (*n.m.*)	*Increase per cent.*
(*a*)	278° 46′	1725	256° 31′	1831	6·14
(*b*)	253° 00′	2033	243° 00′	2041	0·39
(*c*)	222° 03′	4445·5	220° 48′	4450	0·10
(*d*)	236° 06′	3286	237° 44′	3290	0·12
(*e*)	115° 33′	3182	105° 52′	3210	0·88
(*f*)	291° 52′	3900	246° 38′	4185	7·31
(*g*)	298° 22′	2714	270° 51′	2804	3·21
(*h*)	054° 03′	4470	088° 11′	4710	5·37
(*i*)	129° 21′	5030	084° 46′	5448	8·31

12. Distance 2313 n.m. Total alteration of course 47° 12′.
13. Position 55° 30′ N. 33° 44′ W.
L.A.T. March 2 09.38·6 hrs.

CHAPTER VIII

EXERCISE 1

	(*n.m.*)	(*n.m.*)
1.	225° A 4·3	290° B 5·1
2.	120° A 5·1	045° B 6·3
3.	310° A 9·9	020° B 6·8
4.	070° A 4·8	150° B 4·8
5.	285° A 8·0	210° B 11·3
6.	045° A 5·4	320° B 7·0
7.	160° A 3·4	080° B 4·0

EXERCISE 2

	Bearing of A (*T.*)	*Bearing of B* (*T.*)	*Position*	
			(*n.m.*)	(*n.m.*)
1.	090°	145°	270° A 10·0	325° B 11·5
2.	205°	270°	025° A 5·5	090° B 7·3
3.	065°	130°	245° A 8·0	310° B 13·0
4.	290°	350°	110° A 5·1	170° B 4·8
5.	355°	297°	175° A 5·3	117° B 7·1
6.	014°	118°	194° A 8·3	298° B 3·5
7.	078°	012°	258° A 6·7	192° B 9·1

EXERCISE 3

1. Distance (n.m.) (*a*) 0·14, (*b*) 0·5, (*c*) 2·3, (*d*) 3·8.
Vertical angle (*e*) 0° 45′, (*f*) 0° 41′, (*g*) 0° 57′.
2. Distance (n.m.) (*a*) 6·9, (*b*) 10·35, (*c*) 8·6, (*d*) 5·6, (*e*) 12·1.
3. (*a*) 22·7 n.m., (*b*) 21·8 n.m., (*c*) 17·2 n.m., (*d*) 17·4 n.m.
4. 48·4 feet
5. 12·6 knots
6. 21·1 n.m.

EXERCISE 4

	(*n.m.*)	(*n.m.*)
1.	090° A 5·7	139° B 6·1
2.	160° A 3·0	220° B 3·0
3.	225° A 7·2	154° B 8·1
4.	336° A 6·4	271° B 7·5
5.	263° A 7·7	215° B 7·5
6.	104° A 10·5	049° B 5·3
7.	127° A 4·0	205° B 3·4
8.	344° A 6·8	074° B 9·0
9.	202° A 4·3	312° B 7·9
10.	030° A 0–12	210° B 0–12

EXERCISE 5

			(*n.m.*)
1.	150°	B	3·4
2.	219°	B	2·8
3.	039°	B	2·1
4.	281°	B	1·6
5.	090°	B	4·0
6.	053°	B	2·7
7.	180°	B	1·8

EXERCISE 6

	Second Bearing (*T.*)	*Distance* (*n.m.*)	*First Bearing* (*T.*)	*Distance* (*n.m.*)
1.	150°	2·6	060°	3·1
2.	225°	3·8	140°	3·5
3.	220°	6·1	280°	8·7
4.	000°	5·2	080°	10·4
5.	306°	3·1	240°	7·9
6.	068°	6·1	354°	5·4
7.	084°	3·7	130°	5·9
8.	305°	4·2	227°	8·2

EXERCISE 7

	Track, or Course, made good	*Position* (*n.m.*)	*W/V* (*knots*)	*Set and Drift* (*n.m.*)
1.	008°	332° L 6·2	020°/29	200°/2·0
2.	121°	164° L 8·3	342°/15	162°/0·9
3.	002°	048° L 9·9	302°/21	122°/1·9
4.	279°	232° L 8·3	280°/12	100°/0·8
5.	068°	035° L 7·0	129°/25	309°/1·7
6.	214°	175° L 10·5	205°/28	025°/3·1

GENERAL EXERCISES

5. 50° 00·7′ N. 6° 14·4 ′W.
6. Low water.
7. 21·7 n.m.
8. 49° 41·6′ N. 5° 42·0′ W.
9. 49° 59·8′ N. 5° 52·5′ W. Distance 6·8 n.m.
10. 49° 50·0′ N. 2° 03·0′ W. Distance 14·5 n.m.
11. 123° Shipwash light-vessel 4·3 n.m.
12. 061° Orfordness lighthouse 7·6 n.m. (52° 09·2′ N. 1° 46·1′ E.).
13. 52° 55½′ N. 1° 36½′ E.
14. 52° 31¼′ N. 1° 50½′ E.
15. 55° 16·8′ N. 5° 14′ W. 350° Corsewall lighthouse 17 n.m.
16. 170° Oversay Is. lighthouse 13·2 n.m.
17. 51° 49′ N. 7° 41½′ W.
18. 1·8 n.m. and 1·7 n.m.
19. 157° Duncansby Head. 11·4 n.m.
20. 50° 08′ N. 3° 47′ W.
21. 58° 43·2′ N. 3° 07·4′ W. 070° Dunnett Head, 8·5 n.m.
22. 210° Clyth Ness 5·5 n.m. 115° Clyth Ness 1·75 n.m.
23. 50° 18·8′ N. 3° 19′ W. 130° Berry Head 8·0 n.m.
24. 52° 02′ N. 5° 35′ W. 330° S. Bishop lighthouse 12·5 n.m.
25. 50° 17½′ N. 3° 22′ W.
26. 51° 17½′ N. 4° 12′ W. 000° Bull Point 5·5 n.m.
27. First 57° 13·7′ N. 7° 09′ W. Second 57° 19·4′ N. 7° 06·5′ W.
28. 38° 36·2′ N. 74° 55′ W.
29. Track 243° T.
First position 52° 11′ N. 6° 03·5′ W. Second position 52° 07′ N. 6° 16′ W.
Set 030° T. drift 1·6 n.m. (1·9 knots), W/V 210°/38 knots.
30. 2° 37½′.

CHAPTER IX

Exercise 1

1.	Rel. vel. 141°	19·0 knots
2.	Rel. vel. 109°	20·4 knots
3.	Rel. vel. 207°	28·6 knots
4.	Rel. vel. 222°	20·5 knots
5.	Rel. vel. 143°	20·7 knots
6.	Rel. vel. 148°	22·5 knots

For aircraft multiply speeds by 10

Exercise 2

	Course (T.)	*Speed of Opening (knots)*
1.	195°	5·0
2.	227°	8·2
3.	348°	22·7
4.	034°	9·2
5.	202° 160°	13·5 4·0
6.	002°	10·0

For aircraft multiply speeds by 10

Exercise 3

	True Course	*Duration hrs. mins.*	*Speed of Approach (knots)*
1.	350°	4 18	23·3
2.	239°	4 49	15·6
3.	045° 075°	10 21 28 35	11·6 4·2
4.	104°	4 29	20·1
5.	023°	4 22	11·0
6.	193°	5 10	11·6

Exercise 4

	True Course	*Speed of Opening (knots)*
1.	146°	142
2.	113°	106
3.	341°	165
4.	165°	134
5.	071°	146
6.	197°	125

Exercise 5

	True Course	*Speed of Approach (knots)*	*Time (mins.)*
1.	002°	192	15·6
2.	202°	170	42·4
3.	092°	208	23·1
4.	153°	123	51·2
5.	038°	205	28·1
6.	332°	180	50·0

Exercise 6

	Relative W/V		*True W/V*
1.	190°/28½ knots	**4.**	353°/28 knots
2.	169°/21 knots	**5.**	208°/30 knots
3.	108°/45½ knots	**6.**	121°/30 knots

Exercise 7

	True Course	*Time Interval* *hrs.*	*mins.*	*mins.*
1.	099°	1	46	(10·6)
2.	166°	1	40	(10·0)
3.	278°	1	46	(10·6)
4.	270°	2	06	(12·6)
5.	158°	1	52½	(11·25)
6.	056°	2	10	(13·0)

Exercise 8

	Course out (T.)	*Course back (T.)*	*G.S. out (knots)*	*G.S. back (knots)*	*R. of A. (n.m.)*	*T.T.T. hrs.*	*mins.*
1.	159°	321°	125	112	236	*1	53·4
2.	308°	112°	181	210	292	1	36·8
3.	046°	214°	194	162	221	1	08·4

T.T.T.

* for ship multiply by 10 thus:
1. 18 hrs. 54 mins.
2. 16 hrs. 08 mins.
3. 11 hrs. 23·5 mins.

Exercise 9

	Course out (T.)	*Course back (T.)*	*Track back*	*R. of A. (n.m.)*	*T.T.T. hrs.*	*mins.*
1.	109°	320°	309°	216	1	33·4
2.	155°	309°	316°	243	2	10·3
3.	012°	218°	211°	207	0	56·4
4.	350°	161°	159°	339	1	29·1
5.	210°	020°	031°	157	0	51·8

Exercise 10

Course out 084° (T.); G.S. 177 knots. Time 0 hrs. 34 mins.

	Search 1	2	3	4
Track	060°	150°	240°	330°
Course (T.)	060°	162°	240°	318°
G.S. (knots)	180°	147°	120°	147°
8 n.m.	2·7 mins.	3·3 mins.	4·0 mins.	3·3 mins.

Bearing 345° (T.); distance 89 n.m.

General Exercises

1. Course out 324° T. Course back 174° T. R. of A. 56 n.m.
G.S. out 97 knots G.S. back 135 knots T.T.T. 35 mins.
Air position { 53° 51′ N. 0° 07′ E.
D.R. position { 53° 47′ N. 0° 43′ E.

2. Course out 026° T. Course back 224° T. R. of A. 168 n.m.
G.S. out 140 knots G.S. back 94 knots T.T.T. 1 hr. 12 mins.
Air position { 53° 31′ N. 3° 07′ E.
D.R. position { 53° 40′ N. 4° 00′ E.

3. Course out 068° T. Course back 286° T. R. of A. 142 n.m.
G.S. out 145 knots G.S. back 85 knots T.T.T. 59 mins.
Air position 53° 59′ N. 3° 13′ E.
D.R. position 53° 41′ N. 4° 03′ E.

4. Course out 076° T. Course back 289° T. R. of A. 137 n.m.
G.S. out 96 knots G.S. back 159 knots T.T.T. 1 hr. 25½ mins.
Air position 53° 23′ N. 6° 00′ E.
D.R. position 53° 50′ N. 4° 46′ E.

5. Course out 069° T. Course back 275° T. R. of A. 115 n.m.
G.S. out 105 knots G.S. back 128 knots T.T.T. 1 hr. 05 mins.
Air position 54° 17′ N. 4° 33′ E.
D.R. position 53° 51′ N. 4° 24′ E.

6. Course out 056° T. E.T.I. 11.41 hrs. Course back 266° T.

7. Course out 092° T. G.S. 81 knots. E.T.I. 11.42½ hrs. D.R. position 077° A 125 n.m.

8. Course out 075° T. G.S. 112 knots. E.T.I. 14.16½ hrs. W/V 343°/16 knots. Adjusted course 290° T., speed of approach 108 knots. E.T.I. 14.32 hrs., position 079° A. 122 n.m.
Course back 268° T. E.T.A. 15.49 hrs.

9. Course out 098° T. W/V 175°/43 knots. Readjusted course 105° T. Course to intercept 048° T. E.T.I. 08.11 hrs. Course for A 224° T. E.T.A. at A 09.42 hrs.

10. Course to intercept 073° T. 083° M. 076½° C. E.T.I. 02.30 hrs.

11. Course to intercept 124° T. 134° M. 128° C. E.T.I. 19.56 hrs.

12. Course to intercept 063° T. 073° M. 067° C. E.T.I. 13.23 hrs.
Position of interception 078° Spurn 98 n.m.

13. Course out 032° T. E.T.A. 11.20 hrs.

Square Search

	1	2	3	4
Track	150°	240°	330°	060°
Course T.	150°	245°	330°	055°
G.S.	38 knots	34·5 knots	32 knots	34·5 knots

Position at 13.50 hrs. 53° 39′ N. 3° 22′ E.
Course back 218° T.
E.T.A. base 16.52 hrs.

14. Course out 079° T. G.S. 36.5 knots. E.T.A. 13.14 hrs.

Square Search

	1	2	3	4
Track	150°	240°	330°	060°
Course T.	150°	244°	330°	056°
G.S.	38 knots	35·9 knots	34 knots	35·9 knots

Position at 14.30 hrs. 54° 02′ N. 3° 18′ E.
Course back 253° T. E.T.A. 17.46 hrs.

15. Course out 118° T. G.S. 180 knots. E.T.A. 08.38 hrs.

Square Search

	1	2	3	4
Track	070°	160°	250°	340°
Course T.	070°	176°	250°	324°
G.S.	190 knots	144 knots	110 knots	144 knots

Position of raft 094° A 80 n.m.
Course back 268° T. E.T.A. 09.57 hrs.

16. Course out 062° T. G.S. 140 knots. E.T.A. 08.22 hrs.

Square Search

	1	2	3	4
Track	020°	110°	200°	290°
Course T.	020°	122°	200°	278°
G.S.	144 knots	118 knots	96 knots	118 knots

Position of survivors 53° 32′ N. 2° 30′ E.
Bearing and distance of survivors from launch 315° 14 n.m.

CHAPTER XII

EXERCISE 1

	Dec.	*H.A.* *hrs. mins.*	*R.A.* *hrs. mins.*	*S.H.A.*	*Star*
1.	16° N.	20 35 (309°)	4 30	292°	Aldebaran
2.	11° S.	3 45 (056°)	13 22	159°	Spica
3.	45° N.	5 00 (075°)	20 40	50°	Deneb
4.	26° S.	4 10 (063°)	16 26	114°	Antares
5.	40° N.	3 10 (048°)	18 30	82°	Vega
6.	12° N.	4 00 (060°)	10 05	209°	Regulus
7.	10° N.	19 45 (296°)	19 45	64°	Altair
8.	30° S.	21 45 (326°)	22 55	16°	Fomalhaut
9.	10° S.	2 40 (040°)	5 10	282°	Rigel

EXERCISE 2

	H.A. *hrs. mins.*	*Az.*	*Alt.*
1.	22 06 (317°)	N. 107° E.	66°
2.	02 06 (032°)	S. 65° W.	59°
3.	19 28 (292°)	N. 98° E.	21°
4.	05 07 (077°)	S. 72° W.	23°
5.	04 47 (072°)	N. 94° W.	21°
6.	20 29 (307°)	S. 88° E.	40°
7.	22 07 (332°)	N. 145° E.	35°
8.	01 27 (022°)	S. 124° W.	63°
9.	05 31 (083°)	N. 48° W.	20°

EXERCISE 3

	Alt.	*Dec.*
1.	42°	16° S.
2.	33°	27° N.
3.	21°	30° S.
4.	40°	00°
5.	15°	17° N.

EXERCISE 4

	Az.	*Dec.*
1.	N. 126° E.	5° S.
2.	S. 105° W.	12° S.
	or S. 40° W.	or 49° S.
3.	N. 84° W.	26° N.
4.	S. 75° E.	30° S.
5.	N. 29° E.	47° N.
	or N. 140° E.	or 25° S.

EXERCISE 5

	H.A. hrs.	H.A. mins.	Alt.
1.	02	16 (034°)	45°
2.	20	40 (310°)	24°
3.	03	30 (052½°)	43°
4.	04	24 (066°)	30°
5.	21	20 (230°)	42°
6.	22	24 (246°)	37°

EXERCISE 6

	H.A. hrs.	H.A. mins.	Az.
1.	19	36 (294°)	N. 93° E.
2.	02	40 (040°)	S. 124° W.
3.	05	32 (083°)	N. 70° W.
4.	21	00 (315°)	N. 127° E.
5.	20	08 (302°)	S. 78° E.

EXERCISE 7

	Latitude
1.	50° N.
2.	30° S.
3.	20° N.
4.	40° S.
5.	60° N.
6.	55° N.
7.	50° S.
8.	00°

GENERAL EXERCISES

3.

	Lat.	L.H.A.	Dec.	Bearing	Alt.
(a)	—	50°	7½° N.	—	—
(b)	—	333°	53° S.	—	—
(c)	—	—	—	080°	50°
(d)	—	—	—	152°	28½°
(e)	—	—	—	336°	18½°
(f)	—	—	0°	224°	—
(g)	—	335°	—	—	62°
(h)	—	—	36° N.	—	43°
(i)	—	306°	—	084°	—
	—	54°	—	276°	—
(j)	30° N.	—	—	—	—
(k)	40° N.	—	—	—	—
(l)	35° S.	—	—	—	—
(m)	50° S.	—	—	—	—

4.

(a) Sirius
(b) Capella
(c) Achernar
(d) Procyon
(e) Altair
(f) Regulus
(g) Sirius
(h) Alpheratz
(i) Aldebaran

5.

	Az.	Alt.
(a)	N. 147° W.	35°
(b)	N. 65° E.	41°
(c)	S. 155° W.	55°

CHAPTER XIII

EXERCISE 1

	PZX	*ZPX*	*PXZ*
1.	107° 52¾′	49° 57½′	44° 32½′
2.	112° 21¾′	56° 10′	35° 28¼′
3.	39° 02¼′	37° 42½′	120° 03½′
4.	130° 35½′	30° 33¼′	51° 25¾′
5.	108° 00′	64° 28¼′	61° 17¼′

EXERCISE 2

	Hour Angle (*ZPX*)	*Azimuth* (*PZX*)
1.	81° 09′	63° 31′
2.	33° 09′	68° 25¼′
3.	52° 44′	121° 26′
4.	45° 33′	125° 09½′
5.	74° 47′	61° 13½′

EXERCISE 3

	ZX
1.	38° 43½′
2.	41° 20′
3.	37° 58½′
4.	67° 51′
5.	35° 39′
6.	65° 24′

EXERCISE 4

	Altitude
1.	47° 35½′
2.	47° 31′
3.	56° 29½′
4.	37° 48′
5.	29° 37¼′
6.	26° 29½′

EXERCISE 5

	Apparent time						*Azimuth*	
	Rise			*Set*			*Rise*	*Set*
	hrs.	*mins.*	*secs.*	*hrs.*	*mins.*	*secs.*		
1.	5	38	00	18	22	00	N. 82° 08½′ E.	N. 82° 08½′ W.
2.	5	30	56	18	29	04	N. 65° 47′ E.	N. 65° 47′ W.
3.	5	13	24	18	46	36	S. 69° 05½′ E.	S. 69° 05½′ W.
4.	6	28	44	17	31	16	N. 105° 28′ E.	N. 105° 28′ W.
5.	6	13	24	17	46	36	S. 111° 21′ E.	S. 111° 21′ W.
6.	6	00	00	18	00	00	N. 90° 00′ E.	N. 90° 00′ W.
7.	6	00	00	18	00	00	S. 66° 33′ E.	S. 66° 33′ W.

Amplitude

Rise	*Set*
E. 7° 51½′ N.	W. 7° 51½′ N.
E. 24° 13′ N.	W. 24° 13′ N.
E. 20° 54½′ S.	W. 20° 54½′ S.
E. 15° 28′ S.	W. 15° 28′ S.
E. 21° 21′ N.	W. 21° 21′ N.
E. 00° 00′ N./S.	W. 00° 00′ N./S.
E. 23° 27′ S.	W. 23° 27′ S.

EXERCISE 6

	Civil Twilight *begins* *hrs. mins.*	*Civil Twilight* *ends* *hrs. mins.*	*Nautical Twilight* *begins* *hrs. mins.*	*Nautical Twilight* *ends* *hrs. mins.*
1.	03 35 L.A.T.	04 25 L.A.T.	02 37 L.A.T.	04 25 L.A.T.
	19 35 L.A.T.	20 25 L.A.T.	19 35 L.A.T.	21 23 L.A.T.
2.	04 29 L.A.T.	05 01 L.A.T.	03 55 L.A.T.	05 01 L.A.T.
	18 59 L.A.T.	19 31 L.A.T.	18 59 L.A.T.	20 05 L.A.T.
3.	01 52 L.A.T.	03 16 L.A.T.	20 44 L.A.T.	03 16 L.A.T.
	20 44 L.A.T.	22 08 L.A.T.	Twilight all night	
4.	06 15 L.A.T.	06 49 L.A.T.	05 42 L.A.T.	06 49 L.A.T.
	17 11 L.A.T.	17 45 L.A.T.	17 11 L.A.T.	18 18 L.A.T.
5.	07 24 L.A.T.	08 12 L.A.T.	06 41 L.A.T.	08 12 L.A.T.
	15 48 L.A.T.	16 36 L.A.T.	15 48 L.A.T.	17 19 L.A.T.
6.	08 53 L.A.T.	11 02 L.A.T.	07 35 L.A.T.	11 02 L.A.T.
	12 58 L.A.T.	15 07 L.A.T.	12 58 L.A.T.	16 25 L.A.T.
7.	05 22 L.A.T.	06 00 L.A.T.	04 44 L.A.T.	06 00 L.A.T.
	18 00 L.A.T.	18 38 L.A.T.	18 00 L.A.T.	19 16 L.A.T.
8.	09 32 L.A.T.	14 28 L.A.T.	07 58 L.A.T.	16 02 L.A.T.
	Sun never above horizon No daylight		No daylight	

GENERAL EXERCISES

1. ZPX = 43° 03′. PZX = 111° 11′.
2. ZX = 42° 12′.
3. Hour angle 3 hrs. 37 mins. 18 secs. Azimuth S. 68° 28′ W.
4. Zenith distance 45° 56′. True altitude 44° 04′. Azimuth S. 105° W.
5. True amplitude W. 26° 31′ N. Approx. time of sunset 19 hrs. 11 mins. 08 secs.
Duration of civil twilight 0 hrs. 36 mins. 00 secs.
Duration of nautical twilight 1 hr. 14 mins. 42 secs.
6. Declination 17° 56′ N.
7. Arcturus.
8. Altair.
9. Denebola.
10. Altitude 41° 12′. Azimuth S. 51° E.
11. Altitude 50° 25′. Azimuth S. 41° 12′ W.
12. Latitudes 53° 36½′ N. and 17° 48′ S.

CHAPTER XIV

EXERCISE 1

Time zone:
1. + 5; − 8; + 11; − 12.

		hrs. mins.		*hrs. mins.*
2.	1. September 16	02 38;	4. November 11	19 55
	2. February 5	06 22;	5. January 13	05 15
	3. April 30	22 36;	6. January 1	03 50

EXERCISE 2

1. April 4 20 hrs. 12 mins. 35 secs.
2. June 14 19 hrs. 39 mins. 24 secs.
3. 16 hrs. 11 mins. 24 secs.
4. 5 hrs. 43 mins. 36 secs. or 17 hrs. 43 mins. 36 secs.
5. Losing 0·85 seconds.
6. C 4 mins. 38 secs. slow of A.

Exercise 3

	R			*Sun* *Dec.*	*E*		
	hrs.	*mins.*	*secs.*		*hrs.*	*mins.*	*secs.*
1.	06	42	38·4	23° 01·1′ S.	11	56	26·8
2.	17	05	02·7	22° 26·2′ S.	11	53	51·3
3.	18	45	22·0	22° 57·5′ N.	11	55	57·2
4.	18	54	16·8	22° 45·5′ N.	11	55	33·2
5.	19	02	19·4	22° 33·0′ N.	11	55	13·1

	R.A.			*Dec.*
	hrs.	*mins.*	*secs.*	
		Moon		
6.	18	29	15	18° 12·0′ S.
7.	21	51	57	9° 46·0′ S.
8.	23	57	23	00° 03·2′ N.
		Venus		
9.	16	59	15	22° 37·7′ S.
10.	19	01	45	22° 40·1′ S.
		Mars		
11.	15	46	52	19° 33·2′ S.
12.	16	45	38	22° 12·4′ S.

Exercise 4

	Moon's Meridian Passage *hrs.*	*mins.*
1.	15	31 L.M.T.
2.	20	52 L.M.T.
3.	05	10 L.M.T.
4.	10	13 L.M.T.
5.	00	05 L.M.T.

Exercise 5

	Meridian Passage of:	*hrs.*	*mins.* (*L.M.T.*)
1.	Venus	10	31
2.	Venus	10	46
3.	Mars	08	59
4.	Mars	08	42

Exercise 6

		G.H.A. ♈	*G.H.A. Sun*
1. January 4.	(*a*)	107° 19′	182° 55′
	(*b*)	192° 15′	267° 35′
	(*c*)	1° 05′	75° 54′
July 9.	(*d*)	327° 52′	219° 59′
	(*e*)	1° 02′	253° 04′
	(*f*)	280° 34′	171° 48′

		G.H.A. Moon
2. January 4.	(*a*)	230° 35′
	(*b*)	1° 14′
July 9.	(*c*)	0° 52′
	(*d*)	299° 55′

	G.H.A. Venus	*G.H.A. Mars*	*G.H.A. Jupiter*
3. January 4.	(*a*) 302° 28′	(*c*) 359° 46′	(*e*) 303° 11′
July 9	(*b*) 3° 15′	(*d*) 184° 18′	(*f*) 197° 01′

Exercise 7

L.H.A.

Canopus	46° 38′
Betelgeuse	305° 09′
Regulus	65° 43′
Sun	323° 00′
Moon	298° 29′
Altair	80° 33′
Deneb	58° 06′
Achernar	280° 47′
Venus	284° 21′
Jupiter	73° 13′

General Exercises

hrs. *hrs.*

1.

11 and 13 = 12° 31′
10 and 14 = 25° 34′
9 and 15 = 39° 38½′
8 and 16 = 55° 08′
7 and 17 = 72° 05′
6 and 18 = 90° 00′

8.

		hrs.	*mins.*	
Aldebaran	L.M.T. of upper transit	21	36	visible.
Rigel	L.M.T. of upper transit	22	15	visible.
Capella	L.M.T. of upper transit	22	16	visible.
Sirius	L.M.T. of upper transit	23	46	visible.
Vega	L.M.T. of upper transit	11	40	visible.
Regulus	L.M.T. of upper transit	03	12	visible.
Canopus	L.M.T. of upper transit	23	26	not visible.
α Centauri	L.M.T. of upper transit	07	41	not visible.
Capella	L.M.T. of lower transit	10	18	visible.

9. G.M.T. meridian passage Venus. January 25 18 hrs. 54 mins.
G.M.T. meridian passage Mars. January 1 21 hrs. 12 mins.
10. G.M.T. upper meridian passage Moon. January 30 9 hrs. 03 mins.
11. G.M.T. upper meridian passage Moon. January 23 13 hrs. 37 mins. 36 secs.
12. G.M.T. December 1 18 hrs. 11 mins. 01 secs.
13. July 7.
14. Losing 1·75 secs. daily.
15. Chronometer C 6 mins. 52 secs. fast on Chronometer A.
16. Error 38 mins. 53 secs. slow.

CHAPTER XV

General Exercises

7.

	Latitude		*Latitude*
(*a*)	28° 42′ N.	(*f*)	69° 40′ N.
(*b*)	20° 32′ S.	(*g*)	55° 40′ S.
(*c*)	40° 47′ N.	(*h*)	22° 43′ S.
(*d*)	18° 42′ N.	(*i*)	49° 46′ S.
(*e*)	15° 08′ N.	(*j*)	52° 43′ S.

8.

	H.A.T.S. *hrs. mins. secs.*	*E* *hrs. mins. secs.*	*Longitude*
(*a*)	—	—	43° 02¾′ W.
(*b*)	—	—	41° 32½′ E.
(*c*)	—	11 54 41	165° 37¼′ E.
(*d*)	—	11 56 06	168° 13½′ W.
(*e*)	14 49 33	11 53 10	—
(*f*)	11 06 22	11 55 22	—

9.

	H.A. *hrs. mins. secs.*	*R* *hrs. mins. secs.*	*Longitude*
(*a*)	—	—	147° 00′ E.
(*b*)	02 46 04	—	—
(*c*)	—	—	135° 32′ W.
(*d*)	—	19 03 00	42° 44′ E.
(*e*)	20 20 37	6 58 40	—
(*f*)	21 57 24	18 47 40	—

10.

	H.A.T.S. *hrs. mins. secs.*	*Longitude*	*Azimuth*
(*a*)	20 22 11	7° 44′ E.	S. 81° E. (099°)
(*b*)	20 01 34	62° 17′ W.	N. 99° E. (099°)
(*c*)	03 12 17	64° 23′ E.	S. 126° W. (306°)
(*d*)	02 54 17	62° 53′ W.	N. 137° W. (223°)

11.

	H.A. *hrs. mins. secs.*	*Longitude*	*Azimuth*
(*a*)	3 55 10	41° 59′ W.	N. 77° W. (283°)
(*b*)	21 26 10	106° 13′ E.	S. 28° E. (152°)
(*c*)	21 18 53	57° 54′ W.	N. 82° E. (082°)
(*d*)	6 18 24	130° 38′ E.	S. 81° W. (261°)

12.

H.A. *hrs. mins. secs.*	*Longitude*
20 14 56	3° 51′ E.

13.

	Latitude	*Longitude*
(*a*)	45° 07′ N.	35° 23′ W.
(*b*)	25° 09′ S.	9° 53′ E.
(*c*)	12° 59′ N.	84° 06′ E.
(*d*)	9° 56′ S.	34° 13′ W.
(*e*)	54° 14′ N.	15° 27′ W.

14.

	Latitude	*Longitude*
(*a*)	38° 59′ S.	162° 20′ W.
(*b*)	28° 09′ N.	63° 00′ W.
(*c*)	53° 29′ N.	31° 03′ W.
(*d*)	15° 31′ S.	98° 39′ E.
(*e*)	68° 25′ N.	15° 27′ E.
(*f*)	34° 13′ S.	179° 40½′ E.

15. (*a*)

	H.A. *hrs. mins. secs.*	*Longitude*	*Bearing*
Spica	3 15 40	145° 04′ W.	235°
Altair	20 46 35	145° 53′ W.	109°

Final position: Lat. 35° 56′ N. Long. 145° 37′ W.

(*b*)

	H.A. *hrs. mins. secs.*	*Longitude*	*Bearing*
Canopus	20 48 07	74° 18′ E.	146°
Fomalhaut	4 16 54	74° 26′ E.	243°

Final position: Lat. 13° 56′ S. Long. 74° 24′ E.

17.

	Intercept
(*a*)	9′ towards
(*b*)	7′ away
(*c*)	12′ towards
(*d*)	11′ away
(*e*)	8′ away
(*f*)	9′ away
(*g*)	6′ towards
(*h*)	13′ towards.

18. (*a*)

	G.H.A. Sun	*Longitude*	*L.H.A. Sun*
(i)	—	—	52° 54′
(ii)	—	—	311° 00′
(iii)	—	62° 24′ W.	—
(iv)	—	154° 38′ E.	—
(v)	188° 20′	—	42° 36′
(vi)	318° 52′	—	329° 50′
(vii)	326° 34′	44° 42′ E.	—
(viii)	136° 52′	115° 16′ W.	—
(ix)	22° 12′	38° 19′ W.	—
(x)	179° 55′	—	352° 09′

(*b*)

	G.H.A.♈	*Longitude*	*L.H.A. Star*
(i)	—	—	24° 19′
(ii)	—	—	322° 06′
(iii)	—	71° 08′ W.	—
(iv)	—	108° 16′ E.	—
(v)	131° 20′	—	48° 31′
(vi)	306° 10′	—	316° 50′
(vii)	320° 33′	63° 18′ E.	—
(viii)	254° 19′	75° 36′ W.	—

19.

	Latitude	*Longitude*
(*a*)	40° 14′ N.	35° 38′ W.
(*b*)	37° 39′ S.	131° 16′ E.
(*c*)	46° 05′ S.	80° 49′ W.
(*d*)	10° 48′ N.	59° 12′ E.
(*e*)	32° 19′ N.	64° 33′ W.

20.

	Latitude	*Longitude*
(*a*)	20° 08′ S.	53° 34′ E.
(*b*)	39° 55′ S.	179° 02′ E.
(*c*)	19° 35′ S.	00° 15′ W.
(*d*)	34° 51′ N.	20° 48′ E.

21.

		Calculated Altitude	*Intercept*	*Bearing*	*Position*
(*a*)	Altair	42° 15′	8′ away	156·1°	54° 23′ N.
	Arcturus	29° 35·6′	12·4′ towards	262·3°	00° 29′ E.
(*b*)	Betelgeuse	26° 32′	7′ away	116·5°	54° 20′ N.
	Dubhe	32° 28′	8′ towards	024·8°	04° 31′ E.
(*c*)	Procyon	43° 49′	5′ away	187·4°	51° 26′ N.
	Capella	62° 33′	7′ towards	275·1°	46° 46′ W.
(*d*)	Regulus	49° 43′	8′ away	174·2°	52° 33′ N.
	Aldebaran	19° 24·5′	5·5′ towards	271·9°	146° 38′ W.
(*e*)	Fomalhaut	68° 13′	7′ towards	026·4°	49° 56′ S.
	Achernar	59° 37′	7′ away	124·1°	79° 44′ W.
(*f*)	Antares	64° 34·5′	5·5′ towards	009·3°	51° 23′ S.
	Peacock	53° 58′	8′ away	124·0°	65° 18′ W.

22.

	Calculated Altitude	*Intercept*	*Bearing*
Jupiter	36° 30·5′	9·5′ towards	317·7°
Achernar	47° 11·0′	4·0′ towards	216·1°

Final position : Lat. 30° 17′ S. Long. 49° 54′ W.

23.

		Calculated Zenith Distance	*Intercept*	*Bearing*	*Position*
(*a*)	Capella	25° 37·5′	7·5′ towards	094·2°	54° 54′ N.
	Hamal	32° 19·5′	6·5′ away	195·3°	15° 51′ W.
(*b*)	Aldebaran	38° 49·0′	11·0′ towards	164·1°	54° 12′ N.
	Alpheratz	47° 33·5′	11·5′ away	261·0°	48° 14′ W.
(*c*)	Rigel	46° 36·7′	18·7′ towards	340·0′	53° 09′ S.
	Achernar	37° 33·0′	13·0′ towards	235·0°	77° 56′ W.
(*d*)	Arcturus	61° 05·0′	5·0′ away	093·5°	49° 30′ N.
	Regulus	37° 28·7′	7·3′ away	170·4°	139° 49′ W.
(*e*)	Spica	64° 42·5′	6·5′ towards	156·8°	51° 06′ N.
	Pollux	52° 17·0′	9·0′ away	269·5°	169° 59′ E.

24.

	Calculated Zenith Distance	*Intercept*	*Bearing*	*Position*
Mars	42° 30·0′	1·0′ towards	227·2°	35° 00·0′ N.
Jupiter	35° 51·0′	7·0′ towards	100·1°	41° 31·0′ W.

CHAPTER XVI

EXERCISE 1

	Mean Latitude	*D. Long. (ch. long.)*	*Convergency*
1.	43° 31′ N.	23° 40′	16·3°
2.	25° 20′ N.	50° 52′	21·8°
3.	26° 03′ N.	50° 25′	22·1°
4.	26° 06′ S.	58° 15′	25·6°
5.	00° 25′ N.	40° 33′	0·05°
6.	27° 35′ N.	2° 20′	1·1°
7.	35° 21′ S.	23° 27′	13·6°

GENERAL EXERCISES

	Conversion Angle	*Rhumb-line Bearing*	*Position Line*	*Position*
1.	1·2°	071·2°	072° — 252°	53° 59′ N. 3° 16′ E.
	2·0°	110·0°	112° — 292°	
2.	1·0°	055·0°	056° — 236°	54° 21′ N. 2° 36′ E.
	0·7°	317·3°	317° — 137°	
3.	0·8°	045·8°	047° — 227°	53° 32′ N. 2° 14′ E.
	0·7°	080·7°	081° — 261°	
4.	1·0°	305·0°	304° — 124°	53° 44′ N. 4° 25′ E.
	0·2°	349·8°	350° — 170°	
5.	1·8°	105·8°	108° — 288°	54° 08′ N. 2° 35′ E.
	1·0°	061·0°	062° — 242°	

6.	0·8° 1·4°	219·2° 268·6°	218° – 038° 267° – 087°	53° 25′ N. 4° 07′ E.
7.	2·9° 1·6°	277·1° 218·4°	274° – 094° 217° – 037°	54° 24′ N. 5° 21′ E.
8.	0·4° 1·5°	199·6° 283·5°	199° – 019° 282° – 102°	54° 33′ N. 1° 50′ E.
9.	2·5° 4·3°	247·5° 285·7°	245° – 065° 281° – 101°	50° 30′ N. 15° 58′ W.
10.	1·3° 3·2°	218·7° 276·8°	217° – 037° 274° – 094°	49° 10′ N. 13° 46′ W.
11.	3·7° 2·4°	256·3° 297·6°	253° – 073° 295° – 115°	54° 03′ N. 16° 54′ W.
12.	0·9° 0·9°	080·9° 140·9°	082° – 262° 142° – 322°	37° 14′ N. 73° 08′ W.
13.	3·0° 1·2°	093·0° 026·2°	096° – 276° 027° – 207°	48° 42′ N. 12° 40′ W.
14.	2·3° 3·7°	107·3° 068·7°	110° – 290° 072° – 252°	53° 13′ N. 17° 04′ W.
15.	2·1° 1·5°	297·9° 031·5°	296° – 116° 033° – 213°	56° 47′ N. 3° 15′ E.
16.	6·3° 5·5°	069·3° 129·5°	076° – 256° 135° – 315°	51° 51′ N. 23° 18′ W.

MISCELLANEOUS D.R. AND TACTICAL PLOTTING QUESTIONS

1.

	Track	*Distance (n.m.)*	*Course*	*G.S. (knots)*	*Time hrs.*	*mins.*
S. to W.	344°	187	334°	169	1	06
W. to C.	144°	121	152°	196		37
C. to S.	193°	85	202°	170		30
				Total time	2	13

2.

	Track	*Distance (n.m.)*	*Course*	*G.S. (knots)*	*Time hrs.*	*mins.*
Ch. to Cr.	020°	86	011°	131		39
Cr. to X	332°	109	334°	125		52
X to L.	190°	77	196°	190		24
L. to Ch.	156°	99	155°	195		30
				Total time	2	25

3.

	Track	*Distance (n.m.)*	*Course*	*G.S. (knots)*	*E.T.A. hrs.*
K.L. to R.	108°	157	115°	173	11.04
R. to A.	183°	43	193°	136	11.23
A. to C.	291°	174	283°	126	12.46
C. to K.L.	020°	34	010°	166	12.58

4.

	Track	*Distance*	*Course*	*G.S.*	*E.T.A.*
		(*n.m.*)		(*knots*)	*hrs.*
L. to N.	053°	77	046°	112	09.41
X to N.	053°	12	045°	128	09.36
N. to A	005°	103	006°	123	10.26
A to B	250°	85	259°	162	10.57
B to L.	175°	119	172°	177	11.37

W/V experienced from L. to X 009°/27 knots

5.

	Track	*Distance*		*Course*		*G.S.*	*E.T.A.*
		n.m.	(*T.*)	(*M.*)	(*C.*)	(*knots*)	*hrs.*
W. to Buoy	098°	85	094°	104°	104°	173	14.29
Buoy to C.	204°	140	213°	223°	221°	150	15.25
C. to W.	349°	143	342°	352°	354°	133	16.30
Readjusted for W/V 317°/13 knots experienced from W. to S.							
S. to C.	188°	57	192°	202°	201°	158	15.24
C. to W.	349°	143	346°	356°	357°	138	16.26

6.

	Track	*Distance*		*Course*		*G.S.*	*E.T.A.*
		(*n.m.*)	(*T.*)	(*M.*)	(*C.*)	(*knots*)	*hrs.*
H. to M.	157°	158	149°	159°	161°	125	09.56
M. to L.	294°	75	293°	303°	304°	180	10.21
L. to H.	002°	114	010°	020°	019°	162	11·03
Readjusted for W/V 114°/31 knots experienced from H. to K.L.							
K.L. to M.	157°	93	148°	158°	160°	125	09.56
M. to L.	294°	75	294°	304°	305°	180	10.21
L. to H.	002°	114	013°	023°	022°	159	11.03

Position at 10.38 hrs. 52° 39′ N. 0° 20′ W.
T.M.G. 001° G.S. 145 knots. New course 012° T., 022° M., 021° C.
Final E.T.A. 11.05½ hrs.

7.

	Track	*Distance*		*Course*		*G.S.*	*E.T.A.*
		(*n.m.*)	(*T.*)	(*M.*)	(*C.*)	(*knots*)	*hrs.*
G. to S.	350°	177	344°	354°	355°	162	11.25

G. to C. T.M.G. 348°; T.E. 2° P.; G.S. 157 knots; W/V 220°/19 knots.
New course 348° T., 358° M., 359° C.
Position by observation at 10.46½ hrs. = 52° 29′ N. 00° 03′ E.
Position by D.R. at 10.55 hrs. = 52° 49′ N. 00° 01′ W.
Position by observation at 11.07 hrs. = 53° 18′ N. 00° 17′ W.
Final track 356°; Course 352° T., 002° M., 003° C.
Final E.T.A. 11.30 hrs.

8. 13.15 hrs. Track 018°; distance 75 n.m.
Course 009° T.; G.S. 165 knots.
E.T.A. Lowestoft 13.42 hrs.
13.40 hrs. W/V experienced 232°/51 knots.
Track 000°.
13.50 hrs. G.S. 167 knots.
14.10 hrs. D.R. position 53° 50′ N. 1° 43′ E.
Track 285°; distance 68 n.m.
Course 272° T.; G.S. 123 knots.
Drift 13° Starboard; E.T.A. Flamboro' 14.43 hrs.
15.22 hrs. T.M.G. 179°; G.S. 122 knots; drift 11° port.
T.E. 1° Starboard; W/V experienced 226°/38 knots.
Course 183° T.; E.T.A. Cambridge 15.41 hrs.

9.	First track	118°.
	Course	126° T., 135° M., 137° C.
	G.S.	204 knots.
	Second track	121°.
	T.E.	3° S.
	Drift	5° P.
	G.S.	224 knots.
	D.R. position	50° 58′ N. 3° 29′ E.
	A/C	20° P.
	Third track	277°.
	Distance	156 n.m.
	Course	274° T., 283° M., 283° C.
	G.S.	170 knots.
	E.T.A.	00.13 hrs.
	W/V	260°/36 knots.
	Fourth track	277°.
	W/V	255°/28 knots.
	E.T.A.	00.13 hrs.

10.	First track	085°.
	Distance	256 n.m.
	Course	094° T., 103° M., 103° C.
	G.S.	197 knots.
	E.T.A.	16.00 hrs.
	G.S.	200 knots.
	Drift	12° P.
	Lat. at coast	52° 40′ N.
	Observed position	52° 41′ N. 1° 56′ E.
	D.R. position	52° 49′ N. 4° 14′ E.
	Second track	089°.
	Third track	260°.
	Distance	249 n.m.
	Course	249° T., 257° M., 255° C.
	G.S.	167 knots.
	E.T.A.	17.49 hrs.
	T.M.G.	258°.
	G.S.	162 knots.
	T.E.	2° P.
	W/V	220°/38 knots.
	G.S.	164 knots.
	T.M.G.	254°.
	G.S.	172 knots.
	W/V	209°/23 knots.
	Track	279°.
	Course	272° T., 282° M., 282° C.
	Distance	55 n.m.
	G.S.	180 knots.
	E.T.A.	17.49 hrs.

11.	Track	110°.
	Distance	196 n.m.
	Course	114° T.
	T.M.G.	121°.
	G.S.	145 knots.
	Drift	7° S.
	Position	52° 42′ N. 2° 02′ E.
	T.M.G.	109°.
	G.S.	149 knots.
	W/V	197°/14 knots.
	Track	110°.
	Drift	4° P.
	Course	261° T.
	Track	267°.
	W/V	190°/17 knots.
	G.S.	145 knots.
	E.T.A.	17.12 hrs.

12.	Track	107°.
	Distance	272 n.m.
	T.A.S.	142 knots.
	Course	118° T., 127° M., 129° C.
	G.S.	151 knots.
	Track	101°.
	Position at 10.25 hrs.	52° 03′ N. 2° 36′ E.
	T.M.G.	106°.
	G.S.	154 knots.
	W/V	222°/35 knots.
	A/C	5° S.
	E.T.A.	11.28 hrs.
	Track	268°.
	Distance	251 n.m.
	Course	256° T., 264° M., 267° C.
	T.M.G.	267°.
	G.S.	121 knots.
	Track	268°.
	W/V	210°/37 knots.
	G.S.	128 knots.

13.	Track	094°.
	Distance	158 n.m.
	Course	098° T., 107° M., 108° C.
	G.S.	212 knots.
	E.T.A.	11.09 hrs.
	T.M.G.	100°.
	T.E.	6° S.
	Drift	2° S.
	G.S.	217 knots.
	W/V	290°/30 knots.

New track	091°.
New course	091° T., 100° M., 100° C.
New E.T.A.	11.08 hrs.
Position at 11.01 hrs.	52° 18′ N. 4° 11′ E.
Course	086° T., 094° M., 093° C.
Time to set course	11.56 hrs.
Track	257°.
Course	259° T., 268° M., 267° C.
Track	252°.
G.S.	172 knots.
W/V	300°/32 knots.
D.R. position	51° 56′ N. 2° 55′ E.
Course	250° T.
T.A.S.	208 knots.

14. First course	001° T.
Second course	080° T. Track 091°. G.S. 97 knots.
Course to intercept	249° T. 258° M.
Speed of approach	133 knots.
Distance to approach	35 n.m.
Time to approach	16 mins.
Time of interception	11.26 hrs.
Position of interception	54° 06′ N. 2° 53′ E.
Track back to raft	064°.
Distance back to raft	39 n.m
Time of rescue	12.44 hrs.
Course back	225° T. 234° M. 236° C.
G.S. back	150 knots.
Distance back	136 n.m.
Time taken	54 mins.
E.T.A.	13.38 hrs.

15. W/V 076°/25 knots.
Course to return to track 096° T., 106° M., 100° C.
Readjusted course 112° T., 122° M., 117° C.
G.S. 101 knots. Distance 58 n.m. Time 34 mins.
E.T.A. 07.19 hrs.
Distance to Colchester 107 n.m.
Course to Colchester 227° T. 237° M. 239° C.
T.A.S. 135 knots.

16. Course out 082° T. 092° M. 085° C.
G.S. out 113 knots.
Course back 238° T. 248° M. 251° C.
G.S. back 122 knots.
R. of A. 174 n.m.
T.T.T. back 11.32 hrs.
D.R. position at turn 54° 01′ N. 4° 05′ E.
Course to craft in distress 135° T., 144° M., 141° C.
G.S. 96 knots. Distance 67 n.m.
E.T.A. at craft 12.52 hrs.
Course to aerodrome 264° T. 273° M. 278° C.
G.S. 132 knots. Distance 142 n.m. E.T.A. 13.56 hrs.

17. Course to intercept	014° T.
W/V	302°/48 knots.
Course from Flamboro'	335° T.
G.S. from Flamboro'	143 knots.
Speed of approach	154 knots.
Distance to approach	25 n.m.
Time to approach	10 mins.
E.T.I.	11.25 hrs.
Course on bearing	086° T.

Track for bearing	093° T.
G.S. out	221 knots.
R. of A.	129 n.m.
T.T.T.	12.00 hrs.
Course back	276° T.
Track back	267°.
G.S. back	140 knots.
Reached stationary object	12.18½ hrs.
Readjusted course back	272° T.
Speed of approach	147 knots.
Distance to approach	81 n.m.
Time to approach	33 mins.
E.T.A. at ship	12.56½ hrs.
Position at E.T.A.	54° 15′ N. 0° 05′ E.

18.

Track out	065°
Distance out	115 n.m.
Course out	076° T. 086° M. 080° C.
G.S. out	150 knots.
E.T.A.	12.01 hrs.

Search

	1	2	3	4
Track	160°	250°	340°	070°
Course T.	160°	240°	340°	080°
Course M.	170°	250°	350°	090°
Course C.	169°	254°	350°	083°
G.S. (knots)	120	148	180	148
Time (mins.) to cover 10 n.m.	5	4	3·3	4

Dinghy reached 13.06 hrs.
Position of dinghy 081° Flamboro' Head 73 n.m.

MISCELLANEOUS NAUTICAL CHART QUESTIONS

1. (*a*) Bearing 333° T
(*b*) Bearing 344° T.
(*c*) Bearing 273° T.
2. (*a*) Course 117° T. Distance 81 n.m.
(*b*) Course 083° T. Distance 89 n.m.
(*c*) Course 137° T. Distance 68 n.m.
3. (*a*) Course 152° T. Distance 63 n.m.
(*b*) Course 088° T. Distance 186 n.m.
(*c*) Course 136° T. Distance 52 n.m.
4. (*a*) Course 219° T. 227° C. Distance 107 n.m.
(*b*) Course 258° T. 269° C. Distance 76 n.m.
5. (*a*) First course 122° T. 127° C. Distance 27 n.m.
Second course 135° T. 142° C. Distance 56 n.m.
Third course 195° T. 206° C. Distance 38 n.m.
Fourth course 211° T. 222° C. Distance 16 n.m.
(*b*) 00.00 hrs. S/C 128° T. 133° C. Steaming distance 43 n.m.
04.46 hrs. A/C 138° T. 145° C. Steaming distance 20 n.m.
07.00 hrs. A/C 152° T. 160° C. Steaming distance 38 n.m.
11.15 hrs. A/C 114° T. 118° C. Steaming distance 7 n.m.
12.00 hrs. A/C 118° T. 122° C. Steaming distance 79 n.m.
23.20 hrs. E.T.A.
6. Ship's position 52° 43′ N. 2° 04′ E.
Course steered 130° C. 135° M. 125° T.
Course made good 136° T.
Dutch coast reached in 51° 40′ N. 3° 41′ E.

7. 18.00 hrs. S/C 067° T. 071° C.
21.00 hrs. A/C 088° T. 091° C.
E.T.A. 04.05 hrs. on the following day.

8. First course 132° T. 139° C. Steaming distance 32½ n.m.
Second course 097° T. 100° C. Steaming distance 27½ n.m.
Third course 070° T. 074° C. Steaming distance 74 n.m.
Total time 13 hrs. 24 mins.

9. Position at start 54° 24′ N. 00° 08′ W.
00.00 hrs. S/C 124° T. 130° C.
Distance 210 n.m.; time 21 hrs. 00 mins.
06.00 hrs. Position 53° 41′ N. 01° 07′ E.
A/C 104° T. 110° C. Distance to steam 132 n.m.
19.12 hrs. E.T.A.

10. First course 000° T. 009° C. Distance 39 n.m.
Second course 310° T. 322° C. Distance 28 n.m.
Third position 321° T. 332° C. Distance 30 n.m.
Position by simultaneous bearings 52° 32′ N. 2° 04′ E.
Set 345° T. Drift 2 n.m.
Position by running fix 52° 52′ N. 01° 38′ E.

11. 10.30 hrs. Ship's position 54° 53′ N. 00° 26′ E.
11.20 hrs. Ship's position 54° 41′ N. 00° 33′ E.
S/C to intercept 349° T. 359° C.
E.T.I. 13.39 hrs.
12.00 hrs. Ship's position 54° 31′ N. 00° ′39 E.
Own position 53° 16′ N. 01° 07′ E.
A/C 007° T. 016° C.
13.56 hrs. E.T.I. in position 080°. Flamboro' Head 53 n.m.
14.11 hrs. S/C 189° T. 200° C. Distance 81 n.m.
E.T.A. 16.55 hrs.
14.45 hrs. D.R. position 53° 59′ N. 1° 19′ E.
15.45 hrs. Position by radio 53° 18′ N. 1° 08′ E.
Bearing of Cromer 168° T. Distance 23 n.m.
Course to Cromer 171° T. 181° C.
E.T.A. at Cromer 16.27 hrs.

12. 08.00 hrs. Ship's position 54° 17′ N. 2° 06′ E.
S/C 069° T. 072° C.
E.T.I. 13.00 hrs. in 54° 50′ N. 3° 00′ E.
10.10 hrs. Ship's position 54° 31′ N. 02° 28′ E.
Own position 54° 34′ N. 01° 05′ E.
A/C 087° T. 089° C.
E.T.A. 12.06 hrs.
12.06 hrs. A/C 150° T. 157° C. Track 150°.
12.27 hrs. A/C 247° T. 259° C. Track 240°.
12.51 hrs. A/C 330° T. 339° C. Track 330°.
13.46 hrs. A/C 053° T. 057° C. Track 060°.
13.50 hrs. Ship's position 077° Scarboro' 95 n.m.

MISCELLANEOUS ASTRONOMICAL AND RADIO PLOTTING QUESTIONS

1. Position 53° 05½′ N. 4° 35′ E.
2. Position 53° 24′ N. 1° 21′ E.
3. Position 54° 15′ N. 0° 41′ E.
4. Position 53° 07′ N. 0° 49′ E.
5. Position 53° 57′ N. 0° 28′ E.
6. Position 53° 14′ N. 0° 43′ E.
7. Position 53° 55′ N. 2° 25′ E.
Course 234° T. E.T.A. 04.42 hrs.

8. Position at 23.17 hrs. 54° 52½′ N. 2° 15′ E.
Position at 23.37 hrs. 54° 06′ N. 1° 01′ E.
T.M.G. 223°. W/V 327°/47 knots.
Course 270° T. E.T.A. July 9 00.02 hrs.

9. Position at 00.51 hrs. 50° 16′ N. 12° 48′ W.
T.M.G. 084°. W/V 285°/32 knots.
Course 066° T. E.T.A. 02.32 hrs.

10. Position at 08.42 hrs. 53° 54′ N. 15° 24′ W.
Position at 11.06 hrs. 50° 15′ N. 8° 46′ W.
T.M.G. 132°. G.S. 137½ knots.
W/V 235°/18 knots.

11. First track 262°. G.S. 158 knots. W/V 300°/35 knots.
Second track 336°. G.S. 154 knots.
Position at 11.30 hrs. 52° 33′ N. 13° 10′ W.
Position at 12.02 hrs. 54° 18′ N. 12° 25′ W.
T.M.G. 014°. New W/V 260°/49 knots.

12. Course to intercept 271° T.
Speed of approach 144 knots.
Distance to approach 351 n.m.
Time to approach 2 hrs. 26 mins.
E.T.I. 09.31 hrs.
Position by observation at 09.30 hrs. 51° 17′ N. 15° 35′ W.
T.M.G. 278°.
W/V experienced since 07.05 hrs. 217°/27 knots.

13. W/V 260°/43 knots.
Readjusted course 211° T.
E.T.A. 15.23 hrs.
D.R. position at 15.50 hrs. 49° 09′ N. 4° 24′ W.
Position by observation at 15.50 hrs. 48° 54′ N. 4° 34′ W.
W/V experienced since 14.28 hrs. 290°/38 knots.
New course 190° T.
E.T.A. 16.01 hrs.
Course home 355° T.
G.S. home 134 knots.
Course to vessel 276° T.
G.S. to vessel 108 knots.
E.T.A. at vessel 17.24 hrs.
Position of lifeboat 49° 21′ N. 6° 28′ W.
Course to aerodrome 035° (T.)
E.T.A. at aerodrome 18.35 hrs.

14. Position of ship by radio 56° 51′ N. 13° 31′ W.
Course to intercept 292° T.
G.S. to intercept 172 knots.
Speed of approach 186 knots.
Distance to approach 195 n.m.
E.T.I. 05.43 hrs.
Position by observation at 05.15 hrs. 56° 03′ N. 10° 41′ W.
T.M.G. 304°.
G.S. 168 knots.
Position by D.R. at 05.50 hrs. 56° 59′ N. 13° 11′ W.
Relative wind 190°/29 knots.
Ship's position at 06.40 hrs. 306° Inishtrahull 203 n.m.

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